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Distribution and Application of Root Mersenne Prime

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Abstract

We advance a new idea about existence of so-called root Mersenne primes among Mersenne primes, which include M_p for p=2,3,5,7 and M_p to accord with $p-F_0$ divided by 8 or $p-F_1$ divided by 6, and discover existence of an accurate distribution law of root Mersenne prime i.e. there exist $t_{n+1}=2^n$ root Mersenne primes for $F_n-1<p<F_{n+1}-1(n=0,1,2,3,\ldots)$ and there exist $s_{n+1}=2^{n+1}$ root Mersenne primes for $p<F_{n+1}-1$, where F_n is Fermat number. It will lead to an accurate description for distribution law of Mersenne prime by quantity of root Mersenne prime. We suppose a common law for existence of prime appearing in any three ones including W_2, W_3 among $W_p=2M_p^2-1$ for p=2,3,5,7 will be suitable to W_p produced by at least one root Mersenne prime M_p for $F_n-1<p<F_{n+1}-1(n=0,1,2,3,\ldots)$, so that we may find primes being much larger than the largest known Mersenne prime $M_{43112609}$ from W_p produced by last 4 known root Mersenne primes $M_{25964951}$, $M_{32582657}$, $M_{37156667}$, $M_{43112609}$.

Keywords: Mersenne prime; root Mersenne prime; distribution law; perfect number; odd quadratic form; golden section

Verifications for Distribution Law of Root Mersenne Prime and Distribution Law of Mersenne Prime

Among 47 known Mersenne primes M_p, we discover 8 Mersenne primes to accord with $p-F_0$ divided by 8 i.e. M_{19}, M_{107}, M_{2203}, M_{86243}, M_{216091}, $M_{1257787}$, $M_{20996011}$, $M_{37156667}$ and 18 Mersenne primes to accord with $p-F_1$ divided by 6 i.e. M_{17}, M_{89}, M_{521}, M_{4253}, M_{9689}, M_{9941}, M_{11213}, M_{19937}, M_{21701}, M_{756839}, M_{859433}, $M_{1398269}$, $M_{2976221}$, $M_{3021377}$, $M_{6972593}$, $M_{25964951}$, $M_{32582657}$, $M_{43112609}$, where F_0=3 and F_1=5 are Fermat primes but 6 or 8 is final digit of perfect number, and we also discover the sum of p=2,3,5,7 i.e. 2+3+5+7=17 to accord with $p-F_1$ divided by 6 for M_2, M_3, M_5, M_7. It makes us feel this kind of special Mersenne primes, which includes M_2, M_3, M_5, M_7 and Mersenne primes to accord with $p–F_0$ divided by 8 or $p–F_1$ divided by 6, can be called as root Mersenne prime. We discover existence of an accurate distribution law of root Mersenne prime, that is, there exist $t_{n+1}=2^n$ root Mersenne primes for $F_n–1<p<F_{n+1}–1(n=0,1,2,3,\ldots)$ and there exist $s_{n+1}=2^{n+1}$ root Mersenne primes for $p<F_{n+1}–1$, where F_n is Fermat number.

