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Complex Probability Theory

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Abstract

Probability theory can be understood as a mathematical model for the intuitive notion of uncertainty. Probability theory is in all scientific fields. Also, probability is used in many branches of pure mathematics, even in branches one does not expect this, and like in convex geometry. I developed Complex Probability Theory for analysis of complex and chaotic systems. I think that probability has theoretical and experimental dimensions at the same time. Universe has these probabilities dimensions. I combined them in this theory and constructed four dimensional probabilities on Quaternion Algebra and eight dimensional probabilities on Octonion Algebra and found all probability spaces.

Introduction

The modern period of probability theory is connected with names like S.N. Bernstein (1880-1968), E. Borel (1871-1956), and A.N. Kolmogorov (1903-1987). In particular, in 1933 A.N. Kolmogorov published his modern approach of Probability Theory, including the notion of a measurable space and a probability space.[1],[2],[3],[4],[5]

Notations

Notation of f Complex Probability

Definition-1-1:

Given as set Ω set and subsets A, B, Ω then the following notation is used:

Intersection

$$A \cap B = \{v \in \Omega : v \in A and v \in B\}$$

Union

$$A \cup B = \{v \in \Omega : v \in A or, or both v \in B\}$$

Set Theoretical Minus

$$A \setminus B = \{v \in \Omega : v \in A and v \notin B\}$$

Complement

$$A^{c} = \{v \in \Omega : v \notin A\}$$

Empty Set

$\varnothing$ set without any element. .[1],[2],[3],[4],[5]

Components of Probability Space
Definition-1-2:
Any probability space (Ω, F, P) consists of three components.

1. The elementary events or states v which are collected in a non-empty Ω set .
2. Any σ algebra F , which is the system of observable subsets or events $A \subseteq \Omega$ .The interpretation is that one can usually not decide whether a system is in the particular state $v \in \Omega$,but one can decide whether $v \in A$ or $v \notin A$.
3. Any measure P, which gives a probability to all $A \in F$ .This probability is a number $P(A) \in [0,1]$ that describes how likely it is that the event A occurs. We define the σ algebras F , here we do not need any measure. [1],[2],[3],[4],[5]

σ Algebras
Definition-1-3:
Let be Ω a non-empty set. A system F of subsets $A \subseteq \Omega$ is called σ algebra on Ω if

1. $\varnothing, \Omega, F$
2. $A \in F$ implies that $A^{c} : \Omega \setminus \mathrm{A} \in \mathrm{F}$
3. $A_1, A_2, ... \in F$ implies that $\bigcup_{i=1}^{\infty} A_i \in F$

The pair (Ω, F), where F is a σ algebra on , is called measurable space. The elements $A \in F$ are called events. An event A occurs if $v \in A$ and it does not occur if $v \notin A$.

4. $A, B \in F$ implies that $A \cup B \in F$, then F is called an algebra. Every σ algebra is an algebra. Sometimes, the terms σ field and field are used instead of σ algebra and algebra.

Space Definitions
Probability Space
Definition-2-1:
Let (Ω, F), be a measurable space.

1. A map $P : F \to [0,1]$ is called probability measure if $P(\Omega) = 1$ and for all

$A_1, A_2, ... \in F$ with $A_i \cap A_j = \varnothing$ for $i \neq j$ one has $P\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} P(A_i)$.

The triplet (Ω, F, P) is called probability space.

2. Any map $\mu : F \to [0, \infty]$ is called measure if $\mu(\varnothing) = 0$ and for all $A_1, A_2, ... \in F$ with $A_i \cap A_j = \varnothing$ for $i \neq j$ one has $\mu\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} \mu(A_i)$.

The triplet (Ω, F, μ) is called measure space.

