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Test for Median Using Maple Programme

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Abstract

Here we suggest a general parametric method for testing the median of any distribution using Maple Programme. By deriving the first four central moments of the test statistics, the distribution of sample median is fitted as a member in the Generalized Lambda Distribution (GLD) family. To conduct the test and to evaluate the power of the test, computer programmes in Maple language are provided. The method can be applied for any unimodal continuous distribution.

AMS subject classification:
Keywords: Parametric tests, Maple Programmes, Moments of order statistics, Generalized lambda distribution, Quantiles.

1. Introduction

Let $F(x)$ be the distribution function (cdf) of a continuous random variable X, then the median of X, denoted as Q_2, is defined as

$$Q_2 = inf\left(t : F(t) \geq \frac{1}{2}\right) \tag{1.1}$$

Let $x_{1:n},\ x_{2:n}, \ldots x_{n:n}$ denote the order statistics based on the sample observations. Then the sample median is denoted as q_2 and is defined as

$$\begin{aligned} q_2 = & x_{\frac{n}{2}:n}, \quad \text{if } \frac{n}{2} \text{ is integer} \\ & x_{[\frac{n}{2}]+1:n}, \quad \text{otherwise} \end{aligned} \tag{1.2}$$

where $\left[\frac{n}{2}\right]$ is the integer part of $\frac{n}{2}$.

Detailed descriptions regarding the theory and development of order statistics are available in David and Nagaraja (2003). Balakrishnan and Rao (1998) and Harter and Balakrishnan (1996) described the recent developments in the theory and properties of order statistics. Reiss (1989) described in details the relation between the quantiles and order statistics. Using the cumulative probabilities of binomial distribution, Reiss (1989) described a one sided test for quantiles, which is applicable for one-sided hypotheses only. Kendall and Stuart (1977) described the asymptotic normality of quantiles, using their standard error.

The GLD representation of almost all well known distributions are discussed in details in Karian and Dudewicz (2000). Also numerical results reveals that the GLD representation of any unimodal continuous distribution almost agrees with the true distribution both in terms of cumulative distribution and probability distribution functions. Kumaran and Beena (2005) obtained the expressions for moments and product moments of order statistics from GLD family. Since GLD family yields explicit expressions for moments and product moments of order statistics, we recommend the use of GLD for testing the median through order statistics.

In this article, we suggest a parametric test for population median using order statistics. The method is based on the GLD representation of the distribution of the test statistics. This method consists in deriving the first four moments of the test statistics and fitting its distributions under the null and alternative hypotheses. Since it is possible to fit the distribution of the test statistics under the alternative hypothesis, power of the test can also be evaluated under this method. The method is more reliable as it utilizes maximum information from the sample and it can be used for all types of distributions. To apply this method, computer programmes in Maple language are provided. The most significant aspect of the method is that, by inputting the given set of observations in the programme, one can examine whether the observations are drawn from a population with the specified median against any of the alternatives.

A brief review of the GLD family is discussed in section 2. By deriving the first four central moments of the test statistics, the proposed test is described in section 3. The application of the method on real life data is also provided in this section.

2. Generalized Lambda Distributions (GLD) Family

The generalized lambda distribution (GLD) family is a four parameter family of distributions derived by Ramberg and Schmeiser (1974), as a generalization of the Tukey's (1960) one parameter family. Unlike most other four parameter family of distributions, GLD has no explicit expression for its pdf, instead, members of the family are specified in terms of their quantile function. The quantile function of a four parameter GLD family is given by

$$Q(p) = \lambda_1 + \frac{p^{\lambda_3} - (1-p)^{\lambda_4}}{\lambda_2}, \quad \lambda_2 \neq 0, \quad 0 \leq p \leq 1 \tag{2.1}$$