We have verified the distribution law to be accurately tenable for known root Mersenne primes for $p<F_{3+1}–1$, that is, for $n=0$, there exists $t_{0+1}=2^0=1$ root Mersenne prime i.e. M_3 for $F_0–1<p<F_{0+1}–1$ i.e. $2<p<4$ and there exist $s_{0+1}=2^{0+1}=2$ root Mersenne primes i.e. M_2, M_3 for $p<F_{0+1}–1$ i.e. $p<4$; for $n=1$, there exist $t_{1+1}=2^1=2$ root Mersenne primes i.e. M_5, M_7 for $F_1–1<p<F_{1+1}–1$ i.e. $4<p<16$ and there exist $s_{1+1}=2^{1+1}=4$ root Mersenne primes i.e. M_2, M_3, M_5, M_7 for $p<F_{1+1}–1$ i.e. $p<16$; for $n=2$, there exist $t_{2+1}=2^2=4$ root Mersenne primes i.e. M_{17}, M_{19}, M_{89}, M_{107} for $F_2–1<p<F_{2+1}–1$ i.e. $16<p<256$ and there exist $s_{2+1}=2^{2+1}=8$ root Mersenne primes i.e. M_2, M_3, M_5, M_7, M_{17}, M_{19}, M_{89}, M_{107} for $p<F_{2+1}–1$ i.e. $p<256$; for $n=3$, there exist $t_{3+1}=2^3=8$ root Mersenne primes i.e. M_{521}, M_{2203}, M_{4253}, M_{9689}, M_{9941}, M_{11213}, M_{19937}, M_{21701} for $F_3–1<p<F_{3+1}–1$ i.e. $256<p<65536$ and there exist $s_{3+1}=2^{3+1}=16$ root Mersenne primes i.e. M_2, M_3, M_5, M_7, M_{17}, M_{19}, M_{89}, M_{107}, M_{521}, M_{2203}, M_{4253}, M_{9689}, M_{9941}, M_{11213}, M_{19937}, M_{21701} for $p<F_{3+1}–1$ i.e. $p<65536$. The verified results accurately accord with real existence of known root Mersenne primes for $p<F_{3+1}–1$ i.e. $p<65536$. According to above distribution law of root Mersenne prime, we can expect there should exist $t_{4+1}=2^4=16$ root Mersenne primes for $F_4–1<p<F_{4+1}–1$

i.e. $65536<p<4294967296$ and there should exist $s_{4+1}=2^{4+1}=32$ root Mersenne primes for $p<F_{4+1}–1$ i.e. $p<4294967296$. But there exist only 14 known root Mersenne primes i.e. M_{86243}, M_{216091}, M_{756839}, M_{859433}, $M_{1257787}$, $M_{1398269}$, $M_{2976221}$, $M_{3021377}$, $M_{6972593}$, $M_{20996011}$, $M_{25964951}$, $M_{32582657}$, $M_{37156667}$, $M_{43112609}$ for $65536<p<4294967296$ and there exist only 30 known root Mersenne primes i.e. M_2, M_3, M_5, M_7, M_{17}, M_{19}, M_{89}, M_{107}, M_{521}, M_{2203}, M_{4253}, M_{9689}, M_{9941}, M_{11213}, M_{19937}, M_{21701}, M_{86243}, M_{216091}, M_{756839}, M_{859433}, $M_{1257787}$, $M_{1398269}$, $M_{2976221}$, $M_{3021377}$, $M_{6972593}$, $M_{20996011}$, $M_{25964951}$, $M_{32582657}$, $M_{37156667}$, $M_{43112609}$ for $p<4294967296$, therefore, there should exist 2 unknown root Mersenne primes to be found for $65536<p<4294967296$ from our advanced distribution law of root Mersenne prime.

Above discussion makes us feel distribution law of root Mersenne prime will lead to an accurate description for distribution law of Mersenne prime by using quantity of root Mersenne prime i.e. $s_{n+1}–1$ is just quantity of Mersenne prime for $F_n–1<p<F_{n+1}–1(n=0,1,2,3,\ldots)$. It means that there exist $s_{n+1}–1$ Mersenne primes for $F_n–1<p<F_{n+1}–1(n=0,1,2,3,\ldots)$ and there exist $2(s_{n+1}–1)–n$ Mersenne primes for $p<F_{n+1}–1$, that is, for $n=0$, there exists $s_{0+1}–1=2^1–1=1$ Mersenne prime i.e. M_3 for $F_0–1<p<F_{0+1}–1$ i.e. $2<p<4$ and there exist $2(s_{0+1}–1)–0=2(2^1–1)–0=2$ Mersenne primes i.e. M_2, M_3 for $p<F_{0+1}–1$ i.e. $p<4$; for $n=1$, there exist $s_{1+1}–1=2^2–1=3$ Mersenne primes i.e. M_5, M_7, M_{13} for $F_1–1<p<F_{1+1}–1$ i.e. $4<p<16$ and there exist $2(s_{1+1}–1)–1=2(2^2–1)–1=5$ Mersenne primes i.e. M_2, M_3, M_5, M_7, M_{13} for $p<F_{1+1}–1$ i.e. $p<16$; for $n=2$, there exist $s_{2+1}–1=2^3–1=7$ Mersenne primes i.e. M_{17}, M_{19}, M_{31}, M_{61}, M_{89}, M_{107}, M_{127} for $F_2–$

