Advances in Applied Mathematical Biosciences.
ISSN 2248-9983 Volume 4, Number 1 (2013), pp. 13-23

http://www.irphouse.com

Effect of Blood Perfusion and Metabolism in Temperature Distribution in Human Eye

Gokul K.C.[1], D.B. Gurung[2] and P. R. Adhikary[3]

[1,2,3] Department of Natural Sciences(Mathematics), School of Science
Kathmandu University, Dhulikhel, Kavre, Nepal
[1]Corresponding Author Email: gokulkc2@gmail.com

Abstract

Eye is one of the most sensitive part of the human body when exposed to thermal heat flux. Since there is no barrier (such as skin) to protect the eye against the absorption of an external thermal wave, the flux can readily interact with the cornea. Invasive methods in measuring eye temperature are normally dangerous. Therefore, computational models can be used as a useful tool to study the heat transfer in human eye. Thus, in this work, one dimensional steady state temperature distribution model of human eye is constructed using variational finite element method. We compare the results of temperature distribution along pupillary axis, with and without considering the effect of blood perfusion and metabolism in retinal region using different values of evaporation rates, blood temperatures, ambient temperatures, and lens thermal conductivities. The temperature values so obtained are compared against those reported in the literatures.

Keywords: Finite Element Method, Temperature distribution, Human Eye

Introduction

It is well known that human eye is relatively a small organ and very complex optical system in the human body. It consists of several subdomains with different material properties, usually have complex geometry. Temperature has a profound effect on the eye. The ocular surface has to manage physiologically with the imposed thermal stress of an environment. Temperature changes can affect tissues in several ways; it can kill cells, denature proteins, slow down or speed up metabolism, involved in pathological changes etc of the eye. Heat regulation in human body is characterized by conduction, convection, radiation as well as blood perfusion, metabolism and evaporation. Metabolic heat

generation and blood perfusion play an important role in heat regulation in several parts of human body [3]. Blood perfusion is a physiological term that refers to the process of nutritive delivery of arterial blood to a capillary bed in the biological tissue. Metabolism is the sum of total of all chemical reactions involved in maintaining the living state of the cells, and thus the organism. Evaporation is the conversion of a liquid to a vapor and is always accompanied by cooling.

Current surgical methods in the eye use lasers, which are known to cause local, heating and potentially could lead to loss of corneal clarity as well as opacity of the lens. Because there is no skin layer to absorb radiation from wireless networks or electromagnetic fields, the eye is particularly sensitive to heating from these sources [15]. Thermal disturbances are more pronounced in the eye due to an insufficient blood flow circulation and lack of skin as a protecting layer [4]. Therefore, a mathematical model could be rather useful to medical doctors in minimizing any damage of intraocular tissue due to the heating. This is particularly important in the case of the human eye where the blood flow cannot regulate the heating inside the ocular tissue [14].

Al-Badwaihy and Youssef [13] in 1976 developed a model to analyze the thermal effects of microwave radiation on the eye. A more sophisticated model was developed by Lagendijk *et al.* [10] in 1982, using a simple explicit forward-difference heat balance technique. Guy *et al.* [2] used the finite element method to model the effects of radiation on rabbit eyes. In 1988, Scott [11] proposed a new 2D model of heat transfer through the eye, using the Galerkin finite element method. Amara [6] used a model of the eye based on Scott's work to compute the temperature distributions in the eye when exposed to laser irradiation. Bernardi *et al.*[15] focused on modeling the temperature profiles of the eye exposed to electromagnetic fields and wireless networks. Ooi *et al.*[4] investigated the effects of natural convection in the aqueous humor on the temperature distributions in the eye.

All of the previously developed models have neglected the effects of blood perfusion and metabolism. The significance of blood perfusion and metabolism on the temperature distribution in the eye is debatable, since they takes place only on retina, choroid, iris and cilliary body but it is still necessary to investigate the effects in order to obtain a more accurate model. The objective of this paper is related to the analysis of temperature distribution in the model of the human eye using Pennes bio-heat equation. This model provides the results for steady state temperature distribution including the effect of blood perfusion and metabolism in one dimension.

