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Construction of Continuous Sampling Plan-3 Indexed through AOQ_{cc}

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Abstract

The continuous sampling plans are designed for situations in which production is continuous and lotting is not a natural aspect of the manufacturing situation. In this paper a procedure for constructing Continuous Sampling Plan-3 (CSP-3) indexed through AOQ_{cc}. This plan may safeguard the interests of both producer as well as consumer by properly choosing a right combination using the gain parameter λ. A table is also provided for the easy selection of the plan when $\lambda = 0.2$, $\lambda = 0.4$ and MAPD = 0.01.

Keywords: Operating Characteristic curve, Average Outgoing Quality Limit, Maximum Allowable Percent Defective, Maximum Allowable Average Outgoing Quality, Continuous Sampling Plan, Convex combination.

AMS (2000) Subject Classification: 62P30.

Introduction

In the literature, the average outgoing quality limit (AOQL) is definied as the worst average quality that the consumer will receive in the long run, when the defective items are replaced by non-defective items. The proportion defective corresponding to the inflection point of the OC curve is interpreted as the maximum allowable percent defective (MAPD). Sampling plans indexed through p* (MAPD) which is the quality level corresponding to the inflection point of the Operating Characteristic (OC) curve has been explained by Mandelson (1962), Mayer (1967) and further studied by Soundararajan (1975). The construction of sampling plans based on AOQL is largely consumer oriented and the MAAOQ is the average outgoing quality at the inflection point is a producer oriented, which is the average outgoing quality at MAPD. The advantage of using MAAOQ for designing a sampling plan instead of AOQL is that it reduces the sample size to be inspected which indirectly reduces the total cost. The use of MAAOQ for designing sampling plans was justified by Suresh and Ramkumar (1996). Radhakrishnan (2002) studied various sampling plans including continuous sampling plans indexed through MAPD and MAAOQ. Sampathkumar (2007) constructed mixed sampling plan indexed through AOQL, MAPD, MAAOQ and emphasized the superiority of MAAOQ. Radhakrishnan and Mallika (2008, 2009a, 2009b, 2009c, 2010a, 2010b) constructed single, Double, ChSP-1(Chain Sampling Plan-1) and ChSP-2(Chain Sampling Plan-2)and Link sampling plans indexed through AOQ_{cc}. In this paper a procedure for the construction of CSP-3 plan indexed through AOQ_{cc} which is the convex combination of AOQL and MAAOQ with the gain parameter λ. This plan may safeguard the interests of both producer as well as consumer by choosing a right combination using the gain parameter λ ($0<\lambda<1$).

Glossary of Symbols

The symbols used in this paper are as follows:

N	Lot size
n	sample size
i	clearance number
f	sampling frequency
λ	gain parameter
P	submitted lot quality of lot or process
P*	Maximum allowable percent defective
k	The number of sample units to be found conforming in order that the inspection will continue to be in the sampling mode.
$P_a(p)$	Probability of acceptance for a given quality p
AOQ	Average Outgoing Quality
AOQ_{cc}	Convex combination of AOQL and MAAOQ
AOQL	Average Outgoing Quality Limit
MAAOQ	Maximum allowable average outgoing quality

Definition of AOQ_{cc}

AOQ_{cc} is the convex combination of AOQL and MAAOQ with gain parameter λ suggested by Radhakrishnan and Mallika (2009) as $AOQ_{cc} = \lambda$ AOQL+ (1- λ) MAAOQ

Operating Procedure of CSP-3 Plan

The operating procedure of Continuous Sampling Plan -3 is as follows

Step l: Specify f and i.

Step 2: Begin 100 percent inspection.

Step 3: After i units in succession have been found without a defective, start sampling inspection.

Step 4: Randomly inspect a fraction f of the units.

Step 5: When a defective is found, inspect the next 4 units, if an additional defective is found revert to 100 percent inspection otherwise, continue sampling for k successive sample units. If no defectives is found in k, continue sampling by selecting a fraction f of the units. If a defective is found in the k samples revert to 100 percent inspection immediately.

Operating characteristic function

The OC function for CSP-3 Plan given by Stephens (1979) for k = i is

$$Pa\,(P) = q^{i}\,[1+q^{4}\,(1-q^{i})] / [f\,[1-q^{i}-q^{i+4}(1-q^{i})] + q^{i}\,[1+q^{4}\,(1-q^{i})] + 4pfq^{i}]$$

Construction of CSP-3 Plan indexed through AOQ_{cc}

The general procedure for designing CSP-3Plan indexed through a parameter AOQ_{cc} which is a convex combination of AOQL and MAAOQ is given below:

Step 1: Determine MAPD, MAAOQ and AOQL for CSP-3 for various values of n , i and find R_1 = MAAOQ/MAPD and R_2= AOQL/MAPD.