$1<p<F_{2+1}-1$ i.e. $16<p<256$ and there exist $2(s_{2+1}-1)-2=2(2^3-1)-2=12$ Mersenne primes i.e. M_2, M_3, M_5, M_7, M_{13}, M_{17}, M_{19}, M_{31}, M_{61}, M_{89}, M_{107}, M_{127} for $p<F_{2+1}-1$ i.e. $p<256$; for $n=3$, there exist $s_{3+1}-1=2^4-1=15$ Mersenne prime i.e. M_{521}, M_{607}, M_{1279}, M_{2203}, M_{2281}, M_{3217}, M_{4253}, M_{4423}, M_{9689}, M_{9941}, M_{11213}, M_{19937}, M_{21701}, M_{23209}, M_{44497} for $F_3-1<p<F_{3+1}-1$ i.e. $256<p<65536$ and there exist $2(s_{3+1}-1)-3=2(2^4-1)-3=27$ Mersenne primes i.e. M_2, M_3, M_5, M_7, M_{13}, M_{17}, M_{19}, M_{31}, M_{61}, M_{89}, M_{107}, M_{127}, M_{521}, M_{607}, M_{1279}, M_{2203}, M_{2281}, M_{3217}, M_{4253}, M_{4423}, M_{9689}, M_{9941}, M_{11213}, M_{19937}, M_{21701}, M_{23209}, M_{44497} for $p<F_{3+1}-1$ i.e. $p<65536$. These verified results accurately accord with real existence of known Mersenne primes for $p<F_{3+1}-1$ i.e. $p<65536$. We can expect that there should exist $s_{4+1}-1=2^5-1=31$ Mersenne primes for $F_4-1<p<F_{4+1}-1$ i.e. $65536<p<4294967296$ and there should exist $2(s_{4+1}-1)-4=2(2^5-1)-4=58$ Mersenne primes for $p<F_{4+1}-1$ i.e. $p<4294967296$. But there exist only 20 known Mersenne primes i.e. M_{86243}, M_{110503}, M_{132049}, M_{216091}, M_{756839}, M_{859433}, $M_{1257787}$, $M_{1398269}$, $M_{2976221}$, $M_{3021377}$, $M_{6972593}$, $M_{13466917}$, $M_{20996011}$, $M_{24036583}$, $M_{25964951}$, $M_{30402457}$, $M_{32582657}$, $M_{37156667}$, $M_{42643801}$, $M_{43112609}$ for $65536<p<4294967296$ and there exist only 47 known Mersenne primes for $p<4294967296$, therefore, there should exist 11 unknown Mersenne primes to be found for $65536<p<4294967296$. These results are same as calculated results from distribution law of Mersenne prime advanced by Zhou Haizhong in 1992[1] i.e. there are $2^{n+1}-1$ Mersenne primes for $F_n-1<p<F_{n+1}-1$ $(n=0,1,2,3,\ldots)$ and there are $2^{n+2}-n-2$ Mersenne primes for $p<F_{n+1}-1$.

An Odd Quadratic Form of Root Mersenne Prime and Its Golden Prime

The traditional relation formula between perfect number P_p and Mersenne prime M_p can be expressed as

$$P_p = (M_p^2 + M_p)/2, \tag{1}$$

and (1) must be also suitable to all of root Mersenne primes. From (1) we have

$$W_p = 2(2P_p-M_p) -1, \tag{2}$$

where

$$W_p = 2M_p^2 -1 \tag{3}$$

is an odd quadratic form of root Mersenne prime M_p. (2) shows W_p produced by root Mersenne prime M_p must be a larger number than corresponding perfect number P_p, and implies $W_p=2M_p^2-1$ isn't a new hypothesis but another expression from root Mersenne prime M_p and corresponding P_p. From $M_p=2^p-1$ and (3) we get structure of W_p produced by root Mersenne prime M_p as follows

$$W_p = 2^{2p+1}-2^{p+2}+1. \tag{4}$$

From (3) we get $W_2 =17$, $W_3 =97$, $W_5 =1921$, $W_7 =32257$ produced by first 4 known root Mersenne primes $M_2=3$, $M_3=7$, $M_5=31$, $M_7=127$, which shows W_2, W_3, W_5, W_7 to be larger numbers separately than corresponding perfect numbers $P_2=6$, $P_3=28$, P_5

=496, P_7=8128 but W_2, W_3, W_7 to be larger primes separately than corresponding perfect numbers P_2 , P_3 , P_7 among W_p for p=2,3,5,7.

