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Oscillation of Third–Order Nonlinear Delay Differential Equations with "Maxima"

R. Arul[1] and M. Mani[2]

Department of Mathematics, Kandaswami Kandar's College
Velur–638 182, Namakkal Dt. Tamil Nadu, India

Abstarct

In this paper, we study the oscillatory behaviour of a class of third-order nonlinear delay differential equations with "maxima"

$$(a(t)((b(t)(x(t)+p(t)x(\tau(t)))')')' + q(t)\max_{[\sigma(t),t]} x^{\gamma}(s) = 0, t \geq t_0 \geq 0 \quad (0.1)$$

where γ is the ratio of odd positive integers. We establish sufficient conditions which guarantees that every solution of equation (0.1) is either oscillatory or converges to zero. These results extend some known results in the literature without "maxima". Examples are given to illustrate the main results.
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Keywords and Phrases: Oscillation, nonlinear, third-order, delay, differential equations with "maxima".

1. Introduction

This paper concerned with the oscillation problem of third-order nonlinear delay differential equations with "maxima" of the from

$$(a(t)((b(t)(x(t)+p(t)x(\tau(t)))')')' + q(t)\max_{[\sigma(t),t]} x^{\gamma}(s) = 0, t \geq t_0 \geq 0 \quad (1.1)$$

where γ is the ratio of odd positive integers. Throughout this paper, it is always assumed that:

(C_1) γ is the odd positive integers;

(C_2) $a(t), b(t), p(t), q(t) \in C([t0, \infty), R), a(t) > 0, b(t) > 0, q(t) > 0 \; for \; all \; t \geq t0$;

(C_3) $\sigma(t) \in C([t_0, \infty), R), \sigma(t) < t, \sigma(t)$ is nondecreasing and $\lim_{t\to\infty} \sigma(t) = \infty$.

Further, we will consider the following two cases

$$\int_{t_0}^{\infty} \frac{dt}{a(t)} < \infty, \int_{t_0}^{\infty} \frac{dt}{b(t)} = \infty, \tag{1.2}$$

and

$$\int_{t_0}^{\infty} \frac{dt}{a(t)} < \infty, \int_{t_0}^{\infty} \frac{dt}{b(t)} < \infty. \tag{1.3}$$

By a solution of equation (1.1), we mean a continuous function $x(t) \in C([T_X, \infty)), T_x \geq t_0$, which satisfies (1.1) on $[T_x, \infty)$. We consider only those solutions $x(t)$ of (1.1) which satisfy sup $\{|x(t)|: t \geq T_x\} > 0 \; for \; all \; t \geq T_x$. A solution $x(t)$ of equation (1.1) is called oscillatory if it has arbitrary large zeros on $[T_x, \infty)$,otherwise it is called nonoscillatory. A solution $x(t)$ of equation (1.1) is said to be almost oscillatory if $x(t)$ is either oscillatory or $x(t) \to 0$ as $t \to \infty$.

In the last few years, the qualitative theory of differential equations with "maxima" received very little attention eventhough such equations often arise in the problem of automatic regulation of various real system, see for example [19, 21]. The oscillatory behaviour of solutions of differential equations with "maxima" are discussed in [9-13, 17, 18], and the references cited therein.

The are many results available in the literature on the oscillatory and asymptotic behaviour of third-order differential equations without "maxima" see for example [1, 2, 7, 14, 16, 21], and the references cited therein.

In [3], the present authors studied the oscillatory and asymptotic behaviour of solutions of equation (1.1) when $\int_{t_0}^{\infty} \frac{dt}{a(t)} < \infty, \int_{t_0}^{\infty} \frac{dt}{b(t)} = \infty$. Following this trend, in this paper we discussed the oscillatory behaviour of (1.1) when the condition (1.2) or (1.3) holds.

In section 2, we obtain criteria for the almost oscillation of all solutions of equation (1.1) and in section 3, we present some examples to illustrate the main results.

Remark 1.1. All functional inequalities consider in this paper assumed to hold eventually, that is they are satisfied for all t large enough.

Remark 1.2. Without loss of generality we can deal only with the positive solution of equation (1.1), since the proof for the negative solution is similar.

2. Oscillation Results

In the following, we will establish some oscillation criteria for equation (1.1).To simplify our notation, let us denote

$$z(t) = x(t) + p(t)x(\tau(t)), y(t) = -w(t) = -a(t)(b(t)z'(t))' \text{ and } B(t) = \int_t^\infty \frac{ds}{b(s)}$$

To obtain sufficient condition for the oscillation of solutions of equation (1.1), we need the following lemma.