3. Any measure space (Ω, F, μ) or a measure μ is called σ finite provided that there are $\Omega_k \subseteq \Omega, k = 1,2,...$, such that

a. $\Omega_k \in F$ for all $k = 1,2,...$

b. $\Omega_i \cap \Omega_j = \varnothing$ for $i \neq j$

c. $\Omega = \bigcup_{k=1}^{\infty} \Omega_k$

d. $\mu(\Omega_k) < \infty$

The measure space (Ω, F, μ) or the measure μ are called finite if $\mu(\Omega) < \infty$.[1],[2],[3],[4],[5]

Components of Complex Probability Space

Definition-2-2:

Any complex probability space (Ω, F, P) consists of three components .But probability will have G theoretical and H experimental probability components $P(G, H)$ on complex space. Because of a complex probability space (Ω, F, G, H) consists of four components.

1. The elementary events or states v which are collected in a non-empty Ω set .
2. Any σ algebra F , which is the system of observable subsets or events $A \subseteq \Omega$ .The interpretation is that one can usually not decide whether a system is in the particular state $v \in \Omega$,but one can decide whether $v \in A$ or $v \notin A$.
3. Any measure P ,which gives a probability to all $A \in F$ .This probability is a complex number $P(A) \in [0,1] + e_1[0,1]$ and $P(A) \in C$ that describes how likely it is that the event A occurs. We define the σ algebras F , here we do not need any measure. [1],[2],[3],[4],[5]

G Theoretical Probability Space

Definition-2-3:

G set is a collection of theoretical probability objects.

Let G is a theoretical probability space.

G^k is k dimensional theoretical probability space.

$$G = \{g^0, g^1, ..., g^k\}$$

Theoretical probability is $G(A) \in [0,1]$

Theoretical probability is mathematical values of probability.

H_1 Experimental Probability Space

Definition-2-4:

H_1 set is a collection of experimental probability objects.

Let H_1 is a experimental probability space.

H_1^k is k dimensional experimental probability space.

$$H_1 = \{h^0, h^1, ..., h^k\}$$

Experimental probability is $H_1(A) \in [0,1]$

Experimental probability is experimental values of probability.

P Complex Probability Space

Definition–2–5:

Let P is a complex probability space.

The form of a complex probability number is $\Psi = G + H_1 e_1$ and $\Psi(A) = G(A) + H_1(A)e_1$

$$P = \{\Psi = G + H_1 e_1 \in C | G, H_1, \in R\}$$

$$P = \{\Psi = G + H_1 e_1 \in C | e_1^{\ 2} = -1, G, H_1 \in [0,1]\}$$

$$P = \{\Psi(A) = G(A) + H_1(A)e_1 \in C \mid G(A), H_1(A) \in [0,1]\}$$

Let S is a complex probability set.

$$S = \{\Psi = G + H_1 e_1 \in P | G, H_1, \in R\}$$

$$S = \{\Psi = G + H_1 e_1 \in P | e_1^{\ 2} = -1, G, H_1 \in [0,1]\}$$

$$S = \{\Psi(A) = G(A) + H_1(A)e_1 \in P \mid G(A), H_1(A) \in [0,1]\}$$

Let A and B are events

$\Psi(A) = G(A) + H_1(A)e_1$

$\Psi(B) = G(B) + H_1(B)e_1$ are complex probabilities.

$(\Psi(A) + \Psi(B)) = (G(A) + G(B)) + (H_1(A) + H_1(B))e_1$

$(\Psi(A) + \Psi(B)) = 1 + 1e_1$

$|\Psi(A) + \Psi(B)| = |1 + 1e_1|$

$$|\Psi(A)+\Psi(B)|=\sqrt{2}$$

Probability Definitions

G Theoretical Probability

Definition-3-1:

Let (Ω, F), be a measurable space.

G is mathematical probability space. We will get theoretical probability from mathematical results.

1. A map $G : F \to [0,1]$ is called probability measure if $G(\Omega)=1$ and for all $A_1, A_2, ... \in F$ with $A_i \cap A_j = \varnothing$ for $i \neq j$ one has $G\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} G(A_i)$.

   The triplet (Ω, F, G) is called theoretical probability space.

