Advances in Applied Mathematical Biosciences.
ISSN 2248-9983 Volume 3, Number 1 (2012), pp. 41-48

http://www.irphouse.com

Effects of Shape Parameter and Length of Stenosis on Blood Flow through Improved Generalized Artery with Multiple Stenoses

A.K. Singh

Assistant Professor, Department of Applied Sciences, FET, RBS College, Agra, India
E-mail: aksinghrbs@yahoo.com

Abstract

The effects of the stenosis shape and height on the resistance to flow for different values of parameter β for enhanced comprehensive geometry of multiple stenoses located at the points of equal distance are investigated. It is observed that flow resistance decreases as shape parameter and β increases and it increases as stenosis height and length increase. The study reveals that wall shear stress increases up to mid axial distance and then it decreases for increasing values of β.

Keywords: Stenosis, shape parameter, Flow resistance, Wall shear stress, Axial distance

Glossary of Terms

R_0	Radius of normal artery
l	Length of the artery
l_0	Length of stenosis
d	Distance between the points at the equal distance
δ	Maximum height of stenosis
s	Shape parameter of stenosis ($s \geq 2$)
β	A positive number ≥ 1
m	Number of stenosis in the artery
(r, z)	Radial and axial co-ordinates
(v_F, v_P)	Axial components of fluid and particle velocities

(ρ_F, ρ_P)	Actual densities of material constituting the fluid and particle phases
$\left[(1-k)\rho_F, k\rho_P\right]$	Phase densities of fluid and particle
k	Volume fraction density of the particles which is assumed to be constant
p	Pressure
$\mu_1 \approx \mu_1(k)$	Mixture viscosity (effective of apparent viscosity)
D	Drag coefficient of interaction for the force exerted by one phase on the other
(F, P)	Quantities associated with the fluid and particle phases
(F, P)	Fluid (plasma) viscosity
b_0	Radius of a particle
T	Absolute Temperature

Introduction

Formation of stenosis in blood vessels occurs due to arteriosclerosis plaques or other types of abnormal tissue development. The stenosis can cause turbulence and reduce flow by means of chocking. Very high shear stress near the throat of the stenosis can activate platelets which can totally block blood flow through to the heart or brain. The results of such stenosis on the cardiovascular system can be understood by studying the blood flow in this connection. It is necessary to study the blood flow through such type of stenosis to improve the arterial system. Many mathematicians and medical parishioners have contributed their research works in this direction. Singh et al. [1] assumed the stenosis is mild and radially non-symmetric. They performed the graphical analysis for a single loop of stenosis having maximum depression at different point. They also observed that increasing value of parameter β shows lower variation. Srivastava [2] observed that the resistance to flow decreases with increasing shape parameter but increases with hematocrit (red cell concentration).

Young and Tsai [3] found and verified experimentally an approximate equation for predicting pressure drop across a stenosis. They observed that three dimensionless parameters can be used to characterize the unsteady flow in a stenosis. The influence of geometric characteristic such as shape, length and percent lumen area reduction on the pressure drop across an arterial stenosis is studied by Seeley and Young [4]. They investigated the pressure drop across multiple stenoses by considering two blunt plugs in series. Mandal et al. [5] solved the problem of non-Newtonian and pulsatile flow through an irregularly arterial segment numerically where the non-Newtonian rheology of flowing blood is considered by the universal power law model where both the shear thinning and shear thickening models of the streaming blood are taken in to account. Sapna Ratan Shah [6] shown that the resistance to flow, apparent viscosity and wall shear stress increases with the size of the stenosis. Chakravarty and Chowdhury [7] presented an analytical study of on the behaviour of blood flow in an artery having a stenosis and studied the effect of shape of stenosis on the resistance to blood flow. Varghese and Frankel [8] obtained numerical predictions for computational pulsatile flow through different axisymmetric stenosis within the frame work of two equation turbulence models.