Here, λ_1 and λ_2 represent the location and scale parameters where as λ_3 and λ_4 represent the shape parameters of the distribution. The support of the random variable with the above distribution is $\left[\lambda_1 - \frac{1}{\lambda_2}, \lambda_1 + \frac{1}{\lambda_2}\right]$ when $\lambda_3 > 0$ and $\lambda_4 > 0$. The support is $\left(-\infty, \lambda_1 + \frac{1}{\lambda_2}\right)$ when $\lambda_3 < 0$ and $\lambda_4 = 0$ and it is $\left(\lambda_1 - \frac{1}{\lambda_2}, \infty\right)$ when $\lambda_3 = 0$ and $\lambda_4 < 0$. The parameters λ_3 and λ_4 are independent of change of origin and scale but λ_1 and λ_2 changes according to the changes in the origin and scale. In other words, if X is a member of the GLD family with parameters λ_1, λ_2, λ_3 and λ_4, then $Y = aX + b$ will also be a member of the GLD family with parameters $a\ \lambda_1 + b$, $a\lambda_2$, λ_3 and λ_4.

2.1. Estimation of Parameters and Fitting of GLD

The popular method of fitting GLD to a data set is the method of moments due to Ramberg et al. (1979). In this method the parameters λ_3 and λ_4 are first derived by solving the equations $\alpha_3 = \hat{\alpha_3}$ and $\alpha_4 = \hat{\alpha_4}$, where α_3 and α_4 are the coefficients of skewness and kurtosis of the distribution and $\hat{\alpha_3}$ and $\hat{\alpha_4}$ are their sample estimates. This system of equations are too complex, so that to obtain the solutions, computer programme in Maple language is provided. The program utilizes the values of $\hat{\alpha}_1,\ \hat{\alpha}_2,\ \hat{\alpha}_3,\ \hat{\alpha}_4$ and an initial $(\lambda_3,\ \lambda_4)$ as input and a maximum of 10 iterations were used. By solving the equations $\alpha_1 = \hat{\alpha}_1$, $\alpha_2 = \hat{\alpha}_2$ and using the estimated values of λ_3 and λ_4, the values of λ_1 and λ_2 were determined. It may be noted that skewness and kurtosis are independent of location and scale parameters and moments of all orders exist if λ_3 and λ_4 are of same sign.

Since, corresponding to every admissible pair of skewness and kurtosis measures GLD family contains a member, a wide variety of densities with different tail shapes are available in the family. This family was used for Monte-Carlo simulation studies of robustness of statistical procedures and for sensitivity analysis. The family contains unimodal, U-shaped, J-shaped, symmetric and asymmetric distributions. One of the important advantages of this family is that all its members can be represented by a single quantile function and almost all known distributions can be represented as its member.

3. Test for Median

In this section, we propose a new method for testing the population median based on the GLD representation of the test statistics. To apply this method, assume that the parent distribution is a member of the GLD family with parameters $(\lambda_1,\ \lambda_2,\ \lambda_3,\ \lambda_4)$. Then to test $H_0 : Q_2 = Q_2^0$, the test-statistics used is q_2, the sample median. The first four central moments of the test-statistics are derived and hence the sampling distribution of the test statistics is fitted as a member of the GLD family. From the quantile function of the fitted GLD, percentile points of the distribution of the test-statistics for different values of n and α can be computed. By fitting the distribution of the test statistics under the alternative hypothesis, power of the test can also be evaluated. To conduct the test, computer programmes in Maple language are provided.

3.1. Derivation of the First Four Moments of the Test-Statistics

Let the GLD parameters of the distribution of X be $(\lambda_1,\ \lambda_2,\ \lambda_3,\ \lambda_4)$ and let $\mu_r^{'}(k:n),\ r = 1,\ 2, \ldots$ denote the moments about origin of the k^{th} order statistics $x_{k:n}$ based on a random sample of size n drawn from the population. Kumaran and Beena (2005) derived general expression for moments about origin of order statistics from GLD family and it is given as

$$\mu_r^{'}(k:n) = \frac{1}{\beta(k, n-k+1)} \sum_{i=0}^{r} \binom{r}{i} \lambda_1^{r-i} \lambda_2^{-i} \times \sum_{j=0}^{i} (-1)^j \binom{i}{j} \beta[\lambda_3(i-j)+k, \lambda_4 j + n - k + 1] \tag{3.1}$$

