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Edge Graceful Labeling of Some Trees

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Abstract

Lo [6] introduced the notion of edge graceful graphs : A graph G with q edges and p vertices is said to be edge graceful if there exists a bijection f from the edge set to the set {1, 2, …..q} so that the induced mapping f^+ from the vertex set to the set {0, 1, 2,…..p-1} given by f^+ (x) = ∑ {f (xy) / xy ∈ E (G)} (mod p) is a bijection. In this paper, we investigate edge graceful labeling of bistar graph.

Introduction

All graphs in this paper are finite, simple and undirected. Terms not defined here are used in the sense of Harary [1]. The symbols V(G) and E(G) will denote the vertex set and edge set of a graph G. The cardinality of the edge is called the size of G. A graph with p vertices and q edges is called a (p, q) graph.

A labeling (or valuations) [2] of a graph G is an assignment f of labels to the vertices of G that induces for each edge xy, a label depending on the vertex labels f (x) and f (y).Let G be a graph with q edges. Let f be an injection from the vertices of G to set {0, 1, 2,…….q} is called a graceful labeling of G if when we assign to each edge xy the label |f(x) – f (y)| the resulting edge labels are distinct.

Lo [6] introduced the notion of edge graceful graphs. A graph G with q edges and p vertices is said to be edge graceful if there exists a bijection f from the edge set to the set {1, 2, ….q} so that the induced mapping the vertex set to the set {0, 1, 2…….p-1} given by f^+(x) = ∑ {f (xy/xy ∈ E (G)} (mod p) is a bijection.

The necessary condition for a graph to be edge graceful is q (q+1) ≡ 0 or p/2 (mod p). With this condition one can vertify that even cycles, and paths of even length are not edge graceful. But whether trees of odd order are edge graceful is still open.

On attempting to move towards this conjecture, in this paper we checked it for a special type of trees called bistar graph. Here again the odd order trees turn to be edge graceful graph. Thus it confirms that the conjecture is moving towards affirmative.

Main Results

Definition: 2.1

The graph $B_{n, m}$ is defined as the graph obtained by joining the center u of the star $K_{1, n}$ and the center of another star $K_{1, m}$ to a new vertex w.

Observation: 2.2

The number of vertices in the graph $B_{n, m}$ is p = n+m+3 and the number of edges q = n+m+2.

Theorem: 2.3

The bistar graph $B_{n, m}$ for n≠m where n, m even is an edge graceful graph.

Proof

Let $\{w, v, v^1, v_1, v_2 \ldots\ldots v_n, v_{n+1} \ldots\ldots\ldots v_{n+m}\}$ be the vertices of bistar and edges e_i (see fig 1) are defined as follows

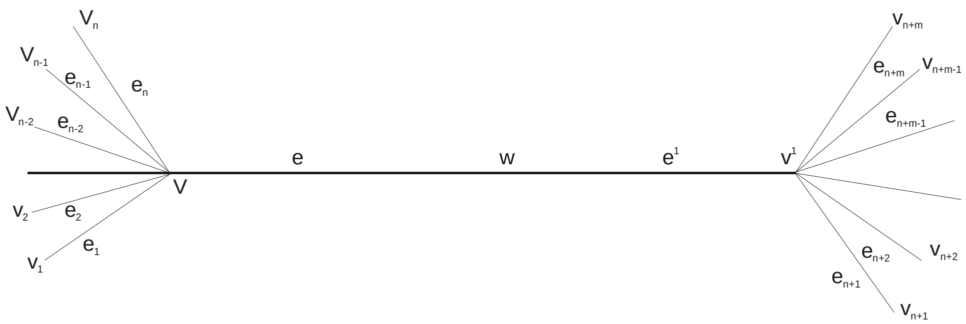

Figure1: $B_{n, m}$ with ordinary labeling.

e = (w, v);$e^1 = (w, v^1)$; $e_i = (v, v_i)$ for i = 1, 2, 3……n and $e_i = (v^1, v)$ for i = n+1, n+2, ……n + m.