Mathematical Formulation

The mathematical expression for the geometry of artery with stenosis is given by

$$\frac{R}{R_0}=1-B\left[l_0^{s-1}\{\beta z-md-(m-1)l_0\}-\{\beta z-md-(m-1)l_0\}^s\right] \quad ; m(d+l_0)-l_0\le \beta z\le m(d+l_0)$$

$$=1 \quad ; \text{Otherwise} \tag{1}$$

Where $B=\frac{\delta}{R_0 l_0^s}\frac{s^{s/(s-1)}}{(s-1)}$ and δ be the maximum height of stenosis at $z=\frac{md+(m-1)l_0+l_0/s^{1/s-1}}{\beta}$

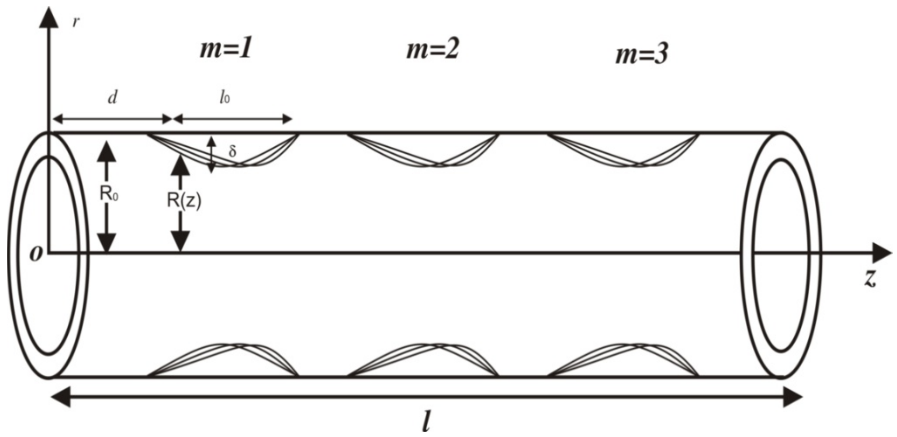

Figure 1: Geometry of Axially symmetric but radially non-Symmetric Artery

Let us consider the axisymmetric laminar, steady one dimensional flow of blood in an artery, the constitutive equations for two phase macroscopic model of blood flow in a mild stenosis case are simplified as

$$(1-k)\frac{dp}{dz}=(1-k)\frac{\mu'}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)v_F+kD(v_P-v_F) \tag{2}$$

$$k\frac{dp}{dz}=kD(v_F-v_P) \tag{3}$$

The expression for the drag coefficients of interaction (D) and empirical relation for the viscosity of suspension μ_1 are

$$D = \frac{4.5\left(\frac{\mu_2}{b_0{}^2}\right)\left\{4 + 3(8k - 3k^2)^{1/2} + 3k\right\}}{(2-3k)^2} \tag{4}$$

$$\mu_1 = \frac{\mu_2}{(1-uk)}; \qquad u = \left[.07\exp\left\{2.49k + 1107^0 \frac{K}{T} e^{-1.69k}\right\}\right] \tag{5}$$

Boundary conditions are

$$v_F = 0 \text{ at } r = R(z) \tag{6}$$

$$\frac{\partial v_F}{\partial r} = \frac{\partial v_P}{\partial r} = 0 \tag{7}$$

Solving Equations (2) and (3) subject to the boundary conditions given in the eq. (6) and (7), we obtained

$$v_F = -\frac{R_0{}^2}{4(1-k)\mu_1}\frac{dp}{dz}\left[\left(\frac{R}{R_0}\right)^2 - \left(\frac{r}{R_0}\right)^2\right] \tag{8}$$

$$v_P = -\frac{R_0{}^2}{4(1-k)\mu_1}\frac{dp}{dz}\left[\left(\frac{R}{R_0}\right)^2 - \left(\frac{r}{R_0}\right)^2 + \frac{4(1-k)\mu_1}{DR_0{}^2}\right] \tag{9}$$

The flow flux can be calculated as

$$q = 2\pi\left[(1-k)\int_0^R r v_F\, dr + k\int_0^R r v_P\, dr\right]$$

$$= -\frac{\pi R_0{}^4}{8(1-k)\mu_1}\frac{dp}{dz}\left[\left(\frac{R}{R_0}\right)^4 + \theta^2\left(\frac{R}{R_0}\right)^2\right] \tag{10}$$