2. Any map $\mu : F \to [0, \infty]$ is called measure if $\mu(\varnothing)=0$ and for all $A_1, A_2, ... \in F$ with $A_i \cap A_j = \varnothing$ for $i \neq j$ one has $\mu\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} \mu(A_i)$.

   The triplet (Ω, F, μ) is called measure space.

3. Any measure space (Ω, F, μ) or a measure μ is called σ finite provided that there are $\Omega_k \subseteq \Omega, k = 1,2,...$, such that

a. $\Omega_k \in F$ for all $k = 1,2,...$

b. $\Omega_i \cap \Omega_j = \varnothing$ for $i \neq j$

c. $\Omega = \bigcup_{k=1}^{\infty} \Omega_k$

d. $\mu(\Omega_k) < \infty$

The measure space (Ω, F, μ) or the measure μ are called finite if $\mu(\Omega) < \infty$.

If we flip a coin, then we have either"heads" or "tails" on top, that means. Probability of head and probability of tail are $\frac{1}{2}$ to each of the two possible outcomes. If the coin is fair, then heads and tails should receive the same probability. The reason for this has to do with our intuitive notion of what a probability means.

In this sample, the two possible outcomes form the set $\Omega = \{head, tail\}$ or $\Omega = \{h, t\}$

$$G(head) = G(tail) = \frac{1}{2}$$

$$G(h) = G(t) = \frac{1}{2}$$
$$G(head) + G(tail) = 1$$

H_1 Experimental Probability

Definition-3-2:

Let (Ω, F), be a measurable space.

H_1 is experimental probability space. We will get experimental or relative probability from experimental results.

1. A map $H_1 : F \to [0,1]$ is called probability measure if $H_1(\Omega) = 1$ and for all $A_1, A_2, \ldots \in F$ with $A_i \cap A_j = \varnothing$ for $i \neq j$ one has $H_1\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} H_1(A_i)$.

   The triplet (Ω, F, H_1) is called experimental probability space.
2. Any map $\mu : F \to [0, \infty]$ is called measure if $\mu(\varnothing) = 0$ and for all $A_1, A_2, \ldots \in F$ with $A_i \cap A_j = \varnothing$ for $i \neq j$ one has $\mu\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} \mu(A_i)$.

   The triplet (Ω, F, μ) is called measure space.
3. Any measure space (Ω, F, μ) or a measure μ is called σ finite provided that there are $\Omega_k \subseteq \Omega, k = 1,2,\ldots$, such that

a. $\Omega_k \in F$ for all $k = 1,2,\ldots$

b. $\Omega_i \cap \Omega_j = \varnothing$ for $i \neq j$

c. $\Omega = \bigcup_{k=1}^{\infty} \Omega_k$

d. $\mu(\Omega_k) < \infty$

The measure space (Ω, F, μ) or the measure μ are called finite if $\mu(\Omega) < \infty$.

If we toss the coin Q times and the number of heads among these Q tosses is Q_h, then relative frequency of heads is equal to $\frac{Q_h}{Q}$.Now if Q is large, then we tend to think about $\frac{Q_h}{Q}$ as being close to probability of heads. The relative frequency of tails can be written as $\frac{Q_t}{Q}$,where Q_t is the number of tails among the Q tosses, and we again think of $\frac{Q_t}{Q}$ as being close to the probability tails. Since $\frac{Q_h}{Q} + \frac{Q_t}{Q} = 1$, we see that at least intuitively, the probabilities of heads and tails should add up to one.

In this sample, the two possible outcomes form the set $\Omega = \{head, tail\}$ or $\Omega = \{h, t\}$

$$H_1(head) = \frac{Q_h}{Q},\ H_1(tail) = \frac{Q_t}{Q}$$

$$H_1(head) + H_1(tail) = 1$$

Ψ Complex Probability

Definition-3-3:

Let (Ω, F), be a measurable space.

Ψ is complex probability space. We will get complex probability from theoretical and experimental results.

1. A map $\Psi : F \to [0,1]$ is called probability measure if $\Psi(\Omega) = 1$ and for all $A_1, A_2, \ldots \in F$ with $A_i \cap A_j = \varnothing$ for $i \neq j$ one has $\Psi\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} \Psi(A_i)$.