In order to increase chances to find larger primes than corresponding perfect number from W_p produced by root Mersenne prime M_p, we try to introduce golden section method to W_p produced by root Mersenne prime M_p. For a given root Mersenne prime M_p, we can get two irrational numbers as follows

$$\alpha_{p-i} = [(\sqrt{5}-1)/2]^{\,i} W_p \ (i = 1,2). \qquad (5)$$

The odd number being closest to α_{p-i} is written as W_{p-i} and called as i-order golden odd number of W_p produced by root Mersenne prime M_p, and W_{p-i} must be larger than corresponding perfect number P_p from (2). If W_{p-i} is prime, W_{p-i} is called as i-order golden prime of W_p produced by root Mersenne prime M_p. From (5) we get W_{2-1}=11 and W_{2-2}=7 being larger than P_2=6 for W_2, W_{3-1}=59 and W_{3-2}=37 being larger than P_3=28 for W_3, W_{5-1}=1187 and W_{5-2}=733 being larger than P_5=496 for W_5, W_{7-1}=19935 and W_{7-2}=12321 being larger than P_7=8128 for W_7, and we know 11, 7, 59, 37, 1187, 733 to be primes but 19935, 12321 not to be primes. From it we see W_{2-1} and W_{2-2} to be larger primes than P_2 for W_2, W_{3-1} and W_{3-2} to be larger primes than P_3 for W_3, W_{5-1} and W_{5-2} to be larger primes than P_5 for W_5 among W_{p-1} and W_{p-2} produced by first 4 known root Mersenne primes M_p for p=2,3,5,7.

Expecting Possible Existence of Larger Primes than The Largest Known Mersenne Prime

Considering p=2,3,5,7 to be the first continuous prime alignment in making 2^p-1 become root Mersenne prime but their sum 2+3+5+7=17 according with p–F_1 divided by 6 and being just the first one of W_p for p=2,3,5,7 i.e. W_2=17, we can call p=2,3,5,7 as original prime alignment for W_p produced by root Mersenne prime M_p and think $W_p=2M_p^2-1=2^{2p+1}-2^{p+2}+1$ as a reasonable odd quadratic form produced by root Mersenne prime M_p, so that we can suppose that a common law for existence of prime appearing in any three ones including W_2, W_3 among W_p for p=2,3,5,7 will be suitable to W_p produced by at least one root Mersenne prime M_p for $F_n-1<p<F_{n+1}-1$(n=0,1,2,3,…).

Basing on this supposition, we can infer that W_p produced by at least one root Mersenne prime M_p for $F_n-1<p<F_{n+1}-1$(n=0,1,2,3,…) will become larger prime than corresponding perfect number P_p because of W_2, W_3, W_7 to be larger primes separately than corresponding perfect numbers P_2 , P_3 , P_7 among W_p for p=2,3,5,7. In fact, we have known W_3 to be larger prime than P_3 for $F_0-1<p<F_{0+1}-1$ i.e. $2<p<4$, W_7 to be larger prime than P_7 for $F_1-1<p<F_{1+1}-1$ i.e. $4<p<16$, and we have verified $W_{17}=2^{35}-2^{19}+1$=34359214081 and $W_{19}=2^{39}-2^{21}+1$=549753716737 produced separately by root Mersenne primes M_{17} and M_{19} to be larger primes separately than P_{17}=8589869056 and P_{19}=137438691328 for $F_2-1<p<F_{2+1}-1$ i.e. $16<p<256$. It implies the possibility of real existence of prime W_p produced by at least one root Mersenne prime M_p to be larger than corresponding perfect number P_p for $F_n-1<p<F_{n+1}-1$(n=0,1,2,3,…), so that