Model Formulation

The eye is assumed a perfectly bonded solid structure with each component homogeneous. The eye is considered having six major components: cornea, aqueous humor, lens, vitreous humor, retina (with choroid), and sclera (Figure 1). As sketched on figure 1, the thickness of cornea, aqueous humor, lens, vitreous humor, retina and sclera have been considered as l_1, $l_2 - l_1$, $l_3 - l_2$, $l_4 -$

l_3, $l_5 - l_4$ and $l_6 - l_5$ respectively and T_0, T_1, T_2, T_3, T_4, T_5 and $T_6 = T_b$ (body core temperature) are the nodal temperatures at a distances $x = 0$, $x = l_1$, $x = l_2$, $x = l_3$, $x = l_4$, $x = l_5$ and $x = l_6$.

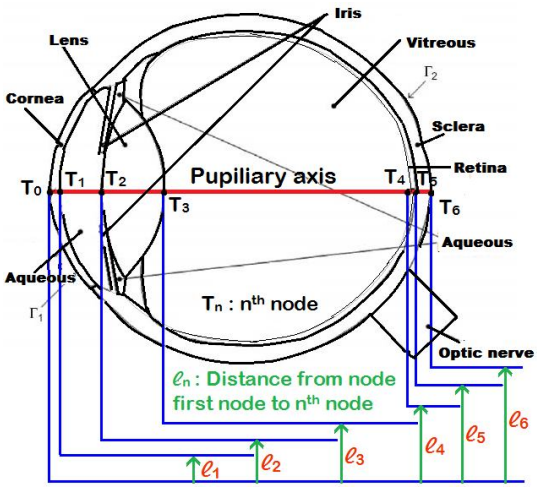

Figure 1: Finite element sketch of human eye

A simple 1D finite element human eye model has been developed to simulate its thermal steady state conditions. The governing differential equation used for heat flow in the human eye due to Pennes bio-heat equation in steady state condition is [8]:

$$\nabla.(k\nabla T) + \omega\rho_b c_b(T_{bl} - T) + Q_m = 0 \qquad (1)$$

where, ρ_b = blood density (kg/m^3), c_b = blood specific heat (J/kg ^{0}C), k = tissue thermal conductivity (W/m ^{0}C), ω = volumetric blood perfusion rate per unit volume (s^{-1}), T_{bl} = blood temperature (^{0}C), T = tissue temperature (^{0}C), t = time (s), Q_m = heat generation due to metabolism (W/m^3).

The three terms on the left-hand side of bio-heat equation are blood conduction, blood perfusion, and metabolism. In our model, blood flow occurs only on retina, so the effect of blood perfusion and metabolism is analyzed on retina only.

Boundary conditions for the system can be defined as follows [5]:

1. In the back of the eye, heat is transferred from blood in the ophthalmic artery to the sclera:

$$\Gamma_2 : -k_s\frac{\partial T}{\partial n} = h_{bl}(T - T_{bl}) \qquad (2)$$

where n is the normal direction to the surface boundary, k_s is the thermal conductivity of sclera, h_{bl} is the heat transfer coefficient between blood and eye (W/m^{20}C), and T_{bl} is blood temperature (^{0}C).

2. At the cornea, heat loss from the eye occurs through convection, radiation, and tear evaporation:

$$\Gamma_1 : \; -k_c \frac{\partial T}{\partial n} = h_\infty (T - T_\infty) + \sigma\varepsilon(T^4 - T_\infty^4) + E \qquad (3)$$

where h_∞ represents the convection heat transfer coefficient between the cornea and ambient environment (W/m^2 ^{0}C), T_∞ is the ambient room temperature (^{0}C), σ is the Stefan Boltzman constant (5.67 x 10^{-8} W/m^2^{0}C ^{4}), ε is the emissivity of the cornea, and E is evaporative heat loss (W/m^2).