Putting $r = 1,\ 2,\ 3,\ 4$ in equation (3.1) and simplifying we obtain the first four raw moments of k^{th} order statistics as

$$\mu_1^{'}(k:n) = \lambda_1 + \frac{A_1}{\lambda_2 A_0} \tag{3.2}$$

$$\mu_2^{'}(k:n) = \lambda_1^2 + \frac{2A_1\lambda_1}{\lambda_2 A_0} + \frac{A_2}{\lambda_2^2 A_0} \tag{3.3}$$

$$\mu_3^{'}(k:n) = \lambda_1^3 + \frac{3A_1\lambda_1^2}{\lambda_2 A_0} + \frac{3A_2\lambda_1}{\lambda_2^2 A_0} + \frac{A_3}{\lambda_2^3 A_0} \tag{3.4}$$

$$\mu_4^{'}(k:n) = \lambda_1^4 + \frac{4A_1\lambda_1^3}{\lambda_2 A_0} + \frac{6A_2\lambda_1^2}{\lambda_2^2 A_0} + \frac{4\lambda_1 A_3}{\lambda_2^3 A_0} + \frac{A_4}{\lambda_2^4 A_0} \tag{3.5}$$

where

$$A_0 = \beta(k, n-k+1), \tag{3.6}$$

$$A_1 = \beta(\lambda_3 + k, n-k+1) - \beta(k, \lambda_4 + n - k + 1), \tag{3.7}$$

$$A_2 = \beta(2\lambda_3 + k, n-k+1) - 2\beta(\lambda_3 + k, \lambda_4 + n - k + 1) + \beta(k, 2\lambda_4 + n - k + 1), \tag{3.8}$$

$$A_3 = \beta(3\lambda_3 + k, n-k+1) - 3\beta(2\lambda_3 + k, \lambda_4 + n - k + 1) + 3\beta(\lambda_3 + k, 2\lambda_4 + n - k + 1) - \beta(k, 3\lambda_4 + n - k + 1), \tag{3.9}$$

$$A_4 = \beta(4\lambda_3 + k, n-k+1) - 4\beta(3\lambda_3 + k, \lambda_4 + n - k + 1) + 6\beta(2\lambda_3 + k, 2\lambda_4 + n - k + 1) - 4\beta(\lambda_3 + k, 3\lambda_4 + n - k + 1) + \beta(k, 4\lambda_4 + n - k + 1) \tag{3.10}$$

Hence, the central moments up to order four of k^{th} order statistics are obtained as

$$\mu_2(k:n) = \lambda_2^{-2}\left[\frac{A_2}{A_0} - \frac{A_1^2}{A_0^2}\right] \tag{3.11}$$

$$\mu_3(k:n) = \lambda_2^{-3}\left[\frac{A_3}{A_0} - \frac{3A_1A_2}{A_0^2} + \frac{2A_1^3}{A_0^3}\right] \tag{3.12}$$

$$\mu_4(k:n) = \lambda_2^{-4}\left[\frac{A_4}{A_0} - \frac{4A_1A_3}{A_0^2} + \frac{6A_1^2A_2}{A_0^3} - \frac{3A_1^4}{A_0^4}\right] \tag{3.13}$$

The mean, variance, skewness and kurtosis of the test statistics q_2 are denoted as $\alpha_1,\ \alpha_2,\ \alpha_3,\ \alpha_4$ and are obtained as

$$E(q_2) = Q_2 = \alpha_1 \tag{3.14}$$

$$\alpha_2 = \mu_2(q_2) = \lambda_2^{-2}\left[\frac{A_2^{'}}{A_0^{'}} - \frac{(A_1^{'})^2}{(A_0^{'})^2}\right] \tag{3.15}$$

$$\alpha_3 = \frac{\mu_3(q_2)}{[\mu_2(q_2)]^{3/2}} = \frac{A_3^{'}(A_0^{'})^2 - 3A_0^{'}A_1^{'}A_2^{'} + 2(A_1^{'})^3}{[A_2^{'}A_0^{'} - (A_1^{'})^2]^{3/2}} \tag{3.16}$$

$$\alpha_4 = \frac{\mu_4(q_2)}{[\mu_2(q_2)]^2} = \frac{A_4^{'}(A_0^{'})^3 - 4(A_0^{'})^2A_1^{'}A_2^{'} + 6A_0^{'}(A_1^{'})^2A_2^{'} - 3(A_1^{'})^4}{[A_2^{'}A_0^{'} - (A_1^{'})^2]^2} \tag{3.17}$$

where $A_0^{'},\ A_1^{'},\ A_2^{'},\ A_3^{'}$ and $A_4^{'}$ are obtained by replacing k as $\frac{n}{2}$ or $\left[\frac{n}{2}\right]+1$ according as when $\frac{n}{2}$ is integer or not in equations (3.6) through (3.10).