Step 2: Find $AOQ_{cc} = \lambda AOQL + (1-\lambda)\,MAAOQ$ for various values of λ and find $R_3 = AOQ_{cc}/MAPD$.

Step 3: The results of Step 1 and Step 2 for MAPD = 0.01, $\lambda = 0.2$ and $\lambda = 0.4$ using Excel Packages are presented in Table 1.

Selection of the plan

Table 1 is used to construct the plan when the MAPD and AOQ_{cc} are specified. One can find the ratio $R_3 = AOQ_{cc}/MAPD$ and locate the value in Table1 under the column R_3 (for fixed values of MAPD = 0.01, $\lambda = 0.2$ and $\lambda = 0.4$) and the corresponding values of n and i are noted.

Table 1: Characteristics of Continuous Sampling Plan-3 for MAPD = 0.01.

								λ = 0.2				λ = 0.4			
R_1	MAAOQ	i	n	R_2	AOQL	i	n	AOQ_{cc}	i	n	R_3	AOQ_{cc}	i	n	R_3
1.99	0.005025	520	95	1.63	0.006135	591	192	0.005247	341	16	1.91	0.005469	397	28	1.83
1.98	0.005051	480	64	1.64	0.006098	577	168	0.00526	348	17	1.9	0.005469	397	28	1.83
1.97	0.005076	428	38	1.65	0.006061	565	149	0.005273	358	19	1.9	0.00547	397	28	1.83
1.96	0.005102	404	30	1.66	0.006024	554	133	0.005286	358	19	1.89	0.005471	397	28	1.83
1.95	0.005128	389	26	1.67	0.005988	543	119	0.0053	363	20	1.89	0.005472	397	28	1.83
1.94	0.005155	358	19	1.68	0.005952	531	106	0.005314	363	20	1.88	0.005474	397	28	1.83
1.93	0.005181	335	15	1.69	0.005917	520	95	0.005329	368	21	1.88	0.005476	397	28	1.83
1.92	0.005208	333	15	1.7	0.005882	510	86	0.005343	368	21	1.87	0.005478	397	28	1.83
1.91	0.005236	326	14	1.71	0.005848	500	78	0.005358	374	22	1.87	0.005481	404	30	1.82
1.9	0.005263	318	13	1.72	0.005814	491	71	0.005373	374	22	1.86	0.005483	404	30	1.82
1.89	0.005291	310	12	1.73	0.00578	480	64	0.005389	374	22	1.86	0.005487	404	30	1.82
1.88	0.005319	300	11	1.74	0.005747	471	58	0.005405	382	24	1.85	0.00549	404	30	1.82
1.87	0.005348	290	10	1.75	0.005714	462	53	0.005421	382	24	1.84	0.005494	404	30	1.82
1.86	0.005376	279	9	1.76	0.005682	454	49	0.005437	389	26	1.84	0.005499	404	30	1.82
1.85	0.005405	268	8	1.77	0.00565	445	45	0.005454	389	26	1.83	0.005503	252	7	1.82
1.83	0.005464	240	6	1.78	0.005618	436	41	0.005495	404	30	1.82	0.005526	411	32	1.81
1.81	0.005525	252	7	1.79	0.005587	428	38	0.005537	411	32	1.81	0.00555	252	7	1.8
1.8	0.005556	252	7	1.8	0.005556	419	35	0.005556	411	32	1.8	0.005556	419	35	1.8
1.75	0.005714	201	4	1.81	0.005525	411	32	0.005676	445	45	1.76	0.005639	436	41	1.77
1.69	0.005917	172	3	1.82	0.005495	404	30	0.005833	491	71	1.71	0.005748	471	58	1.74
1.61	0.006211	160	3	1.83	0.005464	397	28	0.006062	565	149	1.65	0.005913	520	95	1.69

Example 1:

For a specified MAAOQ = 0.005405 and MAPD=0.01 compute the ratio R_1=MAPD/MAAOQ=1.85 which is associated with i = 268, n = 8 in Table 1 and f =1/n =0.125. Hence the CSP-3 Plan for specified MAAOQ = 0.005405 is i=268, k = 268 and f=0.125.

For a specified AOQL = 0.00565 and MAPD=0.01 compute the ratio R_2=MAPD/AOQL=1.77 which is associated with i = 445, n = 45 in Table 1 and f =1/n =0.022. Hence the CSP-3 Plan for specified AOQL = 0.00565 is i=445,k = 445 and f=0.022.

