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Modified Particle Swarm Optimization for Economic Load Dispatch with Valve-Point Effects and Transmission Losses

G. Shabib, Mesalam Abdel Gayed and A.M. Rashwan

Department of Electrical Engineering, High Institute of Energy
South Valley University, Aswan, Egypt
E-mail: engrashwan@yahoo.com

Abstract

A modified particle swarm optimization (PSO) technique is presented in this paper. The proposed method is used to solve the economic power dispatch with valve-point effects and transmission losses. The objective is to minimize the total fuel cost of generation. Two different study cases six and fifteen generators are applied to show the effectiveness and efficiency of the proposed technique. The results is compared with the genetic algorithms technique and traditional (PSO) show that the proposed algorithm produces optimal solution in term of computational time and the optimal cost for economic dispatch problem .

Keywords: Economic Dispatch, Genetic Algorithms, Particle Swarm Optimization.

Introduction

Engineers always concern the cost of products or services. In a power system, minimizing the operation cost is very important. Economic load dispatch (ELD) is a method to schedule the power generator outputs with respect to the load demands, and to operate the power system most economically.

Several conventional and global methods have been applied for solving economic load dispatch (ELD) problems such as lambda iteration, gradient search, Newton’s method and dynamic programming [1]. The conventional methods can find good solutions in a fast manner; however they can only be applied to small scale and simple problems. Recently many techniques based on artificial intelligence has been also used for solving ELD problem such as genetic algorithms (GA) [2], simulated annealing (SA) [7], evolutionary programming (EP) [3], particle swarm optimization (PSO) [4] .One of the technique is the Particle swarm optimization (PSO). Early introduced by Kennedy and Eberhart in 1995[5], [6], It was developed through simulation of a simplified social system; it is one of the modern heuristic algorithms based on the analogy of swarm of birds and fish schooling. The PSO technique can generate high-quality solutions within shorter calculation time and stable convergence characteristic than other stochastic methods. Although the PSO seems to be sensitive to the tuning of some weights or parameters, many researches are still in progress for proving its potential in solving complex power system problems.

Due to its simplicity and good performance, PSO has attracted many attentions and have been applied in various Power system optimization problems such as economic dispatch [5], design of PID controller in AVR system [10], reactive power and voltage control [11] and unit commitment [12].

In this paper, a modified PSO method for solving the ELD problem in power system is proposed. The proposed method considers the nonlinear characteristics of a generator such as valve point effect and transmission losses. The feasibility of the proposed method was demonstrated for two different systems [11], [15].Results obtained show that the proposed approach can obtain more optimum solutions.

Overview of Particle Swarm optimization and its Variation

Natural creatures sometime behave as a Swarm. One of the main streams of artificial life researches is to examine how natural creatures behave as a Swarm and reconfigure the Swarm models inside the computer. Eberhart and Kennedy develop PSO, based on analogy of the Swarm of birds and fish school, each individual exchanges previous experience among themselves [13]. PSO as an optimization tool provides a population based search procedure in which individuals called particles change their position with time. In a PSO system, particles fly around in a multi dimensional search space. During flight each particles adjust its position according its own experience and the experience of the neighboring particles, making use of the best position encountered by itself and its neighbors. In the multidimensional space where the optimal solution is sought, each particle in the swarm is moved toward the optimal point by adding a velocity with its position. The velocity of a particle is influenced by three components, namely, inertial, cognitive, and social. The inertial component simulates the inertial behavior of the bird to fly in the previous direction. The cognitive component models the memory of the bird about its previous best position, and the social component models the memory of the bird about the best position among the particles. The particles move around the multidimensional search space until they find the optimal solution. The modified velocity of each agent can be calculated using the current velocity and the distance from Pbest and Gbest as given in the following equation:

$$V_{ij}^{t} = \omega V_{ij}^{t-1} + C_1.r_1.(P_{best\,ij}^{\,t-1} - X_{ij}^{t-1}) + C_2.r_2.(G_{best\,i}^{\,t-1} - X_{ij}^{t-1}) \qquad (1)$$

i =1, 2…, N_D

j=1, 2…, Npar

Equation (1) show that, a certain velocity can be calculated, which gradually gets close to P_{best} and G_{best}. The current position (searching point in the solution space), each individual moves from the current position to the next one by the modified velocity in (1) using the following equation:

$$X_{ij}^{t} = X_{ij}^{t-1} + V_{ij}^{t} \tag{2}$$

i =1, 2…, N_D; j=1, 2…, Npar

Where,

t Iteration count,

V_{ij} Dimension *i* of the velocity of particle

j at iteration *t*,

X_{ij}^{t} Dimension *i* of the position of particle *j* at iteration *t*,

ω Inertia weight, C1, C2 Acceleration coefficients,

$P_{best\,ij}^{\;t}$ Dimension *i* of the own best position Of particle *j* until iteration *t*,

$G_{best\,ij}^{\;t}$ Dimension *i* of the best particle in the swarm at iteration *t*, ND Dimension of the optimization problem (Number of decision variables),

Npar Number of particles in the swarm,

r_1, r_2 Two separately generated uniformly distributed random numbers in the range [0, 1].