The triplet (Ω, F, Ψ) is called experimental probability space.

2. Any map $\mu : F \to [0, \infty]$ is called measure if $\mu(\varnothing) = 0$ and for all $A_1, A_2, \ldots \in F$ with $A_i \cap A_j = \varnothing$ for $i \neq j$ one has $\mu\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} \mu(A_i)$.

The triplet (Ω, F, μ) is called measure space.

3. Any measure space (Ω, F, μ) or a measure μ is called σ finite provided that there are $\Omega_k \subseteq \Omega, k = 1,2,\ldots$, such that

a. $\Omega_k \in F$ for all $k = 1,2,\ldots$

b. $\Omega_i \cap \Omega_j = \varnothing$ for $i \neq j$

c. $\Omega = \bigcup_{k=1}^{\infty} \Omega_k$

d. $\mu(\Omega_k) < \infty$

The measure space (Ω, F, μ) or the measure μ are called finite if $\mu(\Omega) < \infty$.

Let complex probability is $\Psi(A) = G(A) + H(A)e_1$ and $\Omega = \{head, tail\}$ or $\Omega = \{h, t\}$

G Theoretical Probability

$\Omega = \{head, tail\}$ or $\Omega = \{h, t\}$

$$G(head) = \frac{1}{2}, G(tail) = \frac{1}{2}$$

$G(head) + G(tail) = 1$

H_1 **Experimental Probability**

$\Omega = \{head, tail\}$ or $\Omega = \{h, t\}$

$H(head) = \dfrac{Q_h}{Q}, H(tail) = \dfrac{Q_t}{Q}$

$H(head) + H(tail) = 1$

Ψ **Complex Probability**

$\Omega = \{head, tail\}$ or $\Omega = \{h, t\}$

$\Psi(A) = G(A) + H(A)e_1$

$\Psi(head) = G(head) + H(head)e_1$

$\Psi(tail) = G(tail) + H(tail)e_1$

$\Psi(head) = \dfrac{1}{2} + \dfrac{Q_h}{Q}e_1, \Psi(tail) = \dfrac{1}{2} + \dfrac{Q_t}{Q}e_1$

$\Psi(head) + \Psi(tail) = 1 + 1e_1$

Propositions

Proposition of Complex Probability

Definition-4-1:

Let (Ω, F, P) be a complex probability space. Then the following assertions are

1. If $A_1, A_2, ... \in F$ such that $A_i \cap A_j = \varnothing$ for $i \neq j$ one has $P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} P(A_i)$
2. If $A, B \in F$ then $P(A \setminus B) = P(A) + P(A \cap B)$
3. If $B \in F$ then $P(B^c) = 1 - P(B)$,
4. If $A_1, A_2, ... \in F$ then $P\left(\bigcup_{i=1}^{\infty} A_i\right) \leq \sum_{i=1}^{\infty} P(A_i)$
5. Continuity from below :

If $A_1, A_2, ... \in F$ such that $A_1 \subseteq A_2 \subseteq A_3 \subseteq ...$,then $\lim_{n\to\infty} P(A_n) = P\left(\bigcup_{n=1}^{\infty} A_n\right)$

6. Continuity from above:

If $A_1, A_2, ... \in F$ such that $A_1 \supseteq A_2 \supseteq A_3 \supseteq ...$,then $\lim_{n\to\infty} P(A_n) = P\left(\bigcap_{n=1}^{\infty} A_n\right)$

7. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
8. $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

9. For pair wise events $A_1, A_2, ..., A_n$ it is case that

$$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_i P(A_i) - \sum_{i<j} P(A_i \cap A_j) + \sum_{i<j<k} P(A_i \cap A_j \cap A_k) - ... + (-1)^{n+1} P(A_1 \cap A_2 \cap ... \cap A_n)$$