For a specified value of MAAOQ = 0.005405 , AOQL=0.00565 and λ=0.2 AOQ_{cc}=0.005454. Compute the ratio R_3=MAPD/AOQ_{cc}=1.83 which is associated with i = 389, n = 26 in Table 1 and f =1/n =0.038. Hence the CSP-3 Plan for specified AOQ_{cc} = 0.005454 and λ=0.2 is i=389, k = 389 and f=0.038. The OC curves for the Example 1 is presented in Figure 1

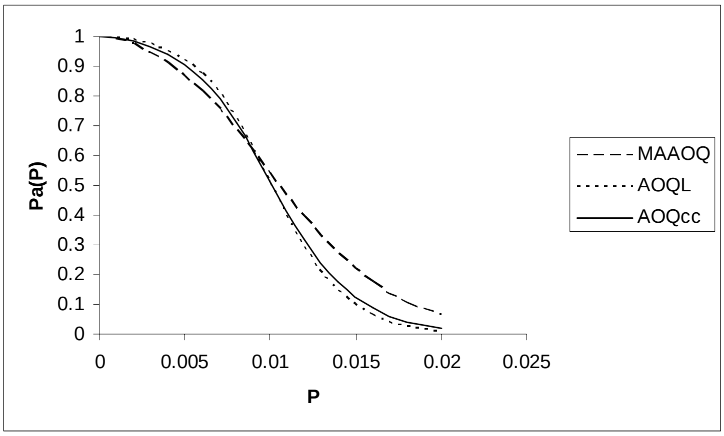

Figure 1: OC curves for i=445, f=0.022 (AOQL).; i=268, f=0.125 (MAAOQ); i=389, f=0.038 (AOQ_{cc}).

Explanation

In a Chalk manufacturing company, if the producer fixes the quality level MAAOQ as 0.005405 (5405 defective chalks out of 1000000) and the consumer fixes the quality level AOQL as 0.00565(565 defective chalks out of 100000) then a compromising quality level AOQ_{cc} can be suggested as 0.005454 (5454 defective chalks out of 1000000). Inspect chalks in the order of their production. If 389 consecutive chalks found conforming, then inspect at the rate of 0.038 (f =1/n) chalks selected at random. If any chalk found defective, inspect the next 4 chalks, if an additional defective is found revert to 100 percent inspection otherwise, continue sampling for 389 successive chalks. If no defective is found continue sampling by selecting a fraction of 0.038 chalks. If a defective is found in the 389 samples revert to 100 percent inspection immediately. The AOQ curves for the Example 1 are presented in Figure 2.

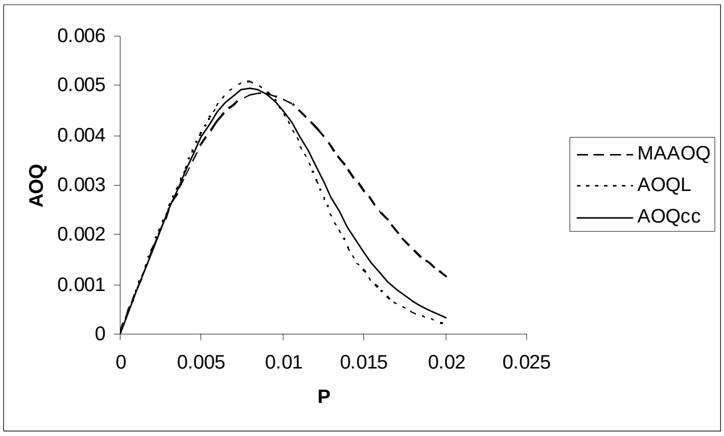

Figure 2: AOQ curves for i=445, f=0.022 (AOQL).; i=268, f=0.125 (MAAOQ); i=389, f=0.038 (AOQ_{cc}).

Example 2

For a specified MAAOQ = 0.005236 and MAPD=0.01 compute the ratio R_1=MAPD/MAAOQ=1.91 which is associated with i = 326, n = 14 in Table 1 and f =1/n =0.071. Hence the CSP-3 Plan for specified MAAOQ = 0.005236 is i=326, k = 326 and f=0.071.

For a specified AOQL = 0.005848 and MAPD=0.01 compute the ratio R_2=MAPD/AOQL=1.71 which is associated with i = 500, n = 78 in Table 1 and f =1/n = 0.013. Hence the CSP-3 Plan for specified AOQL = 0.005848 is i=500, k = 500 and f=0.013.