The presence of nonlinear radiation term in the boundary condition (3) makes the problem difficult to formulate. This difficulty can be resolved by introducing a suitable iterative procedure. For this purpose, we can write the nonlinear boundary condition as:

$$-k_c \frac{\partial T_1}{\partial n} = [h_\infty + \sigma\varepsilon(T_1 + T_\infty)(T_1^2 + T_\infty^2)](T_1 - T_\infty) + E \qquad (4)$$

If the value of the term $(T_1 + T_\infty)(T_1^2 + T_\infty^2)$ is known, the condition (2.4) can be viewed as a generalized convection condition between convection and radiation. Based on this concept, we design the following iterative algorithm:

$$-k_c \frac{\partial T_1^m}{\partial n} = h_{cr}(T_1^m - T_\infty) + E \qquad (5)$$

with

$$h_{cr} = h_\infty + \sigma\varepsilon(T_1^{m-1} + T_\infty)((T_1^{m-1})^2 + T_\infty^2) \qquad (6)$$

$$h_{cr} = h_{convection} + h_{radiation}$$

where T_1^m are temperature sequences for m = 1, 2, 3, … and T_1^0 represents an initial guess of temperature.
The iteration is completed when the convergent condition is satisfied:

$$\|T_1^m - T_1^{m-1}\| < \delta \qquad (7)$$

where δ is iteration tolerance.

The inner body core temperature T_b is assumed to be 37^{0}C. Therefore, the initial boundary condition is

$$T_b = 37^0 C \qquad (8)$$

The partial differential equation (1) together with boundary conditions (2) and (5) in one dimensional variational form is:

$$I = \frac{1}{2}\int_L \left[K\left(\frac{dT}{dx}\right)^2 + \omega\rho_b c_b(T_b - T)^2 + Q_m T\right]dx + \frac{1}{2}h_{bl}(T - T_{bl})^2 + \frac{1}{2}h_{cr}(T - T_\infty)^2 + ET \quad (9)$$

The first term without integral sign is employed for sclera and the last two terms are employed for cornea. We write I separately for the six layers as I_1, I_2, I_3, I_4, I_5, I_6 as

$$I = \sum_{i=1}^{6} I_i \quad (10)$$

Now as a next step of variational finite element method, we differentiate I partially with respect to the nodal temperatures T_o, T_1, T_2, T_3, T_4, and T_5 and equating to zero, we get

$$\frac{\partial I}{\partial \vec{T}} = \begin{bmatrix} \frac{\partial I}{\partial T_1} \\ \frac{\partial I}{\partial T_2} \\ \vdots \\ \frac{\partial I}{\partial T_5} \end{bmatrix} = 0 \quad (11)$$

The equation (11) constitutes a system of linear equations of 5×5 orders, and solving this system of equations, we get the required nodal temperatures.

Results and Discussion

To solve equation (2.11), we consider the following values of parameters [4,5,14]: body core temperature T_6 = 37°C, evaporation rate E = 40 W/m^2, ambient convection coefficient h_{amb} = 10 W/m^2 °C, blood convection coefficient h_{bl} = 65 W/m^2 °C, blood density ρ_b = 1060 Kg/m^3, blood specific heat c_b = 3594 J/Kg °C, thermal conductivities of cornea, aqueous humor, lens, vitreous humor, retina and sclera are 0.580, 0.578, 0.4, 0.594, 0.565, and 0.580 W/m °C respectively, blood perfusion and metabolism at retina ω=35000 s^{-1} and Q_m= 1000 W/m^3, initial guess $T_1^0 = 0$, and tolerance $\delta = 0.0005$.

The steady state temperature distribution is calculated for different parts of human eye with and without taking the effect of blood perfusion and metabolism on the retina and choroid. For this purpose, ambient temperature is taken to be 25°C. The results are shown in figure 2.