Independence of Events
Definition-4-2:
Let (Ω, F, P) be a complex probability space. The events $A_i \subseteq F, i \in K$ is an arbitrary non-empty index set, are called independent, provide that for all distinct $i_1, ..., i_n \in K$ one has that

$$P(A_{i_1} \cap A_{i_2} \cap ... A_{i_n}) = P(A_{i_1}) P(A_{i_2}) ... P(A_{i_n})$$

Given $A_1, A_2, ... \in F$, one can easily see that only demanding
$P(A_{1_1} \cap A_2 \cap ... A_n) = P(A_1) P(A_2) ... P(A_n)$.[1],[2],[3],[4],[5]

Conditional Probability
Definition-4-3:
Let (Ω, F, P) be a complex probability space. Suppose A and B are events in sample space Ω,and suppose that $P(B) > 0$ .The conditional probability of A given B is defined as

$P(A|B) = \dfrac{P(A \cap B)}{P(B)}$ and $A, B \in F$.[1],[2],[3],[4],[5]

Complex Probability Operators
Product
Let

$$\Psi = a_0 + a_1 e_1$$
$$P = \{a_0 + a_1 e_1 \in C | e_1{}^2 = -1, a_0, a_1 \in R\}$$
$$\Psi_1 = a_0 + a_1 e_1$$
$$\Psi_2 = b_0 + b_1 e_1$$

Multiplication is generally commutative $\Psi_1 \times \Psi_2 = \Psi_2 \times \Psi_1$

Conjugate
The conjugate of $\Psi = a_0 + a_1 e_1$ is $\overline{\Psi} = a_0 - a_1 e_1$

$$\Psi = \{a_0 + a_1 e_1 \in C | e_1^2 = -1, a_0, a_1 \in R\}$$
$$\overline{\Psi} = \{a_0 - a_1 e_1 \in C | e_1^2 = -1, a_0, -a_1 \in R\}$$

Magnitude

The magnitude of $\Psi = a_0 + a_1 e_1$ is $|\Psi| = \sqrt{a_0^{\,2} + a_1^{\,2}}$

Multiplicative Inverse

The multiplicative inverse of $\Psi = a_0 + a_1 e_1$ is

$$\Psi^{-1} = \frac{1}{\Psi}\ , \Psi \neq 0 \text{ and } \Psi^{-1} = \frac{\overline{\Psi}}{\Psi\overline{\Psi}}$$

$$\Psi^{-1} = \frac{a_0 - a_1 e_1}{(a_0 + a_1 e_1)(a_0 - a_1 e_1)}$$

$$\Psi^{-1} = \frac{a_0 - a_1 e_1}{a_0^{\,2} + a_1^{\,2}}$$

Division

Let

$$\Psi = a_0 + a_1 e_1$$

$$\Psi = \left\{a_0 + a_1 e_1 \in C \middle| e_1^2 = -1, a_0, a_1 \in R\right\}$$

$$\Psi_1 = a_0 + a_1 e_1$$

$$\Psi_2 = b_0 + b_1 e_1 \text{ and } \Psi_2 \neq 0$$

$$\frac{\Psi_1}{\Psi_2} = d_0 + d_1 e_1$$

$$\Psi = \left\{d_0 + d_1 e_1 \in C \middle| e_1^2 = -1, d_0, d_1 \in R\right\}$$

The conjugates of $\Psi_1 = a_0 + a_1 e_1$ and $\Psi_2 = b_0 + b_1 e_1$ are

$$\overline{\Psi_1} = a_0 - a_1 e_1 \text{ and } \overline{\Psi_2} = b_0 - b_1 e_1$$

$$\frac{\Psi_1}{\Psi_2} = \frac{(a_0 + a_1 e_1)(b_0 - b_1 e_1)}{(b_0 + b_1 e_1)(b_0 - b_1 e_1)}$$

$$\frac{\Psi_1}{\Psi_2} = \frac{(a_0 + a_1 e_1)(b_0 - b_1 e_1)}{b_0^{\,2} + b_1^{\,2}}$$