we may reasonably expect to find prime W_p produced by at least one root Mersenne prime M_p to be larger than corresponding perfect number P_p among W_p produced by known root Mersenne primes M_p for p=86243, 216091, 756839, 859433, 1257787, 1398269, 2976221, 3021377, 6972593, 20996011, 25964951, 32582657, 37156667, 43112609 for $F_4-1<p<F_{4+1}-1$ i.e. $65536<p<4294967296$ though these known root Mersenne primes don't include all of 16 root Mersenne primes for $65536<p<4294967296$. If there exists at least one larger prime than corresponding perfect number among $W_{25964951}=2^{51929903}-2^{25964953}+1$, $W_{32582657}=2^{65165315}-2^{32582659}+1$, $W_{37156667}=2^{74313335}-2^{37156669}+1$, $W_{43112609}=2^{86225219}-2^{43112611}+1$, we will find at least one prime being much larger than the largest known Mersenne prime $M_{43112609}=2^{43112609}-1$, whose structure can be described by $2^{2p+1}-2^{p+2}+1$.

In addition, we have seen W_{p-1} and W_{p-2} to be larger primes separately than corresponding perfect numbers P_2, P_3, P_5 for W_2, W_3, W_5 produced by root Mersenne primes M_2, M_3, M_5, and this common law for existence of prime appearing in W_p for p=2,3,5 will be suitable to W_p produced by at least one root Mersenne prime M_p for $F_n-1<p<F_{n+1}-1(n=0,1,2,3,\ldots)$ according to our supposition. In fact, we have known W_3 to be able to produce W_{3-1} and W_{3-2} being larger primes than corresponding perfect number P_3 for $F_0-1<p<F_{0+1}-1$ i.e. $2<p<4$, W_5 to be able to produce W_{5-1} and W_{5-2} being larger primes than corresponding perfect number P_5 for $F_1-1<p<F_{1+1}-1$ i.e. $4<p<16$, and we have verified $W_{17-1}=21235162129$ and $W_{17-2}=13124051953$ produced by known root Mersenne prime M_{17} to be larger primes than $P_{17}=8589869056$ for $F_2-1<p<F_{2+1}-1$ i.e. $16<p<256$, so that we can also reasonably infer there may exist at least one W_p among W_p produced by known root Mersenne primes M_p for p=86243, 216091, 756839, 859433, 1257787, 1398269, 2976221, 3021377, 6972593, 20996011, 25964951, 32582657, 37156667, 43112609 to be able to produce prime W_{p-1} and W_{p-2} being larger than corresponding perfect number P_p for $F_4-1<p<F_{4+1}-1$ i.e. $65536<p<4294967296$ as our discussion in at least one prime W_p existing for $65536<p<4294967296$ does. It means we may find prime W_{p-1} and W_{p-2} from at least one W_p among W_p produced by last 4 known root Mersenne primes M_p for p=25964951, 32582657, 37156667, 43112609 by searching for i-order golden prime of W_p, which will be larger than corresponding perfect number and much larger than the largest known Mersenne prime $M_{43112609}$.

Conclusion

Discussion in this paper shows existence of an accurate distribution law of root Mersenne prime, which will lead to an accurate description for distribution law of Mersenne prime by using quantity of root Mersenne prime, and every root Mersenne prime M_p will lead to appearing of a reasonable odd quadratic form $W_p=2M_p^2-1=2^{2p+1}-2^{p+2}+1$. From our supposition we can infer there exists at least one W_p produced by root Mersenne prime M_p to be larger prime than corresponding perfect number P_p for $F_n-1<p<F_{n+1}-1(n=0,1,2,3,\ldots)$, and there exists at least one W_p produced by root Mersenne prime M_p to be able to produce W_{p-1} and W_{p-2} being larger

primes than corresponding perfect number P_p for $F_n-1<p<F_{n+1}-1(n=0,1,2,3,\ldots)$. Hence we can reasonably expect that prime W_p produced by at least one root Mersenne prime M_p existing for $65536<p<4294967296$ to be larger than corresponding perfect number P_p and much larger than the largest known Mersenne prime $M_{43112609}$ as well as prime W_{p-1} and W_{p-2} produced by at least one root Mersenne prime M_p existing for $65536<p<4294967296$ to be larger than corresponding perfect number P_p and much larger than the largest known Mersenne prime $M_{43112609}$ may be found from W_p produced by last 4 known root Mersenne primes among 30 known root Mersenne primes.

References

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