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Some New Separation Axioms: A Different Approach

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Abstract

This paper studies some new separation axioms for topological spaces defined in terms of a new topology and a continuous function. This new idea gives the notion of Star-T_1 -spaces, Star-T_2 -spaces, Star-regular spaces, Star-normal-spaces etc. It is found that Star-T_1 –axiom lies between T_0 -axiom and T_1 -axiom. T_2 -axiom implies Star-T_2 –axiom but not conversely. Moreover, T_1 -axiom and Star-T_2 -axiom are in-dependent of each other. This is justified with an example that a co-finite topology cannot be Star-T_2. There are two types of star-regular-spaces, one is A-star-regular and another is B-star-regular. It is found that a regular space is both A-star-regular and is B-star-regular. On the other hand, a finite B-star regular space is regular.

Finally, it is observed that for a compact T_1 -space, all the three separation axioms viz., A-star-regular, B-star-regular and star-normal are equivalent.

Keywords: separation axioms, star-regular, star-normal, co-finite, compact.

AMS Subject Classification: 54A05.

Introduction and preliminaries

With the introduction of generalized open sets, like semi-open sets[6],locally closed sets[2] pre-open sets[7], ∂-sets[3],semi-pre-open sets[1] etc., separation axioms using these generalized open sets have been defined and the corresponding topological spaces are studied. So far there have been two ways of defining new separation axioms in topological spaces. one, in terms of "each singleton satisfying certain conditions" and the other, in terms of "generalized open sets". Following are definitions of some known separation axioms.

DEFINITION 1.1 [2] A topological space (X, t) is called a T_D –space if each singleton is locally closed.

DEFINITION 1.2 [4, 5] A topological space (X, t) is called R_0 if for each open set O in t, $x \in O$ implies $\mathrm{cl}\ x \subset O$.

DEFINITION 1.3 [3] A topological space (X, t) is called a T_∂ -space if each singleton is a ∂-set in (X, t).

DEFINITION 1.4 [4,5] A topological space (X, t) is called R_1 if for $x, y \in X$ such that $\mathrm{cl}\ x \neq \mathrm{cl}\ y$, there exists disjoint open sets U and V such that $\mathrm{cl}\ x \subset U$ and $\mathrm{cl}\ y \subset V$.

DEFINITION 1.5 [6] A topological space (X, t) is called semi-T_o if for $x, y \in X$ there exists a semi-open set U such that $x \in U$ and y does not belong to U or $y \in U$ and x does not belong to U.

DEFINITION 1.6 [6] A topological space (X, t) is called semi-T_1 if for $x, y \in X$ there exist semi-open sets U and V such that $x \in U$ and $y \in V$ and x does not belong to V and y does not belong to U.

DEFINITION 1.7 [6] A topological space (X, t) is called semi-T_2 if for $x, y \in X$ there exist disjoint semi-open sets U and V such that $x \in U$ and $y \in V$.

In the present approach, a new topology and a continuous function have been used.

Star-T_1 -spaces

DEFINITION 2.1 A topological space (X, t) is said to be star-T_1 if there exists a topology s on X and a bijective continuous function f: $(X, t) \to (X, s)$ such that for any two distinct points a, b in X there are open sets G and H in s satisfying the following condition:

$$a \in f^{-1}(G),\ b \in H \text{ and } b \notin f^{-1}(G),\ a \notin H$$

or

$$b \in f^{-1}(G),\ a \in H \text{ and } a \notin f^{1}(G),\ b \notin H.$$

NOTE 2.1 If (X, t) is T_1 then it is star-T_1.

For, consider s = t and f the identity function.

NOTE 2.2 A star-T_1 -space may not be T_1.

EXAMPLE 2.1 Let X = {a, b, c}, t = {Ø, X, {a, b}, {b}, {b, c}}, s = {Ø, X, {a}, {a, b}, {a, c}}.

DEFINE f: $(X, t) \to (X, s)$ by f (a) = c, f (b) = a, f (c) = b. Then f is bi-jective and continuous. Now for the points a, b, choose G = {a}, H = {a, c}.

Then $b \in f^{-1}(G)$, $a \in H$, $a \notin f^{-1}(G)$, $b \notin H$. For the points a, c, choose $G = \{a, b\}$, $H = \{a, \}$. Then $c \in f^{-1}(G)$, $a \in H$, $a \notin f^{-1}(G)$, $c \notin H$.

