Global Journal of Mathematical Sciences: Theory and Practical.
ISSN 0974-3200 Volume 3, Number 3 (2011), pp. 201-207

http://www.irphouse.com

On Generalized-α b Spaces

[1]L. Vinayagamoorthi and [2]N. Nagaveni

[1]Department of Mathematics,
Gnanamani College of Technology, Namakkal, India
[2]Associate Professor, Department of Mathematics,
Coimbatore Institute of Technology, India

Abstract

In this paper we study new class is called of generalized α b – Spaces, (denoted by $T_{g\alpha b}$ -spaces) and study some of their properties.

Keywords and phrases: $T_{1/2}$ -space, Semi-$T_{1/2}$ space, Pre-$T_{1/2}$ space, $T_{g\alpha}$ -space, $T_{\alpha g}$ -space, T_{gs} -space.

Introduction

In 1970, N. Levine introduced the $T_{1/2}$ - space if every g-closed set is closed. The aim of this paper is to continue the study of generalized b-spaces. In particular, the notion of generalized α b-spaces and its various characterizations are given in this paper. Throughout this paper all spaces X is (X, τ) stand for topological spaces with no separation axioms assumed unless otherwise stated. Let A⊆X, the closure of A and the interior of A will be denoted by cl(A) and int(A) respectively and the union of all b-open sets X contained in A is called b-interior of A and is denoted by bint(A) and the intersection of all b-closed sets of X containing A is called b-closure of A and is denoted by bcl(A).

Preliminaries

In this section let us recall some definitions and results which are used in this section

Definition 2.1: A subset A of a topological space (X,τ) is called α - open[16] if A⊆ int (cl(int(A))

Definition 2.2: A subset A of a topological space (X, τ) is called semi- open [1]if A $\subseteq$ cl(int (A))

Definition 2.3: A subset A of a topological space (X, τ) is called pre-open[6] if A $\subseteq$ int (cl(A))

Definition 2.4: A subset A of a topological space (X, τ) is called semi-pre open [1] if A $\subseteq$ cl (int (cl(A)))

Definition 2.5: A subset A of a topological space (X, τ) is called b-open [4]if A $\subseteq$ cl (int (A)) $\bigcup$ int (cl(A)).

Definition 2.6: A is said to be generalized closed set (g-closed) [12] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open

Definition 2.7: A is said to be α -generalized closed set (α g-closed) [16] if α cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open.

Definition 2.8: A is said to be generalized pre-closed set (gp-closed) [12]if A* $\subseteq$ U whenever A $\subseteq$ U and U is open.

Definition 2.9: A is said to be generalized semi-preclosed(gsp-closed) set[6] if spcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open.

Definition2.10: A is said to be generalized semi-closed set(gs-closed) set[3] if scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open.

Definition 2.11: A is said to be semi generalized closed set (sg-closed) [3] if scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is semi open.

Definition 2.12: A is said to be generalized b-closed set(gb-closed) [18] if bcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open.

Remark 2.13: The complement of the above open sets are known as their respective closed sets and vice-versa.

Definition 2.14: A space X is said to be semi-$T_{1/2}$ space,[20]if every sg-closed set is semi closed.

Definition 2.15: A space X is said to be pre-$T_{1/2}$ space,[22] if every gp-closed set is pre-closed.

Definition 2.16: A space X is said to be semi-pre-$T_{1/2}$ [23]if every gα -closed set and gsp-closed set is α -closed set and semi-pre-closed set.

Definition 2.17: A space X is said to be $T_{1/2}$ - space,[19] if every g-closed set is closed, or equivalently if every singleton is open or closed.

Definition 2.18: A space X is said to be pre-regular- $T_{1/2}$ - space,[25] iff every gpr-closed set is pre-closed set. Note that a subset A is called gpr-closed whenever pclA $\subset$ U whenever A $\subset$ U and U is regular open.

Definition 2.19: A space X is said to be $T_{g\alpha}$ space[16] if every gα -closed set is α g-closed set.

Definition 2.20: A space X is said to be $T_{\alpha g}$ -space [16]if every α g-closed set is gα -closed set.

T $_{g\alpha b}$ - spaces

In this section we introduce a new space T $_{g\alpha b}$ - spaces in topology and study some of their properties

Definition 3.1: A topological space X is said to be $T_{g\alpha b}$ -space if every gα b-closed subset of X is α -closed in X.