It can be seen from figure 2 that the temperature difference on the cornea with and without taking blood perfusion and metabolism is 0.12°C. As we move from cornea to retina the temperature differences increase up to 0.17°C. This shows that blood perfusion and metabolism on retina increase eye temperature, which play a significant role in maintaining eye temperature. For simplicity, we use Case I to study the effect of blood perfusion and metabolism in retinal region and Case II to study without these effects.

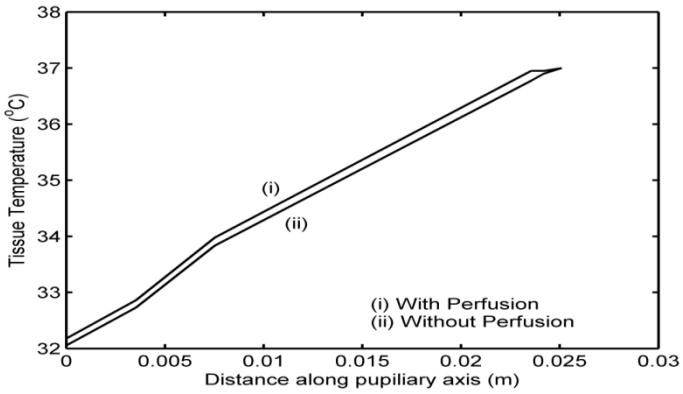

Figure 2: Temperature variation with and without taking the effect of blood perfusion and metabolism

Effect of tear evaporation

The cornea surface contains a three-layered structure: a mucoid layer, a thick aqueous layer and thin oily layer. The function of oily layer is to prevent evaporation of tear from the corneal surface. When the layer destroyed, the evaporation rate increases dramatically. Four sets of data values for E equals 20, 40, 100, 180 (in the range between maximum and minimum values recorded in the experiments [5]) are used in this investigation. The temperature variations are shown in figures 3 and 4.

With Blood Perfusion and Metabolism Case I

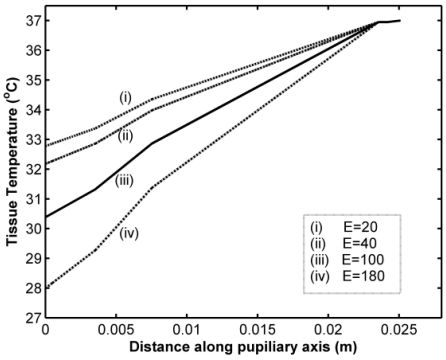

Figure 3: Temperature variation for different evaporation rates

Without Blood Perfusion and Metabolism Case II

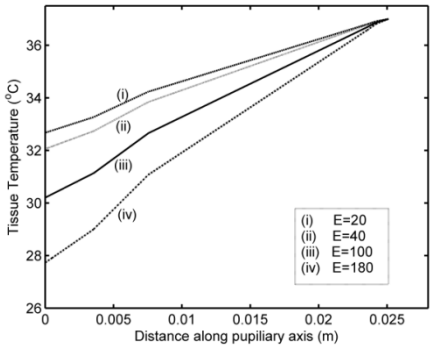

Figure 4: Temperature variation for different evaporation rates

In Case I, the retinal temperature is almost constant, i.e. there is no effect of evaporation rate in retinal region. However, retinal temperature decreases as evaporation rate increases in case II. For different evaporation rates 20, 40, 100, 180, the corneal temperature (sample node T_0) difference between Case I and Case II are 0.10, 0.12, 0.18, and 0.27. The differences shows that as evaporation rate increases the nodal temperature decreases in both cases but decreasing rate

is slow in Case I compare to Case II. From evaporation rate 20W/m^2 to 180W/m^2, the corneal temperature is dropped by 4.78^{o}C in Case I and 4.95^{o}C in Case II.

Effect of blood temperature

In eye, blood vessels present only in retina, choroid, cilliary body, and iris. In normal condition, our blood temperature is 37^{o}C, but when we suffer from fever, our blood temperature increases. In this model, four values of blood temperature 36^{o}C, 37^{o}C, 38^{o}C and 39^{o}C are used for analysis.