Lemma 2.1. Let $x(t)$ be a positive solution of equation (1.1),and let the corresponding $z(t)$

Satisfy $z(t) > 0, z'(t) < 0, (b(t)z'(t))' > 0, (a(t)(b(t)z'(t)))' < 0$ for all $t \geq t_1 \geq t_0$. If

$$\int_{t_0}^\infty \frac{1}{b(v)} \int_v^\infty \frac{1}{a(u)} \int_u^\infty q(s)dsdudv = \infty,$$

then $\lim_{t\to\infty} x(t) = \lim_{t\to\infty} z(t) = 0$

The proof is similar to that of in [15] and hence the details are omitted .

Theorem 2.1. Assume that there exist numbers $\alpha \geq \gamma, \beta \geq \gamma$ and a function $\delta \in c\,([t_0,\infty), R)$ such that α, β are ratio of odd positive integers, $\delta(t)$ is non-decreasing and $\delta(t)$> t. If the condition (2.1) and for all sufficiently large $t_1 \geq t_0$ and for $t_2 \geq t_1$, the first-order delay differential equation

$$w'(t) + c_1^{\gamma-\alpha} q(t) \left(\max_{[\sigma(t),t]} (1-p(s))^\gamma\right) \left(\int_{t_2}^{\sigma(t)} \frac{\int_{t_1}^{s} \frac{du}{a(u)}}{b(s)}\right)^\alpha w^\alpha(\sigma(t)) = 0 \tag{2.2}$$

is oscillatory for all constants $c_1 > 0$, the first-differential equation

$$y'(t) - c_2^{\gamma-\beta} q(t) \left(\max_{[\sigma(t),t]} (1-p(s))^\gamma\right) \left(\int_{\delta(t)}^\infty \frac{ds}{a(s)}\right)^\beta \left(\int_{t_1}^{\sigma(t)} \frac{ds}{b(s)}\right)^\gamma y^\beta(\delta(t)) = 0 \tag{2.3}$$

is oscillatory for all constants $c_2 > 0$, then every solution of equation (1.1) is almost oscillatory.

Proof. Let $x(t)$ be a nonoscillatory solution of equation (1.1). Without loss of generality we

may assume that $x(t)$ is positive , since the proof for the case negative is similar. Then thecorresponding function z(t) satisfies the following three possible cases:

Case(I) $z(t) > 0, z'(t) > 0$, $(b(t)z'(t))' > 0$, $(a(t)(b(t)z'(t))')' < 0$;

Case(II) $z(t) > 0, z'(t) > 0$, $(b(t)z'(t))' < 0$, $(a(t)(b(t)z'(t))')' < 0$;
and

Case (III) $z(t) > 0, z'(t) < 0$, $(b(t)z'(t))' > 0$, $(a(t)(b(t)z'(t))')' < 0$,
for all $t \geq t_1 \geq t_0$. Assume that Case (I) holds .From the definition of $z(t)$ we have

$$z(t) = x(t) + p(t)x(\tau(t))$$

Or

$$x(t) = z(t) - p(t)x(\tau(t)) \geq (1 - p(t))z(t). \tag{2.4}$$

From the equation (1.1) and (2.4) we have

$$(a(t)(b(t)z'(t))')' + q(t)\left(\max_{[\sigma(t),t]}(1 - p(s))^\gamma\right)z^\gamma(t) \leq 0 . \tag{2.5}$$

Using $(a(t)(b(t)z'(t))')' < 0$, we obtain

$$b(t)z'(t) \geq \int_{t_1}^{t} \frac{(a(s)(b(s)z'(s))'}{a(s)}\, ds \;\geq\; (a(t)(b(t)z'(t))' \int_{t_1}^{t} \frac{ds}{a(s)}.$$

Dividing the last inequality by $b(t)$ and then integrating from t_2 to t , we get

$$z(t) \;\geq\; (a(t)(b(t)z'(t))' \int_{t_2}^{t} \left(\frac{\int_{t_1}^{s}\frac{du}{a(u)}}{b(s)}\right) ds. \tag{2.6}$$

From (2.5) and fact that $z'(t) > 0$ we see that there is constant $c_1 > 0$ such that

$$(a(t)(b(t)z'(t))')' + c_1^{\gamma-\alpha} q(t)\left(\max_{[\sigma(t),t]}(1 - p(s))^\gamma\right)z^\alpha(\sigma(t)) \leq 0. \tag{2.7}$$