For the points b, c, choose $G = \{a\}$, $H = \{a, c\}$. Then $b \in f^{-1}(G)$, $c \in H$, $c \notin f^{-1}(G)$, $b \notin H$.

Thus (X, t) is star-T_1 but not T_1.

NOTE 2.3 A star-T_1 -space is T_0.

For, f being continuous, f^{-1} (G) is open in t. Now the result follows.

NOTE 2.4 A T_0 -space may not be star-T_1.

EXAMPLE 2.2 Let $X = \{a, b\}$, $t = \{\emptyset, X, \{a\}\}$. Then (X, t) is T_0. We claim that (X, t) is not star-T_1.

If possible, let (X, t) be star-T_1. Then there exists a topology s on X and a bijective continuous function f: $(X, t) \to (X, s)$ such that for any two distinct points a, b in X there are open sets G and H in s satisfying

1. $a \in f^{-1}(G)$, $b \in H$ and $b \notin f^{-1}(G)$, $a \notin H$

or

2. $b \in f^{-1}(G)$, $a \in H$ and $a \notin f^{-1}(G)$, $b \notin H$.

Now $f^{-1}(G)$ is open in t and hence $f^{-1}(G) = \{a\}$ or X. Since $b \notin f^{-1}(G)$, or $a \notin f^{-1}(G)$, it follows that $f^{-1}(G) = \{a\}$. So certainly (1) holds. Now $G \neq X$, otherwise $f^{-1}(G) = X$, which is not true. So $G = \{a\}$ or $G = \{b\}$.

Case (i) G = {a}.

Since $b \in H$, $a \notin H$, it follows that $H = \{b\}$. Thus s is the discrete topology.

Now f is continuous. So f (a) = a and f (b) = b is not possible, because $f^{-1}(H) = f^{-1}(b) = \{b\}$, which is not open in t Since $f^{-1}(G) = \{a\}$ and since $G = \{a\}$, it follows that f (a) = a. Then f (b) = a must hold. This contradicts the fact that f is surjective.

Case (ii) G = {b}.

Since $f^{-1}(G) = \{a\}$ so f (a) = b. Since G and H are distinct, $H = \{a\}$ or X. But $a \notin H$ implies $H \neq X$. So $H = \{a\}$. Then f (a) = b. Since f is bijective, f (b) = a. But $f^{-1}(H) = \{b\} \notin t$, which contradicts the fact that f is continuous. Thus in any case we have a contradiction. We conclude that (X, t) cannot be star-T_1.

NOTE 2.5 Star-T_1 axiom is lying between T_0 and T_1 axioms.

Star-T_2 -spaces

DEFINITION 3.1 A topological space (X, t) is said to be star-T_2 if there exists a topology s on X and a bijective continuous function f: $(X, t) \to (X, s)$ such that for any two distinct points a, b in X there are open sets G and H in s satisfying the following condition:

$$a \in f^{-1}(G), b \in H, f^{1} (G) \cap H = Ø$$

or

$$b \in f^{-1}(G), a \in H, f^{1} (G) \cap H = Ø.$$

NOTE 3.1 If (X, t) is T_2 then it is star-T_2.
For, consider s = t and f the identity function.

NOTE 3.2 A star-T_2 -space may not be T_2.

EXAMPLE 3.1 Let X = {a, b, c}, t = {Ø, X, {a}, {b}, {a, b}, {b, c}}, s = {Ø, X, {a}, {c}, {a, c}, Define f: (X, t) → (X, s) by f (a) = a, f (b) = c, f (c) = b. Then f is bijective and continuous. Now for the points a, b, choose G = {c}, H = {a}. Then b ∈ f^{-1}(G), a∈ H, f^{-1}(G) ∩H = Ø.

For the points b, c, choose G = {c}, H = {a, c}. Then b ∈ f^{1}(G), c∈ H, f^{-1}(G) ∩ H = Ø. For the points a, c, choose G = {a, c}, H = {c}. Then a ∈ f^{-1}(G), c ∈ H, f^{1} (G) ∩ H = Ø. Hence (X, t) is star-T_2. Also (X, t) is not T_2.

NOTE 3.3 A star-T_2 -space may not be T_1.
In example 3.1, (X, t) is star-T_2 but (X, t) is not T $_1$.

NOTE 3.4 A T_1 -space may not be star-T_2.