With Blood Perfusion and Metabolism Case I

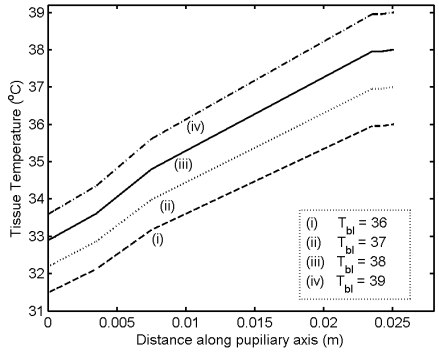

Figure 5: Temperature variation for different blood temperatures

Without Blood Perfusion and Metabolism Case II

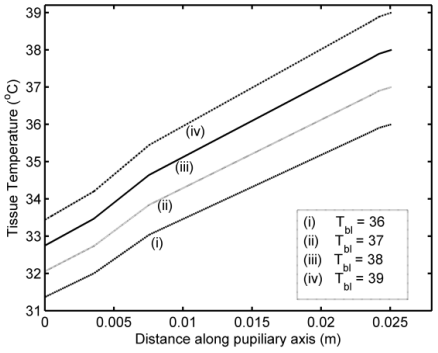

Figure 6: Temperature variation for different blood temperatures

The objective is to provide the possibility of detecting sickness based on the ocular temperature. The numerical results are presented in figures 5 and 6. The corneal temperature increases by 2.10^{o}C in Case I and 2.07^{o}C in Case II when blood temperature varies from 36^{o}C to 39^{o}C. Thus, blood temperature plays an important role in regulating the human body's temperature, and hence eye temperature increases due to increase of blood temperature.

Effect of ambient temperature

Heat loss occurs due to convection and radiation at cornea. This loss is strongly related to ambient temperature. In addition, ambient temperature is one of the factors affecting the amount of tear in the eyes [5]. For this study, four set of ambient temperatures 10^{o}C, 25^{o}C, 40^{o}C, and 50^{o}C are chosen to investigate the effects. The numerical results are presented in figures 7 and 8.

As ambient temperatures increase from 10^{o}C to 50^{o}C the nodal temperatures also increase. The corneal temperature varies from 27.69^{o}C to 39.69^{o}C in Case I and 27.41^{o}C to 39.78^{o}C in Case II. This shows that as ambient temperature increases heat production on retinal region due to perfusion and metabolism decrease. When ambient temperature is beyond blood temperature, the blood flow on retinal region functions to keep eye temperature in thermo state.

With Blood Perfusion and Metabolism Case I

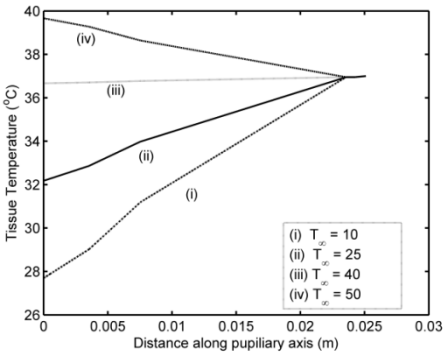

Figure 7: Temperature variation for different ambient temperatures

Without Blood Perfusion and Metabolism Case II

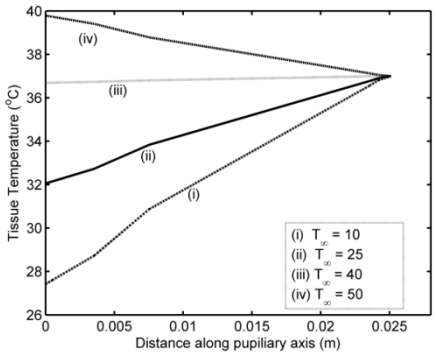

Figure 8: Temperature variation for different ambient temperatures

At ambient temperature 50^{o}C, a constant body core temperature is obtained at nodes T_4 and T_5 in Case I but temperature is constantly decreasing from cornea to body core in Case II. Increase in ambient temperature does not increases the retinal temperature beyond body core temperature in Case I but retinal temperature increases beyond body core temperature in Case II. Hence, our model including the effect of blood perfusion and metabolism in retinal part is more realistic to natural phenomena of the eye than previous models.

