Abstract

The continuous sampling plans are designed for situations in which production is continuous and lotting is not a natural aspect of the manufacturing situation. In this paper a procedure for constructing Continuous Sampling Plan-3 (CSP-3) indexed through AOQ_{cc}. This plan may safeguard the interests of both producer as well as consumer by properly choosing a right combination using the gain parameter \( \lambda \). A table is also provided for the easy selection of the plan when \( \lambda = 0.2 \), \( \lambda = 0.4 \) and MAPD = 0.01.

Keywords: Operating Characteristic curve, Average Outgoing Quality Limit, Maximum Allowable Percent Defective, Maximum Allowable Average Outgoing Quality, Continuous Sampling Plan, Convex combination.


Introduction

In the literature, the average outgoing quality limit (AOQL) is defined as the worst average quality that the consumer will receive in the long run, when the defective items are replaced by non-defective items. The proportion defective corresponding to the inflection point of the OC curve is interpreted as the maximum allowable percent defective (MAPD). Sampling plans indexed through p* (MAPD) which is the quality level corresponding to the inflection point of the Operating Characteristic (OC) curve has been explained by Mandelson (1962), Mayer (1967) and further studied by Soundararajan (1975). The construction of sampling plans based on AOQL is largely consumer oriented and the MAAOQ is the average outgoing quality at the inflection point.
point is a producer oriented, which is the average outgoing quality at MAPD. The advantage of using MAAOQ for designing a sampling plan instead of AOQL is that it reduces the sample size to be inspected which indirectly reduces the total cost. The use of MAAOQ for designing sampling plans was justified by Suresh and Ramkumar (1996). Radhakrishnan (2002) studied various sampling plans including continuous sampling plans indexed through MAPD and MAAOQ. Sampathkumar (2007) constructed mixed sampling plan indexed through AOQL, MAPD, MAAOQ and emphasized the superiority of MAAOQ. Radhakrishnan and Mallika (2008, 2009a, 2009b, 2009c, 2010a, 2010b) constructed single, Double, ChSP-1(Chain Sampling Plan-1) and ChSP-2(Chain Sampling Plan-2) and Link sampling plans indexed through AOQcc. In this paper a procedure for the construction of CSP-3 plan indexed through AOQcc which is the convex combination of AOQL and MAAOQ with the gain parameter \( \lambda \). This plan may safeguard the interests of both producer as well as consumer by choosing a right combination using the gain parameter \( \lambda \) (0<\( \lambda \)<1).

**Glossary of Symbols**
The symbols used in this paper are as follows:

- **N** Lot size
- **n** sample size
- **i** clearance number
- **f** sampling frequency
- **\( \lambda \)** gain parameter
- **P** submitted lot quality of lot or process
- **P*\)** Maximum allowable percent defective
- **k** The number of sample units to be found conforming in order that the inspection will continue to be in the sampling mode.
- **P_a(p)** Probability of acceptance for a given quality \( p \)
- **AOQ** Average Outgoing Quality
- **AOQcc** Convex combination of AOQL and MAAOQ
- **AOQL** Average Outgoing Quality Limit
- **MAAOQ** Maximum allowable average outgoing quality

**Definition of AOQcc**
AOQcc is the convex combination of AOQL and MAAOQ with gain parameter \( \lambda \) suggested by Radhakrishnan and Mallika (2009) as \( AOQ_{cc} = \lambda \cdot AOQL + (1- \lambda) \cdot MAAOQ \)
**Operating Procedure of CSP-3 Plan**
The operating procedure of Continuous Sampling Plan -3 is as follows

**Step 1:** Specify $f$ and $i$.

**Step 2:** Begin 100 percent inspection.

**Step 3:** After $i$ units in succession have been found without a defective, start sampling inspection.

**Step 4:** Randomly inspect a fraction $f$ of the units.

**Step 5:** When a defective is found, inspect the next 4 units, if an additional defective is found revert to 100 percent inspection otherwise, continue sampling for $k$ successive sample units. If no defectives is found in $k$, continue sampling by selecting a fraction $f$ of the units. If a defective is found in the $k$ samples revert to 100 percent inspection immediately.