Where $\theta^2 = \dfrac{8k(1-k)\mu_1}{DR_0{}^2}$

$$-\frac{dp}{dz} = \frac{8(1-k)\mu_1}{\pi R_0{}^4}\frac{1}{\left(\frac{R}{R_0}\right)^4 + \theta^2\left(\frac{R}{R_0}\right)^2} \tag{11}$$

$p = p_1$ at $z = 0$ and $p = p_2$ at $z = l$

Integrating (11) to obtain

$$\Delta p = p_1 - p_2 = \frac{8(1-k)\mu_1}{\pi R_0{}^4} \int_0^l \frac{1}{\left(\frac{R}{R_0}\right)^4 + \theta^2 \left(\frac{R}{R_0}\right)^2} dz \tag{12}$$

The resistance to flow is defined as

$$\lambda = (1-k)\mu' \left[\frac{1}{1+\theta^2}\left(1 - \frac{m_{\max}}{\beta}\frac{l_0}{l}\right) + \frac{1}{l} \sum_{m=1}^{m=m_{\max}} \int_{\frac{m(d+l_0)-l_0}{\beta}}^{\frac{m(d+l_0)}{\beta}} \frac{1}{\left(\frac{R}{R_0}\right)^4 + \theta^2\left(\frac{R}{R_0}\right)^2} dz \right] \tag{13}$$

For $m = 1$, we get

$$\lambda = (1-k)\mu' \left[\frac{1}{1+\theta^2}\left(1 - \frac{l_0}{\beta l}\right) + \frac{1}{l} \int_{\frac{d}{\beta}}^{\frac{d+l_0}{\beta}} \frac{1}{\left(\frac{R}{R_0}\right)^4 + \theta^2\left(\frac{R}{R_0}\right)^2} dz \right] \tag{14}$$

$$\tau_w = \frac{(1-c)\mu'}{\left[\left(\frac{R}{R_0}\right)^3 + \theta^2\left(\frac{R}{R_0}\right)\right]}$$

$$\bar{\tau}_w = \frac{(1-c)\mu'}{\left[\left(1-\frac{\delta}{R_0}\right)^3 + \theta^2\left(1-\frac{\delta}{R_0}\right)\right]} \tag{15}$$

Where $\frac{R}{R_0} = 1 - \frac{\delta}{R_0 l_0{}^s} \frac{s^{s/s-1}}{(s-1)} \left[l_0{}^{s-1}(\beta z - d) - (\beta z - d)^s \right]$

Numerical Results

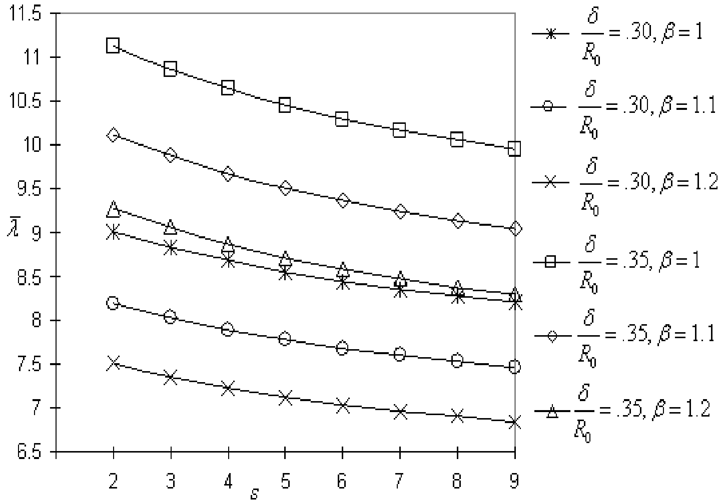

Figure 2: Variation of Resistance to flow with stenosis shape for different stenosis heights