Using (2.6) in (2.7) we obtain

$$w'(t) + c_1^{\gamma-\alpha} q(t)\left(\max_{[\sigma(t),t]}(1 - p(s))^\gamma\right)\left(\int_{t_2}^{\sigma(t)} \frac{\int_{t_1}^{\sigma(t)}\frac{du}{a(u)}}{b(s)}\, ds\right)^\alpha w^\alpha(\sigma(t)) = 0.$$

In view of theorem 2.1 of [20], the associated delay differential equation (2.2) also has a positive solution, which is a contradiction. Assume that Case (II) holds. Since $a(t)(b(t)z'(t))' \leq 0$, we see that $a(t)(b(t)z'(t))'$ is decreasing .Then we get

$$a(s)(b(s)z'(s))' \leq a(t)(b(t)z'(t))' \text{ , for } s \;\geq t \;\geq\; t_1.$$

Dividing the last inequality by $a(s)$ and then integrating the resulting inequality from t to l we have

$$b(l)z'(l) \leq b(t)z'(t) + a(t)(b(t)z'(t))' \int_t^l \frac{ds}{a(s)}.$$

Letting $t \to \infty$, we have

$$b(t)z'(t) \leq -a(t)(b(t)z'(t))' \int_t^l \frac{ds}{a(s)}. \tag{2.8}$$

Using conditions $z(t) > 0$ and $(b(t)z'(t))' \leq 0$, we have

$$z(t) \geq b(t)z'(t) \int_{t_1}^t \frac{ds}{b(s)}. \tag{2.9}$$

Thus,

$$\left(\frac{z(t)}{\int_{t_1}^t \frac{ds}{b(s)}}\right)' \leq 0. \tag{2.10}$$

Combining (2.8) and (2.9) ,we have

$$z(t) \geq -a(t)(b(t)z'(t))' \int_t^\infty \frac{ds}{a(s)} \int_{t_1}^t \frac{ds}{b(s)}. \tag{2.11}$$

On the other hand, we have by (2.10) and $\delta(t) \geq \sigma(t)$ that

$$z^\gamma(\sigma(t)) \geq \left(\frac{\int_{t_1}^{\sigma(t)} \frac{ds}{b(s)}}{\int_{t_1}^{\sigma(t)} \frac{ds}{b(s)}}\right)^\gamma z^\gamma(\delta(t)) = \left(\frac{\int_{t_1}^{\sigma(t)} \frac{ds}{b(s)}}{\int_{t_1}^{\sigma(t)} \frac{ds}{b(s)}}\right)^\gamma z^\gamma(\delta(t) z^{\gamma-\beta}\big(\delta(t)\big). \tag{2.12}$$

By (2.9), there exists a constant c_2 such that $(t) \leq c_2 \int_{t_1}^t \frac{ds}{b(s)}$. By (2.12), we get

$$z^\gamma(\sigma(t)) \geq c_2^{\gamma-\beta} \left(\frac{\int_{t_1}^{\sigma(t)} \frac{ds}{b(s)}}{\int_{t_1}^{\sigma(t)} \frac{ds}{b(s)}}\right)^\gamma z^\beta(\delta(t)) \left(\int_{t_1}^{\delta(t)} \frac{ds}{b(s)}\right)^{\gamma-\beta}$$

$$= c_2^{\gamma-\beta} \left(\int_{t_1}^{\delta(t)} \frac{ds}{b(s)}\right)^{-\beta} \left(\int_{t_1}^{\sigma(t)} \frac{ds}{b(s)}\right)^\gamma z^\beta\big(\sigma(t)\big). \tag{2.13}$$

Combining (2.11) and (2.13) ,we obtain

$$z^\gamma(\sigma(t)) \geq c_2^{\gamma-\beta} \left(\int_{t_1}^{\delta(t)} \frac{ds}{a(s)}\right)^{-\beta} \left(\int_{t_1}^{\sigma(t)} \frac{ds}{b(s)}\right)^\gamma (-w(\delta(t)))^\beta. \tag{2.14}$$

Using (2.14) and (2.5) ,we have

$$w'(t) + c_2^{\gamma-\beta} q(t)\left(\max_{[\sigma(t),t]}(1 - p(s))^{\gamma}\right)\left(\int_{\delta(t)}^{\infty}\frac{ds}{a(s)}\right)^{\beta}\left(\int_{t_1}^{\sigma(t)}\frac{ds}{b(s)}\right)^{\gamma}\left(-w(\delta(t))^{\beta}\right) \le 0.$$

or

$$y'(t) - c_2^{\gamma-\beta} q(t)\left(\max_{[\sigma(t),t]}(1-p(s))^{\gamma}\right)\left(\int_{\delta(t)}^{\infty}\frac{ds}{a(s)}\right)^{\beta}\left(\int_{t_1}^{\sigma(t)}\frac{ds}{b(s)}\right)^{\gamma} y^{\beta}(\delta(t)) \ge 0. \quad (2.15)$$