Theorem 3.2: Every $T_{g\alpha b}$ -space is $T_{1/2}$ - space.

Proof: Let us assume that (X, τ) be $T_{g\alpha b}$ -space. Let A be gα b-closed, every gα b-closed sets are g-closed since X is $T_{g\alpha b}$ -space then A is closed therefore X is $T_{1/2}$ -space.

Remark 3.3: The converse of the above theorem need not be true as seen from the following example.

Example 3.4: Let X ={a,b,c} with τ = {x,ϕ,{a},{c},{a,c}} in this topological space {a} is gα b-closed but not α -closed.

Theorem 3.5: Every semi-$T_{1/2}$ - space is $T_{g\alpha b}$ -space.

Proof: Let us assume that (X, τ) be semi-$T_{1/2}$ - space. Let A be sg-closed, if every sg-closed sets are gα b-closed, since X is semi-$T_{1/2}$ - space A is α - closed therefore X is $T_{g\alpha b}$ -space.

Remark 3.6: The converse of the above theorem need not be true as seen from the following example.

Example 3.7: Let X ={a,b,c} with τ = {X, ϕ,{a},{b},{a,b}} in this topological space {a} is gα b-closed but not α -closed.

Theorem 3.8: Every $T_{g\alpha b}$ -space is pre-$T_{1/2}$ - space.

Proof: Let us assume that (X, τ) be $T_{g\alpha b}$ -space. Let A be gα b-closed, if every gα b-closed gp-closed sets, since X is $T_{g\alpha b}$ -space, A is pre- closed therefore X is pre-$T_{1/2}$ - space.

Remark 3.9: The converse of the above theorem need not be true as seen from the following example.

Example 3.10: Let X ={a,b,c} with τ = {X, ϕ,{a}} in this topological space the subset {a,b} is gp-closed but not pre-closed.

Theorem 3.11: Every αT_d -space is $T_{g\alpha b}$ -space.

Proof: Let us assume that (X, τ) be αT_d -space. Let A be α g-closed, if every α g-closed is gα b-closed, since X is αT_d -space, A is α - closed therefore X is $T_{g\alpha b}$ -space.

Remark 3.12: The converse of the above theorem need not be true as seen from the following example.

Example 3.13: Let X ={a,b,c} with τ = {X, ϕ,{a}{c},{a,c}} in this topological space the subset {a} is gα b-closed but it is not α - closed.

Theorem 3.14: Every $T_{g\alpha b}$ -space is pre-regular-$T_{1/2}$ - space.

Proof: Let us assume that (X, τ) be $T_{g\alpha b}$ -space. Let A be gα b-closed, if every gα b-closed gpr-closed sets, since X is $T_{g\alpha b}$ -space, A is pre-closed therefore X is pre-regular-$T_{1/2}$ - space.

Remark 3.15: The converse of the above theorem need not be true as seen from the following example.

Example 3.16: Let X ={a,b,c} with τ = {X, ϕ,{a},{b,c}} in this topological space the subset {c} is gα b-closed but not α -closed.

Theorem 3.17: Every $T_{g\alpha}$-space is $T_{g\alpha b}$-space.

Proof: Let us assume that (X, τ) be $T_{g\alpha}$-space. Let A be gα-closed, if every gα-closed set is gαb-closed, since X is $T_{g\alpha}$-space, A is α-closed therefore X is $T_{g\alpha b}$-space.

Remark 3.18: The converse of the above theorem need not be true as seen from the following example.

Example 3.19: Let X ={a,b,c} with τ = {X, ϕ, {a}, {b} {a,b}} in this topological space the subset {a} is gαb-closed but not α-closed.

Theorem 3.20: Every $T_{\alpha g}$-space is $T_{g\alpha b}$-space.

Proof: Let us assume that (X, τ) be $T_{\alpha g}$-space. Let A be αg-closed, if every αg-closed set is gαb-closed, since X is $T_{\alpha g}$-space, A is α-closed therefore X is $T_{g\alpha b}$-space.

Remark 3.21: The converse of the above theorem need not be true as seen from the following example.

Example 3.22: Let X ={a,b,c} with τ = {X, ϕ, {a}, {c} {a,c}} in this topological space the subset {a} is gαb-closed but not α-closed.