**Operating characteristic function**
The OC function for CSP-3 Plan given by Stephens (1979) for $k = i$ is

$$P_a (P) = q^i [1+q^4 (1-q^i)]/ [f [1-q^i-q^i+4(1-q^i)] +q^i [1+q^4 (1-q^i)] +4pfq^i]$$

**Construction of CSP-3 Plan indexed through AOQcc**
The general procedure for designing CSP-3 Plan indexed through a parameter AOQcc which is a convex combination of AOQL and MAAOQ is given below:

**Step 1:** Determine MAPD, MAAOQ and AOQL for CSP-3 for various values of $n$, $i$ and find $R_1 = \text{MAAOQ}/\text{MAPD}$ and $R_2 = \text{AOQL}/\text{MAPD}$.

**Step 2:** Find $\text{AOQcc} = \lambda \text{AOQL} + (1- \lambda) \text{MAAOQ}$ for various values of $\lambda$ and find $R_3 = \text{AOQcc}/\text{MAPD}$.

**Step 3:** The results of Step 1 and Step 2 for MAPD = 0.01, $\lambda = 0.2$ and $\lambda = 0.4$ using Excel Packages are presented in Table 1.

**Selection of the plan**
Table 1 is used to construct the plan when the MAPD and AOQcc are specified. One can find the ratio $R_3 = \text{AOQcc}/\text{MAPD}$ and locate the value in Table 1 under the column $R_3$ (for fixed values of MAPD = 0.01, $\lambda = 0.2$ and $\lambda = 0.4$) and the corresponding values of $n$ and $i$ are noted.
Table 1: Characteristics of Continuous Sampling Plan-3 for MAPD = 0.01.

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Example 1:
For a specified MAAOQ = 0.005405 and MAPD=0.01 compute the ratio \( R_1 = \frac{\text{MAPD}}{\text{MAAOQ}} = 1.85 \) which is associated with \( i = 268, n = 8 \) in Table 1 and \( f = \frac{1}{n} = 0.125 \). Hence the CSP-3 Plan for specified MAAOQ = 0.005405 is \( i=268, k = 268 \) and \( f=0.125 \).

For a specified AOQL = 0.00565 and MAPD=0.01 compute the ratio \( R_2 = \frac{\text{MAPD}}{\text{AOQL}} = 1.77 \) which is associated with \( i = 445, n = 45 \) in Table 1 and \( f = \frac{1}{n} = 0.022 \). Hence the CSP-3 Plan for specified AOQL = 0.00565 is \( i=445, k = 445 \) and \( f=0.022 \).

For a specified value of MAAOQ = 0.005405 , AOQL=0.00565 and \( \lambda = 0.2 \) \( \text{AOQ}_{cc} = 0.005454 \). Compute the ratio \( R_3 = \frac{\text{MAPD}}{\text{AOQ}_{cc}} = 1.83 \) which is associated with \( i = 389, n = 26 \) in Table 1 and \( f = \frac{1}{n} = 0.038 \). Hence the CSP-3 Plan for specified \( \text{AOQ}_{cc} = 0.005454 \) and \( \lambda = 0.2 \) is \( i=389, k = 389 \) and \( f=0.038 \). The OC curves for the Example 1 is presented in Figure 1.
Figure 1: OC curves for \(i=445, f=0.022\) (AOQL); \(i=268, f=0.125\) (MAAOQ); \(i=389, f=0.038\) (AOQcc).