The following weighting function is usually utilized:

$$\omega = \omega_{\max} - \frac{\omega_{\max} - \omega_{\min}}{T_{\max}} t \tag{3}$$

Where,

$\omega_{\max}, \omega_{\min}$ Initial and final weights,

T max maximum iteration number

t current iteration number.

A new variation in the classical PSO since $Pbest_i$ is the best solution a particle found so far, it can be considered as the experience a particle acquired in past time. Experience usually helps people learn and accumulate new knowledge. Base on this observation, a new PSO in which the particles explore around its previous best is proposed and therefore the proposed modification in equation (2) is given as:

$$X_{ij}^{t} = X_{ij}^{t-1} + P_{best\,ij}^{t-1} \tag{4}$$

ELD Problem Formulation

The economic dispatch problem is to simultaneously minimize the overall cost rate and meet the load demand of a power system. The power system model consists of N generating units already connected to the system. The economic dispatch is to determine the optimal share of load demand for each unit in the range of 3 to 5 minutes [1].

The basic ED becomes a nonconvex optimization problem if the practical operating conditions are included. The basic cost function used is:

Minimize

$$F_T = \sum_{i=1}^{N} F_i(P_i) \tag{5}$$

$$\sum_{i}^{N} F_i(P_i) = \sum_{i=1}^{N} (a_i P_i^2 + b_i P_i + c_i) \tag{6}$$

Where,

F_T Total generation cost ($/hr),

F_i Cost function of generator i ($/hr)

a_i, b_i, c_i Cost coefficients of generator i,

P_i Power of generator i (MW),

N Number of generators.

Active Power Balance Equation

For power balance, an equality constraint should be satisfied: Total generated power should be the same as total demand plus the total line losses.

$$\sum_{i=1}^{n} P_i = P_{Load} + P_{Loss} \tag{7}$$

Where P_{Load} is the total load in the system (MW) and P_{Loss} is the network loss (MW) that can be calculated by matrix loss formula. However, the transmissions losses considered are governed by the following equation:

$$P_{Loss} = \sum_{i=1}^{m}\sum_{j=1}^{m} P_i B_{ij} P_j + \sum_{i=1}^{m} B_{0i} P_i + B_{00} \tag{8}$$

Where,

B_{ij}, B_{0i}, B_{00} are the B-matrix coefficient, P_i is the power in the line.

Minimum and Maximum Power Limits

Generation output of each generator should lie between maximum and minimum limits. The corresponding inequality constraints for each generator are

$$P_{i,\,min} \le Pi \le P_{i,\,max} \tag{9}$$

Where

$P_{i,\,min}$ and $P_{i,\,max}$ are the minimum and the maximum output of generator i, respectively.

Valve Point Effect (VPE)

The valve opening process of multivalve steam turbines produce a ripple –like effect in the heat rate curve of the generators, and it is taken into consideration in the ELD problem by superimposing the basic quadratic fuel-cost Characteristics with the rectified sinusoidal component as follows:

$$\sum_{i}^{n} F_i\,(P_i) = a_i P_i^{\,2} + b_i P_i + C_i + \left| e_i \sin(f_i\,(P_{i,nin} - P_i)) \right| \tag{10}$$

Where,

F_T Total generation cost ($/hr),

F_i Cost function of generator i ($/hr)

a_i, b_i, c_i ,e_i ,f_i Cost Coefficients of Generator i,

P_i Power of generator i (MW),

N Number of generators.

The objective of economic dispatch VPE is to minimize F_T with the constraints from (5) to (10).

Solving ELD Problem by MPSO

Originally, PSO is designed to solve unconstrained Continuous optimization problems with objective variables bounded in a multidimensional hyperspace. However, except the bounded constraint of variables, economic power dispatch problems are basically nonlinear optimization problems with equality constraints as can be seen from (5) and (6). To apply PSO to solve this kind of problems, it requires the constraints to be treated in advance employing some kind of strategies.