EXAMPLE 3.2 Let X be an infinite set and t be the cofinite topology on X. Then (X, t) is T_1.We claim that (X, t) is not star-T_2. If possible, let it be star-T_2. Then there exists a topology s on X and a bijective continuous function f: (X, t) → (X, s) such that for any two distinct points a, b in X there are open sets G and H in s satisfying the following condition:

(1) $a \in f^{-1}(G), b \in H, f^{1} (G) \cap H = Ø$

or

(2) $b \in f^{-1}(G), a \in H, f^{1} (G) \cap H = Ø$. -1

Let a, b ∈ X and (1) hold. Then f^{1} (G) ∈ t. Then X – f^{1}(G) is a finite -1 set. Since f^{1}(G)∩H = Ø, H ⊂ X – f^{1} (G). So H is a finite set. Since -1 H ≠ Ø and f^{1} (H) is open and f is surjective, so f^{1} (H) is an infinite set with finite complement. But f is injective, so f^{1} (H) cannot be infinite (since H is finite). Thus a contradiction arises.

Hence (1) cannot hold. Similarly we can show that (2) cannot hold. Hence (X, t) cannot be star-T_2..

NOTE 3.5 A star-T_2 -space is star- T_1.

Proof follows from the definitions.

Star-Regular-spaces

Recall that a topological space (X, t) is regular if for any point a in X and any closed set F in (X, t) where $a \notin F$, there exist two open sets G and H in (X, t) satisfying $a \in G$, $F \subset H$, $G \cap H = \emptyset$.

We now have the following definitions:

DEFINITION 4.1 A topological space (X, t) is said to be A-star-regular if there exists a topology s on X and a bijective continuous function f:

$(X, t) \to (X, s)$ such that for any point a in X and any closed set F in

(X, t) where $a \notin F$, there are open sets G and H in s satisfying the condition that $a \in f^{-1}(G)$, $F \subset H$, $f^{-1}(G) \cap H = \emptyset$.

DEFINITION 4.2 A topological space (X, t) is said to be B-star-regular if there exists a topology s on X and a bijective continuous function f: $(X, t) \to (X, s)$ such that for any point a in X and any closed set F in (X, t) where $a \notin F$, there are open sets G and H in s satisfying the condition that $F \subset f^{-1}(G)$, $a \in H$, $f^{-1}(G) \cap H = \emptyset$.

NOTE 4.1 A regular space is A-star-regular.

For, consider s = t and f, the identity function.

NOTE 4.2 An A-star-regular space may not be regular.

EXAMPLE 4.1 Let X = {a, b, c}, t = {Ø, X, {a}, {a, b} }, s = {Ø, X, {c}, {b, c}}.

Then c(t) = {Ø, X, Ø, {b, c}, {c}}. where c(t) denotes the set of all closed sub- sets in (X, t). Define f: $(X, t) \to (X, s)$ by f (a) = c, f (b) = b, f (c) = a.

Then f s bijective and continuous. Now for the point a and closed set F = {b, c}, choose G = {c} ∈ s, H = {b, c} ∈ s then $a \in f^{-1}(G)$, $F \subset H$, $f^{-1}(G) \cap H = \emptyset$.

For the point a and closed set F = {c}, choose G = {c}, H = {b, c}.

For the point b and closed set F = {c}, choose G = {b, c}, H = {c}.

Hence (X, t) is A-star-regular. Clearly it is not regular since {a} and {b, c} cannot be separated by open sets in t.

NOTE 4.3 A regular space is B-star-regular.

For, consider s = t and f, the identity function.

THEOREM 4.1 A finite B-star-regular space is regular.

PROOF: If possible, let (X, t) be a finite B-star regular space which is not regular. Then there exist $a \in X$, $F \in C(t)$ such that $a \notin F$ and for every open set G_a and every open set H_F, $a \in G_a$, $F \subset H_F \Rightarrow G_a \cap H_F \neq \emptyset$. Since (X, t) is B-star-regular, there exists a topology s on X and a bijective continuous function f: $(X, t) \to (X, s)$ satisfying the B-star-regularity condition. So there exist $G\in s$, $H \in s$ such that $F \subset f^{-1}(G)$, $a\in H$, $f^{-1}(G) \cap H = Ø$............ (1).

Now F is not open in (X, t), otherwise $a\in X - F$, $F\subset F$ and $(X-F) \cap F = Ø$, a contradiction. Since F is continuous $f^{-1}(G) \in t$. So $f^{-1}(G) - F \neq Ø$. Let $a_1 \in f^{-1}(G) - F$. Now $f^{-1}(G) \cap cl_t H = Ø$

$\Rightarrow f^{-1}(G) \cap cl_t \{a\} = Ø$.......... (2).