W(e deduce from Lemma 2.3 of [7], the associated advanced differential equation (2.3) also has a positive solution, which is a contradiction. Next assume that Case (III) holds. Then by Lemma 2.1, we have $\lim_{t\to\infty} x(t) = 0$. This completes the proof.

Corollary 2.1. Let γ =1. Assume that there exist a function $\delta \in c([t_0,\infty), R)$ such that $\delta(t)$

is nondecreasing $\delta(t) > t$. If (2.1)

and for all sufficiently large $t_1 \ge t_0$ and $t_2 > t_1$,

$$\liminf_{t\to\infty} \int_{\sigma(t)}^{t} q(s)\left(\max_{[\sigma(s),s]}(1-p(s)\right)\int_{t_2}^{\sigma(t)}\left(\frac{\int_{t_1}^{v}\frac{du}{a(u)}}{b(v)}\right)dv ds > \frac{1}{e}, \quad (2.16)$$

$$\liminf_{t\to\infty} \int_{t}^{\delta(t)} q(s)\left(\max_{[\sigma(s),s]}(1-p(s)\right)\int_{\delta(s)}^{\infty}\left(\frac{du}{a(u)}\right)\int_{t_1}^{\sigma(s)}\left(\frac{du}{b(u)}\right) > \frac{1}{e}, \quad (2.17)$$

Then the equation (1.1) is almost oscillatory.

Proof.Let $= \beta = \gamma = 1$, then we have

$$w'(t) + q(t)\left(\max_{[\sigma(t),t]}(1-p(s)\right)\int_{t_2}^{\sigma(t)}\left(\frac{\int_{t_1}^{s}\frac{du}{a(u)}}{b(s)}\right) w(\sigma(t)) = 0 \ and$$

$$y'(t) - q(t)\left(\max_{[\sigma(t),t]}(1-p(s)\right)\int_{\delta(t)}^{\infty}\left(\frac{ds}{a(s)}\right)\int_{t_1}^{\sigma(t)}\left(\frac{ds}{b(s)}\right) y(\delta(t)) = 0,$$

respectively. Applications of Theorem 2.1 with Lemma 2.1 of [16], we obtain the desired result. This completes the proof.

Corollary 2.2. Let γ< 1.Assume that there exist a function $\beta > 1 \ and \ a \ function \ \delta \in$

$c([t_0,\infty),R)$such that β is the ratio of odd positive integers $\delta(t)$ is nondecreasing and

$\delta(t) > t$. If (2.1) and for all sufficiently large $t_1 \geq t_0$ and $t_3 > t_2 > t_1$, and

$$\int_{t_3}^{\infty} q(t)\left(\max_{[\sigma(t),t]}(1-p(s))^{\gamma}\right)\left(\int_{t_2}^{\sigma(t)} \frac{\int_{t_1}^{s}\frac{du}{a(u)}}{b(s)}\, ds\right)^{\gamma} dt = \infty, \qquad (2.18)$$

and

$$\int_{t_2}^{\infty} q(t)\left(\max_{[\sigma(t),t]}(1-p(s))^{\gamma}\right)\left(\int_{\delta(t)}^{\infty}\frac{ds}{a(s)}\right)^{\beta}\left(\int_{t_1}^{\sigma(t)}\frac{ds}{b(s)}\right)^{\gamma} dt = \infty,$$

then every solution of equation (1.1) is almost oscillatory.

Proof. Let $\alpha = \gamma$. Applications of Theorem 2.1 with Lemma 2.1 of [17], we obtain the desired result. This completes proof.