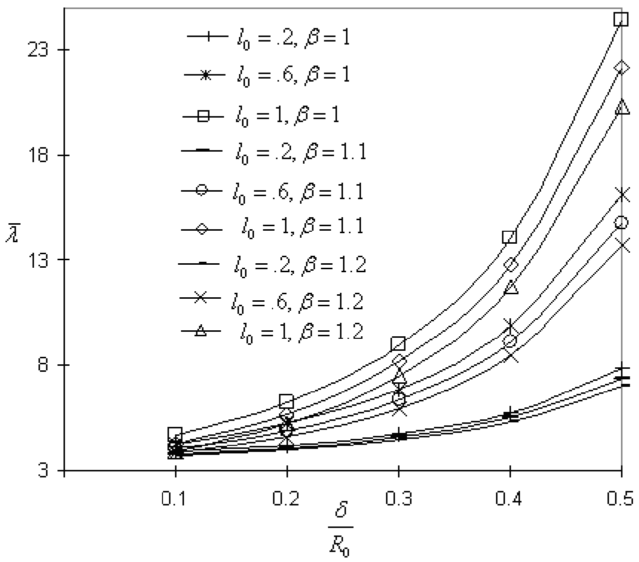

Figure 3: Variation of Resistance to flow with stenosis length for different values of stenosis lengths and β

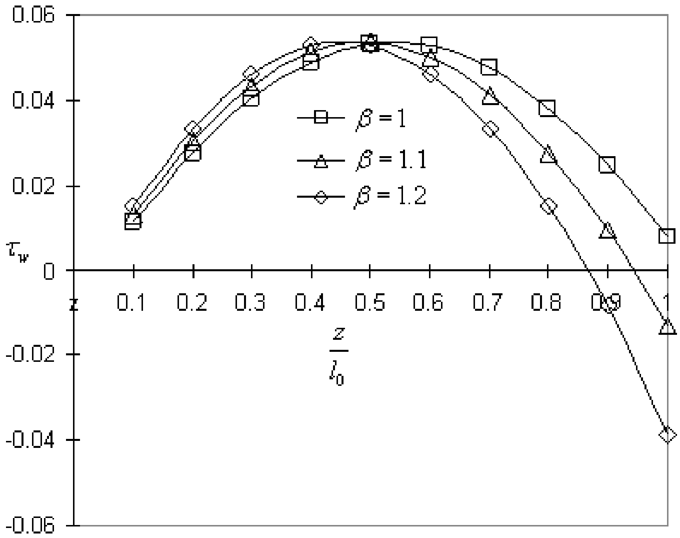

Figure 4: Variation of Wall shear stress with axial distance for different value of β

Conclusion

In this study, we have formulated a mathematical model to study the effects of shape parameter and stenosis length on the resistance to flow and wall shear stress for different values of parameter β under stenotic conditions by considering ,laminar, steady, one dimensional, non-Newtonian and fully developed flow of blood through axially symmetric but radially non-symmetric artery. The effects of variations in axial distance on the wall shear stress for different β has also been investigated. The numerical results for these expressions also have been carried out.

References

[1] Singh, B.; Joshi, P.; Joshi, B.K. (2010): “Blood flow through an artery having radially non-symmetric mild stenosis”, Applied Mathematical Sciences, 4(22): 1065-1072

[2] Srivastava, V.P. (2002): “Particulate suspension blood flow through stenotic arteries: Effects of hematocrit and stenosis shape”, Indian J. Pure appl. Math., 33(9):1353-1360]

[3] Young, D.F.; Tsai, F.Y. (1973): “Flow characteristic in models of arterial stenoses-II. Unsteady flow”, Journal of Biomechanics, 6(5):547-559

[4] Seeley, B.D.; Young, D.F. (1976): “Effect of geometry on pressure losses across models of arterial stenoses”, Journal of Biomechanics, 9(7): 447-448

[5] Mandal, P.K.; Chakravarty, S; Mandal, A. (2007): “Numerical study of unsteady flow of non-Newtonian fluid through differently shaped arterial stenoses”, International Journal of Computer Mathematics, 84(7): 1059-1077

[6] Shah, S.R. (2011): "Non-Newtonian Flow of Blood through an Atherosclerotic Artery", Research Journal of Applied Sciences, 6(1):76-80.

[7] Chakravarty, S.; Chowdhury, A.G. (1988): "Response of Blood flow through an artery under stenotic conditions", Rheologica Acta, 27(4): 418-427

[8] Varghese, S.S.; Frankel, S.H (2003): "Numerical Modeling of pulsatile turbulent flow in stenotic vessels", Journal of Biomechanical Engineering, 125:445-460