This implies $a_1 \notin cl_t \{a\}$. Now consider the closed set $F \cup cl_t \{a\}$ in (X, t) where $a_1 \notin F \cup cl_t \{a\}$. Again by B-star-regularity condition, there exists $G_1 \in s$, $H_1 \in s$ such that $F \cup cl_t \{a\} \subset f^{-1}(G_1)$, $a_1 \in H_1$, $f^{-1}(G_1) \cap H_1 = Ø$............ (3).

Now we claim that $F \cup cl_t \{a\} \notin t$. Otherwise, if $F \cup cl_t \{a\} \in t$ then $[F \cup cl_t \{a\}] - cl_t \{a\} \in t \Rightarrow F \in t$ (since $F \cap cl_t \{a\} = Ø$), a contradiction.

Hence $F \cup cl_t \{a\} \notin t \Rightarrow f^{-1}(G_1 - [F \cup cl_t \{a\}] \neq Ø$. Let $a_2 \in f^{-1}(G_1 - [F \cup cl_t \{a\}]$.

Now $f^{-1}(G) \cap cl_t H_1 = Ø$.

$f^{-1}(G) \cap cl_t \{a_1\} = Ø$.......... (4).

This implies $a_2 \notin cl_t \{a_1\}$. Also $a_2 \notin F \cup cl_t \{a\}$

Now consider the closed set $F \cup cl_t \{a\} \cup cl_t \{a_1\}$ in (X, t) where $a_2 \notin F \cup cl_t \{a\} \cup cl_t \{a_1\}$. Again by B-star-regularity condition, there exists $G_2 \in s$, $H_2 \in s$ such that $F \cup cl_t \{a\} \cup cl_t \{a_1\} \subset f^{-1}(G_2)$, $a_2 \in H_2$ $f^{-1}(G_2) \cap H_2 = Ø$.

Now we claim that $F \cup cl_t \{a\} \cup cl_t \{a_1\} \notin t$. Otherwise, if it is open in (X, t) then $F \cup cl_t \{a\} \cup cl_t \{a_1\} - [cl_t \{a\} \cup cl_t \{a_1\} \in t$. This implies $F\in t$, (since $F \cap [cl_t \{a\} \cup cl_t \{a_1\}] = Ø$), a contradiction. Thus $F \cup cl_t \{a\} \cup cl_t \{a_1\} \notin t$.

This implies $f^{-1}(G_2) - [F \cup cl_t \{a\} \cup cl_t \{a_1\}] \neq Ø$. Let $a_3 \in f^{-1}(G_2) - [F \cup cl_t \{a\} \cup cl_t \{a_1\}]$. Now $f^{-1}(G_2) \cap cl_t H_t = Ø \Rightarrow f^{-1}(G_2) \cap cl_t \{a_2\} = Ø \Rightarrow a_3 \notin cl_t \{a_2\}$. Also $a_3 \notin F \cup cl_t \{a\} \cup cl_t \{a_1\}]$.

Thus $a_3 \notin F \cup cl_t \{a\} \cup cl_t \{a_1\} \cup cl_t \{a_2\}$, where $F \cup cl_t \{a\} \cup cl_t \{a_1\} \cup cl_t \{a_2\}$ is a closed set in (X, t). We can now use B-star-regularity condition and note that $F \cup cl_t \{a\} \cup cl_t \{a_1\} \cup cl_t \{a_2\} \notin t$. Proceeding in this manner after a finite number of steps we must have $F \cup cl_t \{a\} \cup cl_t \{a_1\} \cup cl_t \{a_2\}$....... $cl \{a_n\} = X$ (since X is finite). This implies F is open in (X, t), a contradiction. This completes the proof of the theorem.

NOTE 4.4 It follows from above theorem that in example 4.1, the space (X, t) is not B-star-regular. Thus the class of A-star regular spaces is different from the class of B-star-regular spaces.

THEOREM 4.2 A star-T_1 and A-star-regular space is star- T_2.

PROOF. Since (X, t) is star-T_1, there exists a topology s on X and a bijective continuous function f: (X, t) $\rightarrow$ (X, s) such that for any two distinct points a, b in X there are open sets G and H in s satisfying

(1) $a \in f^{-1}(G)$, $b \in H$ and $b \notin f^{-1}(G)$, $a \notin H$

or

(2) $b \in f^{-1}(G)$, $a \in H$ and $a \notin f^{1}$ (G), $b \notin H$.