For a given set of observations and under the null hypothesis, the values of $(\alpha_1,\ \alpha_2,\ \alpha_3,\ \alpha_4)$ can be estimated using the following programme and are denoted as $(a_1,\ a_2,\ a_3,\ a_4)$.

3.1.1 Programme-P_1

Procedure to determine the values of $(a_1,\ a_2,\ a_3,\ a_4)$ of sample median via observations
Function: Findalphas of the sample median via observations
Purpose:– Compute a-values of the sample median
Arguments:$a,\ b,\ c,\ d$–The GLD parameters of the parent distribution;
n–number of observations
Q_2^0–The specified value of the median under the null hypothesis;

Findalphas:= Proc(n::Numeric, a::Numeric, b::Numeric, c::Numeric, d::Numeric, Q_2^0:Numeric)
Local $f,\ k,\ s,\ u,\ B1,\ B2,\ B3,\ B4,\ B5,\ B6,\ B7,\ B8,\ B9,\ B10,\ B11,\ B12,\ B13,\ B14,\ B15, M1,\ M2,\ M3,\ M4,\ a_1,\ a_2,\ a_3,\ a_4,\ Ah;$
$f := convert\left(\frac{n}{2},\ fraction\right);\qquad k := trunc(f);$
$B1 := evalf(Beta(k,\ n-k+1));\qquad B2 := evalf(Beta(c+k,\ n-k+1));$
$B3 := evalf(Beta(k,\ d+n-k+1));\qquad B4 := evalf(Beta(2c+k,\ n-k+1));$
$B5 := evalf(Beta(c+k,\ d+n-k+1));\qquad B6 := evalf(Beta(k,\ (2d+n-k+1)));$
$B7 := evalf(Beta(3c+k,\ n-k+1));\qquad B8 := evalf(Beta(2c+k,\ d+n-k+1));$
$B9 := evalf(Beta(c+k,\ 2d+n-k+1));\qquad B10 := evalf(Beta(k,\ 3d+n-k+1));$
$B11 := evalf(Beta(4c+k,\ n-k+1));\qquad B12 := evalf(Beta(3c+k,\ d+n-k+1));$
$B13 := evalf(Beta(2c+k,\ 2d+n-k+1));\qquad B14 := evalf(Beta(c+k,\ 3d+n-k+1));$
$B15 := evalf(Beta(k,\ 4d+n-k+1));$
$M1 := evalf\left(a+\frac{B2-B3}{bB1}\right);$
$M2 := evalf\left(b^{-2}\left(\frac{B4-2B5+B6}{B1}-\frac{(B2-B3)^2}{B1^2}\right)\right);$
$M3 := evalf\left(b^{-3}\left(\frac{B7-3B8+3B9-B10}{B1}-\frac{3(B4-2B5+B6)(B2-B3)}{B1^2}+\frac{2(B2-B3)^3}{B1^3}\right)\right);$
$M4 := evalf\left(b^{-4}\left(\frac{B11-4B12+6B13-4B14+B15}{B1}-\frac{4(B2-B3)(B7-3B8+3B9-B10}{B1^2}+\frac{6(B4-2B5+B6)(B2-B3)^2}{B1^3}-\frac{3(B2-B3)^4}{B1)^4}\right)\right);$
$a_1 := q_2;\qquad a_2 := M2;\qquad a_3 := evalf\left(\frac{M3}{a2^{1.5}}\right);\qquad a_4 := evalf\left(\frac{M4}{a2^2}\right);$
$Ah := [a1, a2, a3, a4];$ end:

Using these values of $(a_1,\ a_2,\ a_3,\ a_4)$ as arguments in the following programme, the GLD parameters of the distribution of the test statistics and the percentile points can be evaluated.