Consider the Diophantine equation $x_1+x_2 = p$. The solutions are of the form (t, p-t) where $1 \leq t \leq q/2$. There will be $\frac{q}{2}$ pair of solutions.

With these pairs label the edges of $K_{1, n}$ and $K_{1, m}$ by the coordinates of the pair in any order so that adjacent edges receives the coordinates of the pairs. Also, label $f^+(w) = 0$;$f^+(v^1) = 1$; $f^+(v) = q$.

Now the pendant vertices will have labels of the edges with which they are incident and they are distinct. Hence the graph $B_{n, m}$ & n ≠ m n, m even is edge graceful.

Illustration

Consider the bistar graph $B_{4, 2}$. Here p = 9

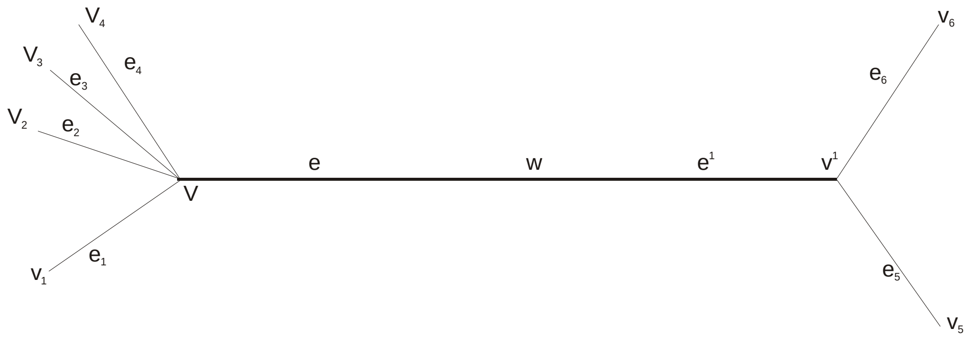

Figure3: Edge graceful labeling of $B_{4,2}$.

Consider the Diophantine equation $x_1+x_2 = p = 9$ The pair of solutions are : (t, p-t) where $t \in (1, q/2)$: (1, 8), (2, 7), (3, 6), (4, 5). We label the edges e & e^1 as follows: f (e) = q = 8, $f(e^1) = 1$. Pair of solutions labels the edges of $K_{1,4}$ & $K_{1,2}$ by the coordinate of the pairs. The edge graceful labeling of bistar $B_{4,2}$ is given in fig 3.

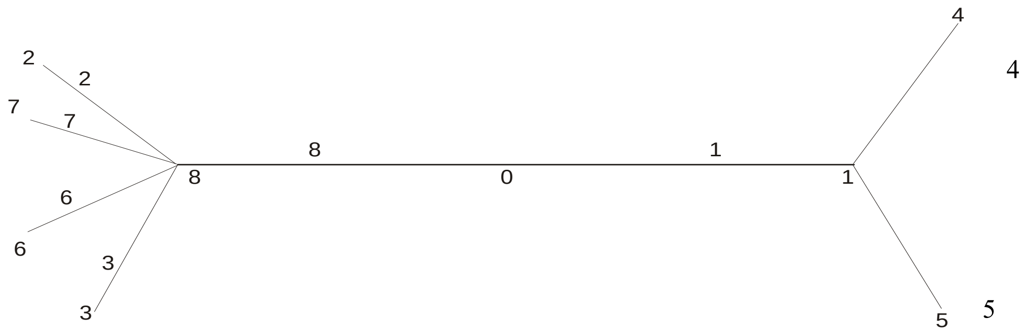

Figure4: Edge graceful labeling of $B_{4,2}$.

Theorem: 2.4
The bistar graph $B_{n,m}$ for $n \neq m$ where n, m odd is a edge graceful graph.

Proof
Consider the ordinary labeling of the vertices & edges of $B_{n,m}$ as was done in Theorem 2.3.

Consider the Diophantine equation $x_1+x_2 = p$. The solutions are of the form (t, p-t) where $1 \leq t \leq q/2$. There will be $\frac{q}{2}$ pair of solutions.