Let a, b $\in$ X and a = b. Suppose that (1) holds. Then $a \notin X - f^{-1}(G) = F$ say, where F is closed in (X, t). Since (X, t) is A-star-regular there exists a topology μ on X and a bijective continuous function g: (X, t) $\rightarrow$ (X, μ) such that for any point p in X and any closed set V in (X, t) where $p \notin V$, there are open sets M and W in μ satisfying $p \in g^{=1}(M)$, $V \subset W$, $g^{-1}(M) \cap W = \emptyset$.

Now $a \in X$ and F is closed in (X, t) and $a \notin F$. Then there are open sets M and W in μ satisfying $a \in g^{=1}(M)$, $F \subset W$, $g^{-1}(M) \cap W = \emptyset$. -1

Now $b \notin f^{-1}(G) \Rightarrow b \in F$. So $a \in g^{-1}(M)$, $b \in W$, $g^{-1}(M) \cap W = \emptyset$.

Hence if (1) holds then the condition for the space (X, t) to be star-T_2 is satisfied. If (2) holds then it can similarly be shown that the condition for the space (X, t) to be star-T_2 is satisfied. This completes the proof of the theorem.

THEOREM 4.3 A star-T_1 and B-star-regular space is star- T_2.

PROOF. Since (X, t) is star-T_1, there exists a topology s on X and a bijective continuous function f: (X, t) $\rightarrow$ (X, s) such that for any two distinct points a, b in X there are open sets G and H in s satisfying

(1) $a \in f^{-1}(G)$, $b \in H$ and $b \notin f^{-1}(G)$, $a \notin H$

or

(2) $b \in f^{-1}(G)$, $a \in H$ and $a \notin f^{1}$ (G), $b \notin H$.

Suppose that a and b are two distinct points in X and let (1) hold. Then $a \notin X - f^{-1}(G) = F$ say, where F is closed in (X, t). Since (X, t) is B-star-regular, by the B-star regularity condition, there exists a topology μ on X and a bijective continuous function g: (X, t) $\rightarrow$ (X, μ) such that there are open sets M and W in μ satisfying $F \subset g^{-1}(M)$, $a \in W$, $g^{-1}(M) \cap W = \emptyset$.

Now $b \notin f^{1}(G) \Rightarrow b \in F \Rightarrow b \in g^{-1}(M)$. Hence We have a topology μ -1 on X and a bijective continuous function g: (X, t) $\rightarrow$ (X, μ) such that for the points a and b there are open sets M and W in μ satisfying $b \in g^{-1}(M)$ $a \in W$, $g^{-1}(M) \cap W = \emptyset$. Hence (X, t) is star-$T_2$.

DEFINITION 4.2 A topological space (X, t) is said to be A-star-T_3 (respectively, B-star-T_3) if it is star-T_1 and A-star-regular (respectively, B-star-regular).

NOTE 4.3 An A- star-T_3 space may not be T_2. In example 4.1, the space (X, t) is star-T_1 and A-star-regular and hence star-T_3. But (X, t) is not T_2.

Star-Normal-spaces

DEFINITION 5.1 A topological space (X, t) is said to be star-normal if there exists a topology s on X and a bijective continuous function f: $(X, t) \to (X, s)$ such that for any two disjoint closed sets A and B in (X, t), there are open sets G and H in s satisfying the condition that $A \subset f^{-1}(G)$, $B \subset H$, $f^{-1}(G) \cap H = \emptyset$.

NOTE 5.1 It is clear from the definition that a T_1 star-normal space is A-star-regular and B-star-regular.

THEOREM 5.1 Let (X, t) be compact and T_1. Then the following statements are equivalent:

- (X, t) is A-star-regular.
- (X, t) is star-normal.
- (X, t) is B-star-regular.

Proof: We shall prove that (i) $\Rightarrow$ (ii) $\Rightarrow$ (iii) $\Rightarrow$ (ii) $\Rightarrow$ (i).

Let (i) hold. let A and B be any two disjoint closed sets in (X, t). Since (X, t) is A-star-regular, there exists a topology s on X and a bijective continuous function f: $(X, t) \to (X, s)$ such that for any point a in X and any closed set F in (X, t) where $a \notin F$, there are open sets G and H in s satisfying the condition that $a \in f^{-1}(G)$, $F \subset H$, $f^{-1}(G) \cap H = \emptyset$.