3.1.2 Programme-P_2

Procedure to determine lambdas and the percentile points from sample A_h-values
#Function: Findlambdas
#Purpose: Estimation of GLD parameters by Newton's approx.
#Arguments: A_h–list of $a_1,\ a_2,\ a_3,\ a_4$;
$I_3,\ I_4$–Initial approx. of λ_3 and λ_4
Findlambdas := Proc(A_h::list, I_3::Numeric, I_4::Numeric)

Local $A,\ B,\ C,\ D_1,\ D_2,\ D,\ \alpha_1,\ \alpha_2,\ \alpha_3,\ \alpha_4,\ F,\ \hat{A}_1,\ \hat{A}_2,\ \hat{A}_3,\ \hat{A}_4,$
$V,\ J,\ err3,\ err4,\ F_k,\ J_k,\ Y,\ Eq3,\ Eq4,\ A_1,\ A_2,\ L, FirstL,\ SecondL,\ l,$
$R_1,\ R_2,\ R_3,\ R_4, R_5,\ R_6,\ R_7,\ R_8;$
with(linalg, vector, matrix, jacobian, linsolve):
$\hat{A}_1 := 0$; $\hat{A}_2 := 1$; $\hat{A}_3 := evalf(Ah[3])$; $\hat{A}_4 := evalf(Ah[4])$; $L_3 := I_3$; $L_4 := I_4$;
$A := \dfrac{1}{1+\lambda_3} - \dfrac{1}{1+\lambda_4};$
$B := \dfrac{1}{1+2*\lambda_3} + \dfrac{1}{1+2*\lambda_4} - 2*Beta(1+\lambda_3, 1+\lambda_4);$
$C := \dfrac{1}{1+3*\lambda_3} - \dfrac{1}{1+3*\lambda_4} - 3*Beta(1+2*\lambda_3, 1+\lambda_4) + 3*Beta(1+\lambda_3, 1+2*\lambda_4);$
$D_1 := \dfrac{1}{1+4*\lambda_3} + \dfrac{1}{1+4*\lambda_4} + 6*Beta(1+2*\lambda_3, 1+2*\lambda_4);$
$D_2 := -4*Beta(1+3*\lambda_3, 1+\lambda_4) - 4*Beta(1+\lambda_3, 1+3*\lambda_4)$; $D := D_1 + D_2$;
$\alpha_1 := \lambda_1 + A/\lambda_2;\quad \alpha_2 := abs(B - A^2)/\lambda_2^2;$
$\alpha_3 := \dfrac{C - 3*A*B + 2*A^3}{abs((B-A^2)^{3/2})};$
$\alpha_4 := \dfrac{d - 4*A*C + 6*B*A^2 - 3*A^4}{(B-A^2)^2};$
$Eq3 := \alpha_3 - \hat{A}_3$; $Eq4 := \alpha_4 - \hat{A}_4$;
$F := vector([Eq3, Eq4])$; $V := vector([\lambda_3, \lambda_4])$;
$j := evalf(jacobian(F, V))$; $err3 := 1$; $err4 := 1$;
while ($err3 > .0001$ or $err4 > .0001$) do
$F_k := vector([evalf(subs(\lambda_3 = L_3, \lambda_4 = L_4, -Eq3)),$
$evalf(subs(\lambda_3 = L_3, \lambda_4 = L_4, -Eq4))]);$
$J_k := matrix([[subs(\lambda_3 = L_3, \lambda_4 = L_4, j[1,1]), subs(\lambda_3 = L_3, \lambda_4 = L_4, j[1,2])],$
$[subs(\lambda_3 = L_3, \lambda_4 = L_4, j[2,1]), subs(\lambda_3 = L_3, \lambda_4 = L_4, j[2,2])]]);$
$Y := linsolve(J_k, F_k)$; $L_3 := L_3 + Y[1]$;
$L_3 := L_3 + Y[1]$; $L_4 := L_4 + Y[2]$;
$err3 := evalf(abs(subs(\lambda_3 = L_3, \lambda_4 = L_4, Eq3)));$
$err4 := evalf(abs(subs(\lambda_3 = L_3, \lambda_4 = L_4, Eq4)));$
od;
$print(L_3, L_4, err3, err4);$
$A_1 := evalf(subs(\lambda_3 = L_3, \lambda_4 = L_4, A));$
$A_2 := evalf(subs(\lambda_3 = L_3, \lambda_4 = L_4, B));$
$L_2 := abs(sqrt(((A_2 - A_1^2))/\hat{A}_2));$
$L_1 := \hat{A}_1 - A_1/L_2;$
$FirstL := [L_1, L_2, L_3, L_4];$
if $L_3 < 0$ then $SecondL := [-FirstL[1], FirstL[2], FirstL[4], FirstL[3]]$ else
$SecondL := FirstL$ fi;
if $evalf(Ah[3]) < 0$ then $L := [-SecondL[1], SecondL[2], SecondL[4], SecondL[3]]$
else $L := SecondL$ fi ;
$l := [L[1]*sqrt(Ah[2]) + Ah[1], (L[2])/(sqrt(Ah[2])), L[3], L[4]];$