Polar Notation

Let

$$\Psi = a_0 + a_1 e_1$$

$$\Psi = \left\{a_0 + a_1 e_1 \in C \middle| e_1^2 = -1, a_0, a_1 \in R\right\}$$

The magnitude of $\Psi = a_0 + a_1 e_1$ is $|\Psi| = \sqrt{a_0^{\,2} + a_1^{\,2}}$

$Arg(\Psi) = \{\theta_1 + 2\pi k\}$ and $\theta = \left\{0 \leq \theta_1 < 360^0 \mid \theta_1 \in R\right\}$, The radius set is $r = \left\{ r_1 = \sqrt{{a_0}^2 + {a_1}^2} \mid r_1 \in R \right\}$,

The polar notation is $\Psi = r_1(\cos\theta_1 + e_1 \sin\theta_1)$

$$\cos\theta = \left\{ \cos\theta_1 = \frac{a_0}{\sqrt{{a_0}^2 + {a_1}^2}} \mid \cos\theta_1 \in R \right\}$$

$$\sin\theta = \left\{ \sin\theta_1 = \frac{a_1}{\sqrt{{a_0}^2 + {a_1}^2}} \mid \sin\theta_1 \in R \right\}$$

The polar notation is $\Psi = r_1(\cos\theta_1 + e_1 \sin\theta_1)$

Its conjugate is $\overline{\Psi} = r_1(\cos\theta_1 - e_1 \sin\theta_1)$

Exponential Form

Let

$Arg(\Psi) = \{\theta_1 + 2\pi k\}$ and $\theta = \{0 \le \theta_1 < 360^0 \mid \theta_1 \in R\}$

The radius set is $r = \left\{ r_1 = \sqrt{{a_0}^2 + {a_1}^2} \mid r_1 \in R \right\}$

The polar form is $\Psi = r_1(\cos\theta_1 + e_1 \sin\theta_1)$

The exponential form is

$$e^{e_1\theta_1} = (\cos\theta_1 + e_1 \sin\theta_1)$$

Its conjugate is

$$e^{-e_1\theta_1} = (\cos\theta_1 - e_1 \sin\theta_1)$$

Power Form

Let

$Arg(\Psi) = \{\theta_1 + 2\pi k\}$ and $\theta = \{0 \le \theta_1 < 360^0 \mid \theta_1 \in R\}$,

The radius set is $r = \left\{ r_1 = \sqrt{{a_0}^2 + {a_1}^2} \mid r_1 \in R \right\}$

The polar form is $\Psi = r_1(\cos\theta_1 + e_1 \sin\theta_1)$

The power form is from degree nth power and $n \in Z$

$$\Psi^n = r_1^n(\cos n\theta_1 + e_1 \sin n\theta_1)$$

Root Form

Let

$Arg(\Psi) = \{\theta_1 + 2\pi k\}$ and $\theta = \{0 \le \theta_1 < 360^0 \mid \theta_1 \in R\}$,

The radius set is $r = \left\{ r_1 = \sqrt{{a_0}^2 + {a_1}^2} \mid r_1 \in R \right\}$

The root form is from degree nth root, $k = 0,1,2,\ldots, n-1$ and $k, n \in Z$

$$\Psi_k = \sqrt[n]{r_1}\left(\cos\frac{\theta_1 + 2k\pi}{n} + e_1 \sin\frac{\theta_1 + 2k\pi}{n}\right)$$

Its roots are $\Psi_k = \{\Psi_0, \Psi_1, ..., \Psi_{n-1}\}$

Addition

$$\Psi_1 = a_0 + a_1 e_1$$

$$\Psi_2 = b_0 + b_1 e_1$$

$$\Psi_1 + \Psi_2 = (a_0 + b_0) + (a_1 + b_1) e_1$$

$$\Psi_1 + \Psi_2 = m_0 + m_1 e_1$$

$$\Psi_1 + \Psi_2 = \left\{ m_0 + m_1 e_1 \in C \,\middle|\, e_1^2 = -1, m_0, m_1 \in R \right\}$$ [6],[7],[8],[9],[10],[11],[12]

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