Among $\frac{q}{2}$ pair of solutions, choose the pair (q, 1). We now label the edges as follows, f (e) = q ;$f(e^1) = 1$; $f(e_1) = 2$; $f(e_{n+1}) = p - 2$. As we have labeled the edges e_1 and e_{n+1} there will be even number of edges incident with each v and v^1 to be labeled. We label these edges as was done in Theorem 2.3.

Then the induced vertex labels are $f^+(w) = 0$; $f^+(v) = q + 2$; $f^+(v^1) = p-1$. The pendant vertices will have the labels of the edges with which they are incident. They are distinct. Hence the graph bistar$B_{n,m}$for $n \neq m$, n, m odd is an edge graceful graph.

Illustration

Consider the bistar graph $B_{3,5}$. Here p = 11, q = 10.

Figure6: $B_{3,5}$ with ordinary labeling.

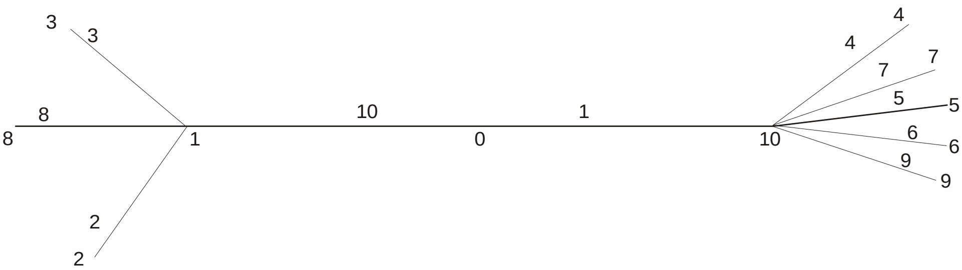

Figure7: Edge graceful labeling of $B_{3,5}$.

Consider the Diophantine equation $x_1 + x_2 = 11$. The pair of solutions are: (1, 10), (2, 9), (3, 8), (4, 7)and (5,6). Let us choose the pair(q,1)=(1,10) Define f (e) = q; $f(e^1)$ = 1. From the remaining pair label the edges as discussed in Theorem 2.3. The edge graceful labeling of bistar $B_{3,5}$ is given in Fig. 7.

Theorem: 2.5

The bistar graph $B_{n,m}$ for n = m where n, m even is an edge graceful graph.

Proof

Consider the ordinary labeling of the vertices and edges of $B_{n,m}$ as was done in Theorem 2.3.

Consider the Diophantine equation, $x_1 + x_2 = p$. The solutions are of the form (t, p-t) where $1 \leq t \leq \frac{q}{2}$. There will be $\frac{q}{2}$ pair of solutions.

With these pairs label the edges of $K_{1,n}$ and $K_{1,m}$ by the coordinates of the pair in any order so that adjacent edges receives the coordinates of the pairs. Also, label f(e)

= q; f (e^1) = 1. Then the induced vertex labels are f^+ (w) = 0; f^+ (v^1) = 1; f^+ (v) = q.

Now the pendant vertices will have labels of the edges with which they are incident and they are distinct. Hence the graph $B_{n,\,m}$ & n = m even is edge graceful.

Illustration

Consider the bistar graph $B_{2,\,2}$. Here p = 7, q = 6 consider the Diophantine equation $x_1+x_2 = 7$. The pair of solutions are (t, p-t) where t ∊ (1, q/2) (1, 6), (2, 5), (3, 4). We label the edges e & e^1 as follows f(e) = q = 6; f(e^1) = 1 we use the pair of solutions to label the edges of $K_{1,\,2}$ &$K_{1,\,2}$ and we label them by the coordinate of the pairs. The edge graceful labeling of bistar $B_{4,\,2}$ is given in fig 3.