$t_{0.05} := l[1] + \dfrac{(0.05)^{l[3]} - (0.95)^{l[4]}}{l[2]};$
$t_{0.95} := l[1] + \dfrac{(0.95)^{l[3]} - (0.05)^{l[4]}}{l[2]};$
$t_{0.025} := l[1] + \dfrac{(0.025)^{l[3]} - (0.975)^{l[4]}}{l[2]};$
$t_{0.975} := l[1] + \dfrac{(0.975)^{l[3]} - (0.025)^{l[4]}}{l[2]};$
$t_{0.01} := l[1] + \dfrac{(0.01)^{l[3]} - (0.99)^{l[4]}}{l[2]};$
$t_{0.99} := l[1] + \dfrac{(0.99)^{l[3]} - (0.01)^{l[4]}}{l[2]};$
$t_{0.005} := l[1] + \dfrac{(0.005)^{l[3]} - (0.995)^{l[4]}}{l[2]};$
$t_{0.995} := l[1] + \dfrac{(0.995)^{l[3]} - (0.005)^{l[4]}}{l[2]};$
end:

To find the Power of the test, first fit the distribution of the test statistics under the alternative hypothesis using the programme P_2 and then using these lambda values in the following programme, power is obtained.

3.2. Programme-P_3

Procedure to determine the power values of the test;
#Function: Power via iteration;
#Purpose: Compute power values of the test;
#Arguments: L–list of lambda values of sample median under the alternative hypothesis;
#Q_0, Q_1– median values under H_0, H_1;
#P_0–initial approx. of power;
#K–critical point of the test;
FindPower:= Proc(L:: list,Q_0::Numeric, Q_1::Numeric, P_0::Numeric, K::Numeric)
Local l_1,Q,E,err,P,p;
$l_1 := evalf(L[1] + Q_1 - Q_0);$
$Q := l_1 + (((p)^((L[3])) - (1 - p)^((L[4]))))/(L[2]);$
$P := P_0;$
$E := K - Q;$
$err := 0.5;$
while $(err > .00001) doP := P + .0002;$
err:=evalf(subs$(p = P, E)$);
od;
if the test is lower tailed then print(P,err) else print(1-P,err) fi;
end.

3.3. Numerical Illustration

The proposed method of testing is illustrated based on sample observations from a log-normal distribution with parameters (0,1/3) ($LN(0, 1/3)$). That is to test $H_0 : Q_2 = 1$, we have the GLD parameters of $LN(0, 1/3)$ as $\lambda_1 = 0.8451,\ \lambda_2 = 0.1085,\ \lambda_3 = 0.01017$ and $\lambda_4 = 0.03422$. The values of ($\alpha_1,\ \alpha_2,\ \alpha_3,\ \alpha_4$), the corresponding GLD parameters of the distribution of the distribution of the test statistics q_2 and the power values of the test at $\alpha = 0.05$ are evaluated using programmes given above and tabulated below.