**Explanation**

In a Chalk manufacturing company, if the producer fixes the quality level MAAOQ as 0.005405 (5405 defective chalks out of 1000000) and the consumer fixes the quality level AOQL as 0.00565 (565 defective chalks out of 1000000) then a compromising quality level AOQcc can be suggested as 0.005454 (5454 defective chalks out of 1000000). Inspect chalks in the order of their production. If 389 consecutive chalks found conforming, then inspect at the rate of 0.038 (\(f=1/n\)) chalks selected at random. If any chalk found defective, inspect the next 4 chalks, if an additional defective is found revert to 100 percent inspection otherwise, continue sampling for 389 successive chalks. If no defective is found continue sampling by selecting a fraction of 0.038 chalks. If a defective is found in the 389 samples revert to 100 percent inspection immediately. The AOQ curves for the Example 1 are presented in Figure 2.

Figure 2: AOQ curves for \(i=445, f=0.022\) (AOQL); \(i=268, f=0.125\) (MAAOQ); \(i=389, f=0.038\) (AOQcc).
Example 2
For a specified MAAOQ = 0.005236 and MAPD=0.01 compute the ratio $R_1=\text{MAPD/MAAOQ}=1.91$ which is associated with $i = 326$, $n = 14$ in Table 1 and $f =1/n =0.071$. Hence the CSP-3 Plan for specified MAAOQ = 0.005236 is $i=326$, $k = 326$ and $f=0.071$.

For a specified AOQL = 0.005848 and MAPD=0.01 compute the ratio $R_2=\text{MAPD/AOQL}=1.71$ which is associated with $i = 500$, $n = 78$ in Table 1 and $f =1/n = 0.013$. Hence the CSP-3 Plan for specified AOQL = 0.005848 is $i=500$, $k = 500$ and $f=0.013$.

For a specified value of MAAOQ = 0.005236, AOQL=0.005848 and $\lambda=0.4$ AOQcc=0.005481. Compute the ratio $R_3=\text{MAPD/AOQcc}=1.82$ which is associated with $i = 404$, $n = 30$ in Table 1 and $f =1/n =0.033$. Hence the CSP-3 Plan for specified AOQcc = 0.005481 and $\lambda=0.4$ is $i=404$, $k = 404$ and $f=0.033$. The OC curves for the Example 2 is presented in Figure 3.

![OC curves for i=500, f=0.013 (AOQL); i=326, f=0.071 (MAAOQ); i=404, f=0.033(AOQcc).](image)

**Figure 3:** OC curves for $i=500$, $f=0.013$ (AOQL); $i=326$, $f=0.071$ (MAAOQ); $i=404$, $f=0.033$(AOQcc).

**Explanation**
In a pen manufacturing company, if the producer fixes the quality level MAAOQ as 0.005236 (5236 defective pens out of 1000000) and the consumer fixes the quality level AOQL as 0.005848 (5848 defective pens out of 1000000) then the compromising quality level AOQcc can be suggested as 0.005481 (5481 defective pens out of 1000000). Inspect pens in the order of their production. If 404 consecutive pens found conforming, then inspect at the rate of 0.033 ($f =1/n$) pens selected at random. If any pen found defective, inspect the next 4 pens, if an additional defective is found revert to 100 percent inspection otherwise, continue sampling for 404 successive pens. If no defective is found continue sampling by selecting a fraction of 0.033 pens. If a defective is found in the 404 samples revert to 100 percent inspection immediately. The AOQ curves for the Example 2 are presented in Figure 4.
Construction of Continuous Sampling Plan-3 Indexed

Conclusion
In this paper a procedure for the construction and selection of CSP-3 Plan indexed through gain parameter $\lambda$, which is a convex combination of AOQL and MAAOQ is stated. A table also constructed for the easy selection of the plans when the indexing parameters and gain parameter are known. Readymade table is also provided in this paper for $\lambda = 0.2$, $\lambda = 0.4$ for MAPD of 0.01 to take quick decisions on the nature of the sampling plan when the quality level of the producer and consumer are known. Similar tables can also be generated for various values of $\lambda$ and MAPD based on the choice of the consumer and producer.

References