The search procedures of the proposed method where as shown below.

Step 1: Specify the lower and upper bound generation power of each unit, and calculate F_{max} and F_{min}. Initialize randomly the individuals of the population according to the limit of each unit including individual dimensions, searching points, and velocities. These initial individuals must be feasible candidate solutions that satisfy the practical operation constraints.

Step 2: To each individual P_i of the population, employ the B -coefficient loss formula to calculate the transmission loss P_{Loss}

Step 3: Calculate the evaluation value of each individual in the population using the evaluation function given by (9).

Step 4: Compare each individual's evaluation value with its P_{best}. The best evaluation value among P_{best} the is denoted as $G_{best.}$

Step 5: Modify the member velocity of each individual according to (1).

Step 6: If $V_{ij}^{t-1} > V_i^{max}$, then $V_{ij}^{t-1} = V_i^{max}$,If $V_{ij}^{t-1} < V_i^{min}$, then $V_{ij}^{t-1} = V_i^{min}$.

Step 7: Modify the member position of each Individual according to (3)

Step 8: If the evaluation value of each individual is better than the previous P_{best} , the current value is set to be P_{best} .If the best P_{best} is better than G_{best} , the value is set to be G_{best}.

Step 9: if the number of iterations reaches the maximum, then go to Step 10. Otherwise, go to Step 2.

Step 10: the individual that generates the latest G_{best} is the optimal generation power of each unit with the minimum total generation cost.

Numerical Examples and Results

To verify the feasibility of the proposed PSO method, two different power systems were tested. In these examples, the transmission losses and valve point effect of units were taken into account in practical application, so the proposed PSO method was compared with an Elitist GA search method and the traditional PSO [9].At each sample system, under the same evaluation function and individual definition, we performed 50 trials using the proposed method to observe the variation during the evolutionary processes and to compare their solution quality, convergence characteristic.

A reasonable B loss coefficients matrix of power system network was employed to draw the transmission line loss and satisfy the transmission capacity constraints. A software program was written in Matlab language to solve this problem.

Case 1: Six-unit system: The system contains six thermal units, 26 buses, and 46 transmission lines [15]. The load demand is 1263 MW. The characteristics of the six thermal units are given in table I. In normal operation, the loss coefficients B with the 100-MVA base capacity are as follow:

$$B_{ij} = \begin{bmatrix} 0.0017 & 0.0012 & 0.0007 & -0.0001 & -.0005 & -0.0002 \\ 0.0012 & 0.0014 & 0.0009 & 0.0001 & -0.0006 & -0.0001 \\ 0.0007 & 0.0009 & 0.0031 & 0.0000 & -0.0010 & -0.0006 \\ -0.0001 & 0.0001 & 0.0000 & 0.0024 & -0.0006 & -0.0008 \\ -0.0005 & -0.0006 & -0.0010 & -0.0006 & 0.0129 & -0.0002 \\ -0.0002 & -0.0001 & -0.0006 & -0.0008 & -0.0002 & 0.0150 \end{bmatrix}$$

$$B_{0i} = 1.0e^{-03} * [-0.3908 \quad -0.1297 \quad 0.7047 \quad 0.0591 \quad -0.6635] \; B_{00} = 0.056.$$