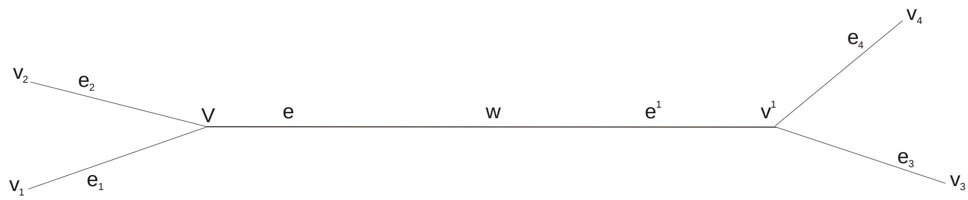

Figure8: $B_{2,\,2}$ with ordinary labeling.

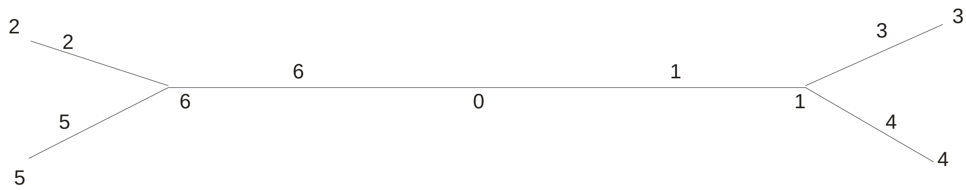

Figure9: Edge graceful labeling of $B_{2,\,2}$.

Theorem: 2.6

The bistar graph $B_{n,\,m}$ for n = m where n, m odd is an edge graceful graph.

Proof

Consider the ordinary labeling of the vertices and edges of $B_{n,\,m}$ as was done in Theorem 2.3.

Consider the Diophantine equation $x_1+x_2=p$. The solutions are of the form (t,p-t) where$1\le t\le q/2$. There will be q/2 pair of solutions.

Among $\frac{q}{2}$ pair of solutions, choose the pair (q, 1). We now label the edges as follows, f(e) = q; f (e^1) = 1; f (e_1) = 2; f (e_{n+1}) = p-2. As we have labeled the edges e_1 and e_{n+1} there will be even number of edges incident with each v and v^1 to be labeled. We label these edges as was done in Theorem 2.3.

Then the induced vertex labels are f^+(w) = 0; f^+(v) = q + 2 $f^+(v^1)$=p-1. The pendant vertices will have the labels of the edges with which they are incident. They are distinct. Hence the graph bistar $B_{n,m}$, n,m odd is an edge graceful graph.

Illustration

Consider the bistar graph $B_{3,3}$. Here p = 9; q = 8

Figure10: $B_{3,3}$ with ordinary labeling.

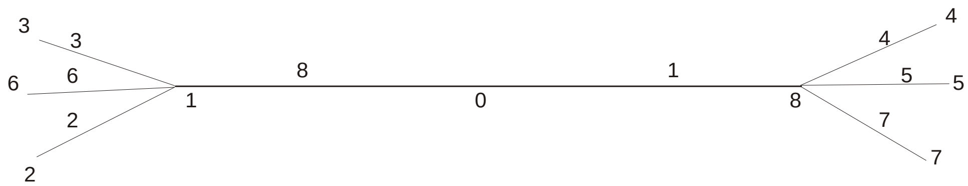

Figure11: Edge graceful labeling of $B_{3,3}$.

Consider the Diophantine equation $x_1+x_2 = 9$. The pair of solutions are; (1,8), (2, 7), (3, 6), (4, 5). Let us choose the pair (1, q) = (1, 8) label the edges as follows f(e) = q; $f(e^1)=1$. From the remaining pairs label the edges as discussed in Theorem 2.3. The edge graceful labeling of bistar $B_{3,3}$ is given in Fig. 11

Theorem: 2.7 [Lo] [6]

If a graph is edge graceful then $q\,(q+1) \equiv \dfrac{p(p-1)}{2} \pmod{p}$

Theorem: 2.8

Any tree of even order is not edge graceful

Proof

By Theorem 2.7

Theorem: 2.9

The graph bistar $B_{n,m}$ of even order is not an edge graceful graph.

Proof

The graph $B_{n,m}$ will be even order in the following cases.

1. n is odd and m is even
2. n is even and m is odd

In both cases p = n + m + 3, which is even and hence by Theorem 2.8 it is not an edge graceful.

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