Corollary 2.3. Let $\gamma = 1$ *and* $\sigma \in c'([t_0,\infty),R)$. Assume that there exist functions $\xi, \delta \in c'([t_0,\infty),R)$ such that $\xi(t) > t$, $\xi(t)$ is nondecreasing , $\sigma(t)$,$(\xi(\xi(t))) < t$, $\delta(t)$ is nondecreasing and $\delta(t) > t$. Suppose also there exists a function $\varphi \in c'([t_0,\infty),R)$ such that $\varphi'(t) > 0$, $\lim_{t\to\infty} \varphi(t) = \infty$. If (2.1) and

$$\lim_{t\to\infty}\sup \frac{\varphi'(\sigma(t))(\sigma'(t))}{\varphi'(t)} \leq 1,$$

$$\lim_{t\to\infty} inf \frac{\left(\frac{1}{b(t)}\int_{t}^{\xi(t)}\frac{1}{a(s_2)}\int_{s_2}^{\xi(s_2)} q(s_1)ds_1 ds_2\right)}{\varphi'(t)} e^{-\varphi(t)} > 0.$$

If for all sufficiently large $t_1 \geq t_0$ and $t_3 > t_2 > t_1$,

$$\int_{t_2}^{\infty} q(t)\left(\max_{[\sigma(t),t]}(1-p(s))^{\gamma}\right)\left(\int_{\delta(t)}^{\infty}\frac{ds}{a(s)}\right)^{\beta}\left(\int_{t_1}^{\sigma(t)}\frac{ds}{b(s)}\right)^{\gamma} dt = \infty,$$

then every solution of equation (1.1) is almost oscillatory.

Proof.Let $= \gamma$. Applications of Theorem 2.1 with Lemma 2.1 of [17], we obtain the desired result. This completes proof.

Theorem 2.2. Let all conditions of Theorem 2.1 hold with (1.2) replaced by (1.3). If

$$\int_{t_0}^{\infty}\frac{1}{b(v)}\int_{t_0}^{v}\frac{1}{a(u)}\int_{t_0}^{u} q(s)\left(\max_{[\sigma(t),t]}(1-p(s))^{\gamma}\right)B^{\gamma}(\sigma(s))dsdudv = \infty, \qquad (2.19)$$

then every solution of equation (1.1) is almost oscillatory.

Proof. Suppose $x(t)$ be a nonoscillatory solution of equation (1.1).

Without loss of generality that $x(t)$ is positive.Then there exists four possible Cases(I),(II),(III) (as those of Theorem 2..1), andCase(IV)

$$z(t) > 0, z'(t) < 0, (b(t)z'(t))' < 0, (a(t)(b(t)z'(t))')' < 0,$$

for $t \geq t_1$, where $t_1 \geq t_0$ is large enough. From the proof of Theorem 2.1, we can eliminate Cases (I),(II) and (III). Consider now the Case (IV). Since , $(b(t)z'(t))' \leq 0$, we get

$$z'(s) \geq \frac{b(t)z'(t)}{b(s)} \text{ for } s \geq t.$$

Integrating this inequality from t to ℓ and letting $\ell \to \infty$ implies that

$$z(t) \geq -B(t)b(t)z'(t) \geq LB(t)$$

for some constants $L > 0$.From equation(2.3) we have,

$$(a(t)(b(t)z'(t))')' + L^\gamma \int_{t_1}^{u} q(t)\left(\max_{[\sigma(t),t]}(1-p(s))^\gamma\right) B^\gamma(\sigma(s))ds \leq 0$$

Integrating again, we have

$$z'(t) \geq L^\gamma \int_{t_1}^{t} \frac{1}{b(v)} \int_{t_1}^{v} \frac{1}{a(u)} \int_{t_1}^{u} q(s)\left(\max_{[\sigma(t),t]}(1-p(s))^\gamma\right) B^\gamma(\sigma(s))dsdudv + z(t),$$

which contradicts (2.19).This completes the proof.

3. Examples

In this section we provide some examples to illustrate the main results.

Examples 3.1. Consider the neutral differential equation

$$\left(e^t\left((x(t)+\tfrac{1}{2}x(t-1)\right)''\right)' + \sqrt{2}\,e^t \max_{\left[t-\frac{15}{2},t\right]} x(s) = 0, t \geq 1, \quad (3.1)$$

By taking $\delta(t) = t + 1$, one can easily verify that all conditions of Corollary 2.1 are satisfied, hence every solution of equation (3.1) is either oscillatory or tends to zero monotonically.

Example 3.2. Consider the neutral differential equation

$$\left(t^2\left(\left(x(t)+\frac{1}{3}x(t-2)\right)''\right)'\right)+t\max_{\left[\frac{t}{8},t\right]}x^{\frac{1}{3}}(s)=0, t\geq 1. \quad (3.2)$$

By taking $\beta = 9/7$ and $\delta = 2t$, it is easy to see that all conditions of Corollary 2.2 are satisfied and hence every solution of equation (3.2) is either oscillatory or tends to zero monotonically.

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