Effect of lens thermal conductivity

It is well known that the water content of lens decreases as age increases, decreasing the water level in lens increasing its hardness. This process changes the thermal conductivity of lens due to age [2]. For the purpose of comparison, we take four sets of values 0.21W/m^{o}C, 0.30 W/m ^{o}C, 0.40 W/m ^{o}C and 0.54 W/m ^{o}C. The numerical results are presented in figures 9 and 10.

As lens thermal conductivity increases from 0.21 W/m^{o}C to 0.54W/m^{o}C, the corneal temperature increases by 0.87^{o}C in Case I and 0.85^{o}C in Case II but vitreous humor temperature decreases by 0.23^{o}C and 0.25^{o}C in Case I and Case II respectively. However, no significant changes occur in retinal region. As thermal conductivity increases, lens will permit more heat transfer from the rear of the eye (high temperature region) to the corneal surface (low temperature region), and thus cause the corneal surface temperature to increase.

With Blood Perfusion and Metabolism Case I

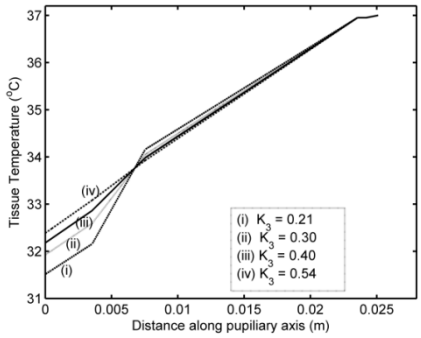

Figure 9: Temperature variation for different Lens thermal conductivity

Without Blood Perfusion and Metabolism Case II

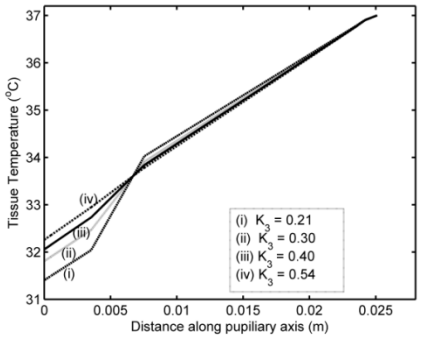

Figure 10: Temperature variation for different Lens thermal conductivity

Conclusion

In this model, we presented a comparative study for the effect of blood perfusion and metabolism in retinal part with and without considering it. The corneal temperature is found to be 32.17^{o}C and 32.05^{o}C with and without considering the effects. Due to the lack of experimental data and clinical observations, the verification of results was only performed using available literatures including previous simulations. Various authors [5, 6, 7, 11, 14] reported similar temperature distribution along the eye pupillary axis. The reported values range from 30.92 ^{0}C to 33.7 ^{0}C on corneal surface. Hence, our steady state results are in a good agreement with the results of many previous results. In earlier studies [5, 6, 7, 11, 14], the effect of blood perfusion and metabolism are assumed negligible in retinal part. Although blood perfusion and metabolism occur only at retina, choroid, iris, and cilliary body, which constitute a very small part of human eye, their effects play an important role in maintaining the eye temperature.

We found from the above results that when evaporation rate increases the nodal temperature decreases in both cases but decreasing rate is slow in Case I compare to Case II. It is due to effect of blood perfusion and metabolism in retinal part. It is also observed that when ambient temperature is beyond body core temperature, the blood flow in retinal region plays important role to maintain the eye temperature. On the other hand, we observed that increase in blood temperature increases the corneal temperature approximately at the same rate in both cases. The increase in thermal conductivity of lens increases corneal temperature, decreases vitreous temperature, but no significant changes occur in retinal region. When thermal conductivity increases, it permits more heat transfer from high temperature region to low temperature region. Hence, increase in lens thermal conductivity, transfers more heat from vitreous towards

cornea. This causes decrease in vitreous temperature and increase in corneal temperature.