$$B_{ij} = \begin{bmatrix} 0.0014 & 0.0012 & 0.0007 & -0.0001 & -0.0003 & -0.0001 & -0.0001 & -0.0001 & -0.0003 & 0.0005 & -0.0003 & -0.0002 & 0.0004 & 0.0003 & -0.0001 \\ 0.0012 & 0.0015 & 0.0013 & 0.0000 & -0.0005 & -0.0002 & 0.0000 & 0.0001 & -0.0002 & -0.0004 & -0.0004 & -0.0000 & 0.0004 & 0.00010 & -0.0002 \\ 0.0007 & 0.0013 & 0.0076 & -0.0001 & -0.0013 & -0.0009 & -0.0001 & 0.0000 & -0.0008 & -0.0012 & -0.0017 & -0.0000 & -0.0026 & 0.0001 & -0.0028 \\ -0.0001 & 0.0000 & -0.0001 & 0.0034 & -0.0007 & -0.0004 & 0.0011 & 0.0050 & 0.0029 & 0.0032 & -0.0011 & -0.0000 & 0.0001 & 0.0001 & -0.0026 \\ -0.0003 & -0.0005 & -0.0013 & -0.0007 & 0.0090 & 0.0014 & -0.0003 & -0.0012 & -0.0010 & -0.0013 & 0.0007 & -0.0002 & -0.0002 & -0.0024 & -0.0003 \\ -0.0001 & -0.0002 & -0.0009 & -0.0004 & 0.0014 & 0.0016 & -0.0000 & -0.0006 & -0.0005 & -0.0008 & 0.0011 & -0.0001 & -0.0002 & -0.0017 & 0.0003 \\ -0.0001 & 0.0000 & -0.0001 & 0.0011 & -0.0003 & -0.0000 & 0.0015 & 0.0017 & 0.0015 & 0.0009 & -0.0005 & 0.0007 & -0.0000 & -0.0002 & -0.0008 \\ -0.0001 & 0.0001 & 0.0000 & 0.0050 & -0.0012 & -0.0006 & 0.0017 & 0.0168 & 0.0082 & 0.0079 & -0.0023 & -0.0036 & 0.0001 & 0.0005 & -0.0078 \\ -0.0003 & -0.0002 & -0.0008 & 0.0029 & -0.0010 & -0.0005 & 0.0015 & 0.0082 & 0.0129 & 0.0116 & -0.0021 & -0.0025 & 0.0007 & -0.0012 & -0.0072 \\ -0.0005 & -.0.0004 & -0.0012 & 0.0032 & -0.0013 & -0.0008 & 0.0009 & 0.0079 & 0.0116 & 0.0200 & -0.0027 & -0.0034 & 0.0009 & -0.0011 & -0.0088 \\ -0.0003 & -0.0004 & -0.0017 & -0.0011 & 0.0007 & 0.0011 & -0.0005 & -0.0023 & -0.0021 & -0.0027 & 0.0140 & 0.0001 & 0.0004 & -0.0038 & 0.0168 \\ -0.0002 & 0.0000 & -0.0000 & -0.0000 & -0.0002 & -0.0001 & 0.0007 & -0.0036 & -0.0025 & -0.0034 & 0.0001 & 0.0054 & -0.0001 & -0.0004 & 0.0028 \\ 0.0004 & 0.0004 & -0.0026 & 0.0001 & -0.0002 & -0.0002 & -0.0000 & 0.0001 & 0.0007 & 0.0009 & 0.0004 & -0.0001 & 0.0103 & -0.0101 & 0.0028 \\ 0.0003 & 0.0010 & 0.0111 & 0.0001 & -0.0024 & -0.0017 & -0.0002 & 0.0005 & -0.0012 & -0.0011 & -0.0038 & -0.0004 & -0.0101 & 0.0578 & -0.0094 \\ -0.0001 & -0.0002 & -0.0028 & -0.0026 & -0.0003 & 0.0003 & -0.0008 & -0.0078 & -0.0072 & -0.0088 & 0.0168 & 0.0028 & 0.0028 & -0.0094 & 0.1283 \end{bmatrix}$$

$$B_{0i} = 1.0e^{-03} * [-0.0001 \; -0.0002 \; 0.0028 \; -0.0001 \; 0.0001 \; -0.0003 \; -0.0002 \; -0.0002 \; 0.0006 \; 0.0039 \; -0.0017 \; 0 \; -0.0032 \; 0.0067 \; -0.006]$$

$$B_{00} = 0.0055;$$

In this case, each individual Pg contains six generators power outputs, such as P_1, P_2, P_3, P_4, P_5, and P_6, which are generated randomly. The dimension of the population is equal to 6 x100. Through the evolutionary process of the proposed method. Table II shows the mean, maximum and minimum cost acquired by the proposed method. These results are obtained out of 50 runs for comparison with the results obtained by genetic algorithms and traditional PSO techniques [9]. It is obvious that, in terms of minimum and mean costs, the proposed strategy perform better than genetic algorithms method and traditional PSO. Table III list the optimum dispatch of generator obtained by proposed method. Note that the outputs of the generators are all within the generator's permissible output limit.

Case 1: 15-unit system: The system contains 15 thermal units, [11], the load demand is 2630 MW. The characteristics of the 15 thermal units are given in Table IV. In normal operation of the system, the loss coefficient B with the 100-MVA base capacity was shown in the Appendix.

In this case, each individual Pg contains 15 generators are generated randomly. The dimension of the population is equal to 15 x100. Through the evolutionary process of the proposed method. Table V shows the mean, maximum and minimum cost acquired by the proposed method .these results show that the proposed method are feasible and indeed capable of acquiring better solution. The optimal dispatch of generators is listed in table VI also notes that all generators' outputs are within the permissible limits.