Then for each $a \in A$, there exist open sets G_a, $H_a \in s$ such that

$$a \in f^{-1}(G_a), B \subset H_a, f^{-1}(G_a) \cap H_a = \emptyset.$$

Since f is continuous, $\{f^{-1}(G_a)\}_{a \in A}$ is an open covering of A in (X, t). Since (X, t) is compact and A is closed in (X, t), A is compact in (X, t). So there exist $a_1, a_2, \ldots, a_n \in A$ such that $A \subset f^{-1}(G_{a1}) \cup f^{-1}(G_{a2}) \cup f^{-1}(G_{a3}) \cup \ldots f^{-1}(G_{an}) = f^{-1}(G_{a1} \cup G_{a2} \cup G_{a3} \cup \ldots G_{an}) = f^{-1}(G)$ say, where $G = G_{a1} \cup G_{a2} \cup G_{a3} \cup \ldots G_{an}$.

Let $H = H_{a1} \cap H_{a2} \cap H_{a3} \cap \ldots H_{an}$. Then $B \subset H$.

$A \subset f^{-1}(G)$, $f^{-1}(G) \cap H = \emptyset$. Hence (X, t) is star-normal. So (ii) follows.

Now if (ii) holds then (iii) follows easily.

Let (iii) hold. Then since (X, t) is B-star-regular, let us call the s-topology on X under consideration as B-star-regular-s-topology on X.

We now have the following lemma.

LEMMA 5.1 If (X, t) is B-star-regular then for any two disjoint closed sets A and B in (X, t), $A \cap cl_s B = \emptyset$, and $B \cap cl_s A = \emptyset$, where $cl_s B$ denotes closure of B w. r. t. the B-star regular s-topology under consideration.

PROOF OF LEMMA: Let A and B be any two disjoint closed sets in (X, t). Let $x \in A$. Then $x \notin B$. By the B-star-regularity condition there exist G, H belonging to the B-star regular s-topology on X such that $B \subset f^{-1}(G)$, $x \in H$ and $f^{-1}(G) \cap H = \emptyset$.

Since $H \in s$, $cl_s\ f^{-1}(G) \cap H = \emptyset$, and $cl_s B \subset cl_s f^{-1}(G)$ implies $cl_s B \cap H = \emptyset$. Thus $x \in A \Rightarrow x \notin cl_s B$. This implies $A \cap cl_s B = \emptyset$. Similarly we can show that $B \cap cl_s A = \emptyset$. This completes the proof of the lemma.

Now consider two disjoint closed sets A and B in (X, t). By the above lemma, $A \cap cl_s B = \emptyset$, and,where $cl\ _sB$ denotes closure of B w. r. t. the B-star regular s-topology under consideration.

Let $x \in cl\ _sB$. Then $x \notin A$. Then there exist $G_x, H_x \in s$ such that $A \subset f^{-1}(G_x)$, $x \in H_x$ and $f^{-1}(G_x) \cap H_x = \emptyset$.

Consider $U = \{H_x : x \in cl\ _sB\}$. Since (X, t) is compact and f is continuous and surjective, (X, s) is compact. Then $cl_s B$ is compact in s. Now U is an open covering of $cl\ _sB$ in s. So there exist $x_1, x_2, \ldots, x_n$ in $cl_s B$ such that $cl_sB \subset H_{x1} \cup H_{x2} \cup \ldots \cup H_{xn}$. Let $M = H_{x1} \cup H_{x2} \cup \ldots \cup H_{xn}$ and $P = G_{x1} \cap G_{x2} \cap \ldots \cap G_{xn}$. Then $M \in s$ and $P \in s$.

Now $f^{-1}(G_{x1} \cap G_{x2} \cap \ldots \cap G_{xn}) = f^{-1}(G_{x1}) \cap f^{-1}(G_{x2}) \cap \ldots \cap f^{-1}(G_{xn})$ and $A \subset f^{-1}(G_{x1}) \cap f^{-1}(G_{x2}) \cap \ldots \cap f^{-1}(G_{xn})$.

This Implies $A \subset f^{-1}(P)$.

Thus $A \subset f^{-1}(P)$ and $B \subset M$. Also $f^{-1}(P) \cap M = \emptyset$.

Hence (X, t) is star-normal. So (ii) holds.

Now if (ii) holds then (i) follows easily. This completes the proof of the theorem.

The following problem can be raised which remains unsolved in this paper.

Problem

To construct an example of an infinite B-star-regular space which is not regular.

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