For a specified value of MAAOQ = 0.005236, AOQL=0.005848 and λ=0.4 AOQcc=0.005481. Compute the ratio R_3=MAPD/ AOQ_{cc} =1.82 which is associated with i = 404, n = 30 in Table 1 and f =1/n =0.033. Hence the CSP-3 Plan for specified AOQ_{cc} = 0.005481 and λ=0.4 is i=404, k = 404 and f=0.033. The OC curves for the Example 2 is presented in Figure 3

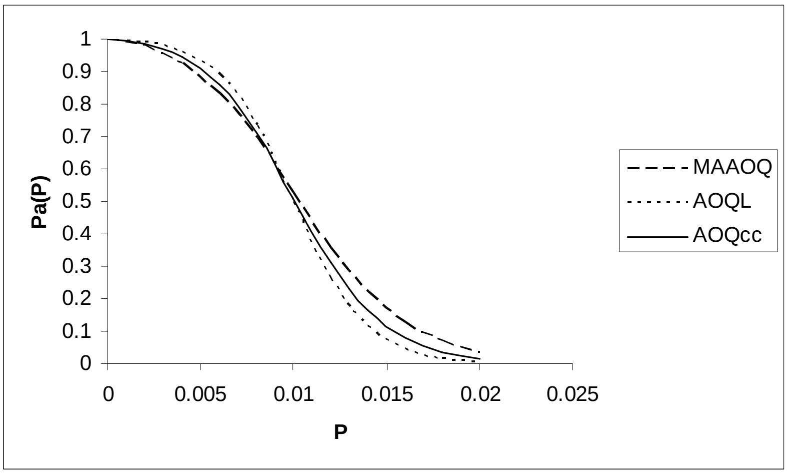

Figure 3: OC curves for i=500, f=0.013 (AOQL); i=326, f=0.071 (MAAOQ); i=404, f=0.033(AOQ_{cc}).

Explanation

In a pen manufacturing company, if the producer fixes the quality level MAAOQ as 0.005236 (5236 defective pens out of 1000000) and the consumer fixes the quality level AOQL as 0.005848 (5848 defective pens out of 1000000) then the compromising quality level AOQ_{cc} can be suggested as 0.005481 (5481 defective pens out of 1000000). Inspect pens in the order of their production. If 404 consecutive pens found conforming, then inspect at the rate of 0.033 (f =1/n) pens selected at random. If any pen found defective, inspect the next 4 pens, if an additional defective is found revert to 100 percent inspection otherwise, continue sampling for 404 successive pens. If no defective is found continue sampling by selecting a fraction of 0.033 pens. If a defective is found in the 404 samples revert to 100 percent inspection immediately. The AOQ curves for the Example 2 are presented in Figure 4.

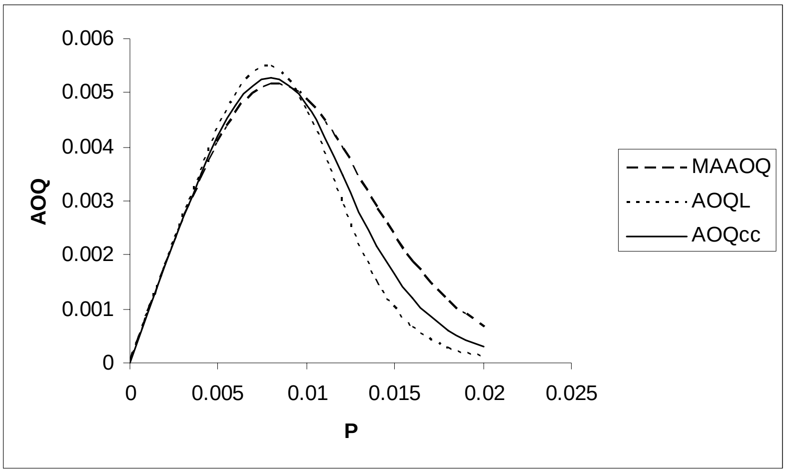

Figure 4: AOQ curves for i=500, f=0.013 (AOQL); i=326, f=0.071 (MAAOQ); i=404, f=0.033(AOQ_{cc}).

Conclusion

In this paper a procedure for the construction and selection of CSP-3 Plan indexed through gain parameter λ, which is a convex combination of AOQL and MAAOQ is stated. A table also constructed for the easy selection of the plans when the indexing parameters and gain parameter are known. Readymade table is also provided in this paper for λ =0.2, λ =0.4 for MAPD of 0.01 to take quick decisions on the nature of the sampling plan when the quality level of the producer and consumer are known. Similar tables can also be generated for various values of λ and MAPD based on the choice of the consumer and producer.

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