Scott [11], E.Y.K.Ng [7] and Li Eric [5] observed that the main parameters affecting the eye temperature are blood temperature, ambient temperature, and evaporation rate. They found the results by neglecting the effect of blood perfusion and metabolism in retinal part. We studied the same case with considering the effect of blood perfusion and metabolism in retinal region. Therefore, the results obtained are more accurate, physiologically more complete, and realistic.

The model could be useful for the researchers to study the effects of heat flux inside the eye and medical scientists to improve the diagnosis and treatment.

Acknowledgement

This work was supported by University Grants Commisson, Nepal.

References

[1] Taflove, A., and Brodwin, M. E., 1975, "Computation of the Electromagnetic Fields and Induced Temperatures Within a Model of the Microwave-Irradiated Human Eye", IEEE Trans Microw Theory Tech MTT, , 23, pp. 888-896.

[2] Guy, A.W., Karmar, P., Emery, A.F. , and Lin, J.C., 1975, "Microwave Induced Temperature Rises in Rabbit Eyes in Cataract Research", Trans ASME- J. Heat Transfer C97, 123-131.

[3] Gurung, D.B., and Saxena, V.P., 2010, "Transient Temperature Distribution in Human Dermal Part with Protective Layer at Low Atmospheric Temperature", International Journal of Biomathematics, 3(4), pp. 439-451.

[4] Ooi, E. H., and Ng, E.Y., 2007, "Simulation of aqueous humor hydrodynamics in human eye heat transfer", Comput. Biol. Med., 38, pp. 252-262.

[5] Li, E., Liu, G. R., Tan, V., and He, Z. C., 2010, "Modeling and simulation of bioheat transfer in the human eye using the 3D alpha finite element metheod(αFEM)", International journal for Numerical Methods in Biomedical Engineering, 26,pp. 955-976.

[6] Amara, E.H., 1995, "Numerical investigations on thermal effects of laser-ocular media interaction", Int J Heat Mass Transfer, 38, pp. 2479-2488.

[7] Ng, E.Y.K., and Ooi, E. H., 2006, "FEM simulation of the eye structure with bio-heat analysis", Computational Methods Programs Biomed, 82, pp. 268-276.

[8] Pennes, H. H., 1948, "Analysis of tissue and arterial blood temperatures in the resting human forearm", J. Appl. Physiol., 85, pp. 5-34.

[9] Heys, J. J., and Barocas, V.H., 2002, "A boussinesq model of natural

convection in the human eye and the formation of Krukenberg's spindle", Ann Biomed Eng., 30, pp. 392-401.

[10] Lagendijk, J. J., 1982, "A mathematical model to calculate temperature distributions in human and rabbit eyes during hyperthermia treatment", Phys Med Biol, 27, pp. 1301-1311.

[11] Scott, J.A., 1988, " A finite element model of heat transport in the human eye", Phys Med Biol, 33, pp. 227-241.

[12] Reedy, J.N., 2010, An introduction to the Finite Element Method, Tata Mc-Graw Hill, India.

[13] Al-Badwaihy, K.A., and Youssef, A.B.A., 1976, "Biological Effects of Electromagnetic Waves", HEW.

[14] Cvekovic, M., Poljak, D., and Peratta, A., 2011, "FETD Computation of the Temperature Distribution Induced into a Human Eye by a Pulsed Laser", Progress In Electromagnetics Research, 120, pp. 403-421.

[15] Bernardi, P., Cavagnaro, M., Pisa, S., and Piuzzi, E., 1998, "SAR distribution and temperature increase in an anatomical model of the human eye exposed to the field radiated by the user antenna in a wireless LAN", IEEE Trans Microw Theory Tech, 46, pp. 2074-2082.

[16] Kumar, S., Acharya, S., Beuerman R., and Palkama, A., 2006, " Numerical solution of ocular fluid dynamics in a rabbit eye: parametric effects, Ann Biomed Eng, 34, pp. 530-544.