Table 1: Generating Units Capacity and Coefficients (6 Unit).

unit	P_{min}(MW)	P_{max}(MW)	a	b	c	e	f
1	100	500	0.0070	7.0	240	300	0.035
2	50	200	0.0095	10.0	200	200	0.042
3	80	300	0.0090	8.5	220	200	0.042
4	50	150	0.0090	11.0	200	150	0.063
5	50	200	0.0080	10.5	220	150	0.063
6	50	120	0.0075	12.0	190	150	0.063

Table II: Comparison between Both Methods (50 Trails).

Method	Min.cost	Max.cost	Mean cost	Average cpu time
GA	15,459	15,524	15,469	41.58
PSO	15,450	15,492	15,454	14.89
MPSO	15,427.9	15,460	15,435	4.26

Table III: Best Solution of 6-Unit System.

Unit power output	GA method	PSO method	MPSO method
P_1(MW)	474.8066	447.4970	445.7020
P_2(MW)	178.6363	173.3221	174.0720
P_3(MW)	262.2089	263.0594	261.4100
P_4(MW)	134.2826	139.0594	133.0682
P_5(MW)	151.9039	165.4761	152.0234
P_6(MW)	74.1812	87.1280	107.5671
Total power output(MW)	1276.03	1276.01	1273.8
Ploss (MW)	13.0217	12.9584	10.8452
Total generation cost ($/h)	15,459	15,450	15,427.9

Table IV: Generatig Units Capacity and Coeficients (15-Unit).

unit	P_{min}(MW)	P_{max}(MW)	a	b	c	e	f
1	150	455	0.000299	10.1	671	100	0.084
2	150	455	0.000183	10.2	574	100	0.084
3	20	130	0.001126	8.8	374	100	0.084
4	20	130	0.001126	8.8	374	150	0.063
5	150	470	0.000205	10.4	461	120	0.077
6	135	460	0.000301	10.1	630	100	0.084
7	135	465	0.000364	9.8	548	200	0.042
8	60	300	0.000338	11.2	227	200	0.042

9	25	162	0.000807	11.2	173	200	0.042
10	25	160	0.001203	10.7	175	200	0.042
11	20	80	0.003586	10.2	186	200	0.042
12	20	80	0.005513	9.9	230	200	0.042
13	25	85	0.000371	13.1	225	300	0.035
14	15	55	0.001929	12.1	309	300	0.035
15	15	55	0.004447	12.4	323	300	0.035

Table V: Comparison between both Methods (50 Trails).

Method	Min.cost	Max.cost	Mean cost	Average cpu time
GA	33,113	33,337	33,228	49.31
PSO	32,858	33,331	33,039	26.76
MPSO	32,780	33,193	32,959	10.34

Table VI: Best Solution of 15-Unit System.

Unit power output	GA method	PSO method	MPSO method
P_1(MW)	415.3108	439.1162	455.0000
P_2(MW)	359.7206	407.9729	390.8112
P_3(MW)	104.4250	119.6324	112.7000
P_4(MW)	74.9853	129.9925	124.3310
P_5(MW)	380.2844	151.0681	356.6001
P_6(MW)	426.7902	459.9978	443.3111
P_7(MW)	341.3164	425.5601	433.1601
P_8(MW)	124.7876	98.5699	91.1211
P_9(MW)	133.1445	113.4936	66.0001
P_{10}(MW)	89.2567	101.1142	30.2511
P_{11}(MW)	60.0572	33.9116	24.1401
P_{12}(MW)	49.9998	79.9583	51.6001
P_{13}(MW)	38.7713	25.0042	45.0300
P_{14}(MW)	41.4140	41.4140	23.3000
P_{15}(MW)	22.6445	36.6140	15.0000
Total power output(MW)	2668.4	2662.4	2662.4
Ploss (MW)	38.2782	32.4306	32.4306
Total generation cost ($/h)	33,113	32,858	32,780

Conclusion

This paper present a new approach for solving ELD problem with valve-point effect and transmission line losses based on modified PSO (MPSO) algorithms. The suggested method includes a simple variation in the position equation which is simple concept, easy implementation, better effectiveness than previous method and applicable to large scale systems.

In the study cases the proposed method has been applied to economic dispatch problem with 6 generators and 15 generators. The results were compared with GA method and traditional PSO method and it show that the proposed method was indeed capable of obtaining higher quality solution efficiently in ELD problem.

Appendix

The B loss coefficients matrix of 15-unit system with a base capacity of 100 MVA is shown as follow: (see equation at the bottom of the page).

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