Table 1: First four moments and the GLD parameters of q_2 from $LN(0, 1/3)$

n	α_1	α_2	α_3	α_4	λ_1	λ_2	λ_3	λ_4
5	1	0.0319	0.6433	3.8396	0.9190	0.5654	0.0363	0.0879
10	1	0.0147	0.4409	3.4507	0.9565	1.0959	0.0566	0.1127
15	1	0.0107	0.3880	3.3868	0.9663	1.3347	0.0622	0.1154
20	1	0.0077	0.3292	3.1400	0.9702	2.0062	0.0812	0.1560
25	1	0.0064	0.3028	3.3697	0.9801	1.6825	0.0663	0.1058

Table 2: Percentile points of the distribution of q_2 from $LN(0, 1/3)$

	Percentage probability levels							
n	95	97.5	99	99.5	5	2.5	1	0.5
5	1.3251	1.4071	1.5071	1.5771	0.7449	0.7014	0.6485	0.6105
10	1.2154	1.2656	1.3255	1.3666	0.8193	0.7870	0.7480	0.7204
15	1.1829	1.2249	1.2747	1.3088	0.8435	0.8150	0.7807	0.7566
20	1.1542	1.1872	1.2252	1.2503	0.8665	0.8431	0.8154	0.7963
25	1.1395	1.1711	1.2089	1.2349	0.8762	0.8527	0.8243	0.8043

Table 3: Power values of the distribution of q_2 from $LN(0, 1/3)$

n	Q_2: for lower tailed test					Q_2: for upper tailed test				
	0.9	0.8	0.7	0.6	0.5	1.1	1.2	1.3	1.4	1.5
5	0.184	0.416	0.646	0.810	0.908	0.108	0.216	0.396	0.634	0.852
10	0.260	0.598	0.844	0.954	0.990	0.164	0.414	0.754	0.952	0.994
15	0.304	0.688	0.910	0.982	0.998	0.200	0.538	0.884	0.989	0.996
20	0.372	0.786	0.961	0.997	1	0.254	0.684	0.964	0.999	1
25	0.400	0.836	0.978	0.999	1	0.291	0.777	0.983	0.999	1

4. Conclusion

We have demonstrated a general method of testing the median of any distribution. The most important aspect of the procedure is that, this method is applicable for testing the median of any type of distribution. Since the computer programmes are provided, by inputting the given set of observations in the programmes, one can easily test whether the observations are drawn from a population with given median against any of the alternatives. Using this method, both the null and alternative distributions of the test statistics can be fitted and the power of the test can be evaluated. As this method doesn't make any rigorous assumption on the distribution of the population, is applicable to almost all practical cases.

References

[1] Balakrishnan, N. and Rao, C. R. (1998). *Order Statistics Theory and Methods*, Elsevier, NewYork.

[2] David, H. A. and Nagaraja, H. N. (2003). *Order Statistics*, John Wiley & Sons, 3rd Edn., NewYork.

[3] Harter, H. L. and Balakrishnan, N. (1996). *C.R.C Handbook of Tables for the Use of Order Staistics in Estimation*, CRC Press, Boca Raton, FL.

[4] Karian, Z. A. and Dudewicz, E. J. (2000). *Fitting Statistical Distributions-The Generalised Lambda Distribution and Generalised Bootstrap Methods*, CRC press, NewYork.

[5] Kendall, M. G. and Stewart, A. (1977). *The Advanced Theory of Statistics*, Fourth ed., Vol.1 Griffin, London.

[6] Kumaran, M. and Beena, K. K. (2005). *On Moments of Order Statistics from Generalised Lambda Distribution and Some Applications*, Journal of Kerala Statistical Association, 16, 20–30.

[7] Ramberg, J. and Schmeiser, B. (1974). *An Approximate Method for Generating Asymmetric Random Variables*, Communications of the ACM, 17, 78–82.

[8] Ramberg, J. Tadikamalla, P. Dudewicz , E. J. and Mykytka, E. (1979). *A Probability Distribution and Its Uses in Fitting Data*, Technometrics, 21(2), 201–214.

[9] Reiss, R. D. (1989). *Approximate Distributions of Order Statistics With Application to Non-parametric Statistics*, Springer-Verlag, NewYork.

[10] Ross, S. M. (1987). *Introduction to Probability and Statistics for Engineers and Scientists*. John Wiley & Sons, New York.