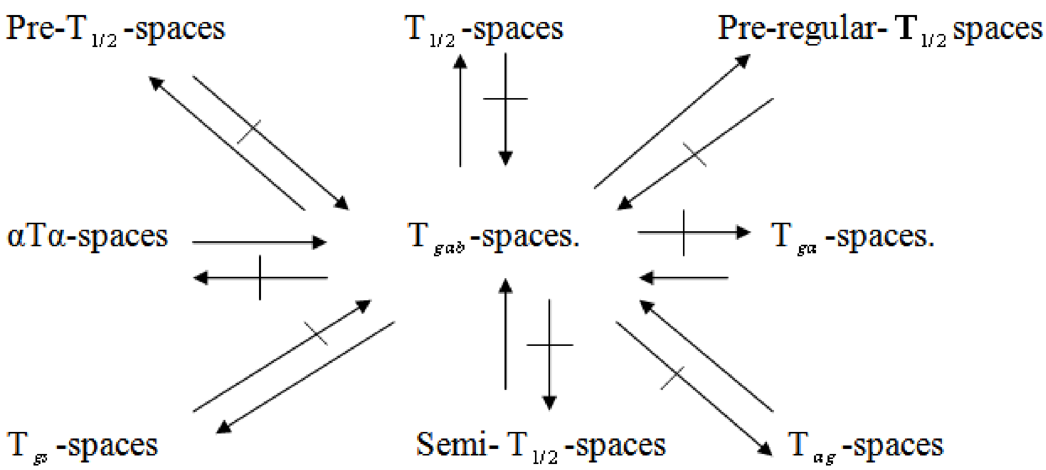
Theorem 3.23: Every $T_{g\alpha b}$-space is T_{gs}-space.

Proof: Let us assume that (X, τ) be $T_{g\alpha b}$-space. Let A be gαb-closed, if every gαb-closed set is gs-closed, since X is T_{gs}-space, A is sg-closed therefore X is-T_{gs}-space.

Remark 3.24: The converse of the above theorem need not be true as seen from the following example.

Example 3.17: Let X ={a,b,c} with τ = {X, ϕ, {a}} in this topological space the subset {a,b} is gs-closed but not sg-closed.

Remark 3.18: By the above theorem and results we obtain the following relations:



References

[1] D. Andrijevic, semipreopen sets, Mat.Vesnik 38, no.1, 24-32.

[2] D. Andrijevic, On b-open sets, Mat. Vesnik 48(1996), no.1-2,59-64.

[3] P. Bhattacharya and B.K. Lahiri, Semigeneralized closed sets in topology, Indian J. Math 29(1987), no.3, 375-382(1988).

[4] M. Caldas and S. Jafari, On some applications of b-open sets in topological spaces, Kochi. J.Math. 2(2007),11-19.

[5] J. Cao, M. Ganster and I. Reilly, Submaximality, extremal disconnectedness and generalazied closed sets, Houston J. Math 24(1998), no.4, 681-688.

[6] J. Dontchev, On generalizing semi-preopen sets, Mem. Fac. Sci. Kochi Univ Ser. A. Math 16(1995), 35-48.

[7] J. Dontchev, contra-continuous function and strongly s-closed spaces, Internal.J.Math.Math.Sci 19(1996), 303-310.

[8] J. Dontchev, M. Ganster and T. Noiri, On p-closed spaces, Int. J. Math. Sci.24(2000)no.3,203-212.

[9] M. Ganster and M. Steiner, On some questions about b-open sets, Questions Answers Gen. Topology 25(2007), no.1, 45-52.

[10] M. Ganster and M. Steiner, On b τ-closed sets. Appl. Gen. Topol 8(2007) no.2, 243-247.

[11] E.EkiciandM.Caldas,Slightly γ continuousfunctions,Bol.Parna,Mat.(3)22(2004).

[12] N. Levine, Generalized closed sets in topology, Rend. Circ. Mat. Palermo(2)19(1970),89-96.

[13] N. Levine, Semi-open sets and semi-continuity in topological spaces. Amer. Math. Monthly 70(1963), 36-41.

[14] A.A. Nasef, On-b-locally closed sets and related topics chaos solitons Fractals 12(2001) no.10, 1909-1915.

[15] O. Njastad, On some classes of nearly open sets pacific. J. Math. 15(1965), 961-970.

[16] H. Maki, R. Devi and K. Balachandran, Associated topologies of generalized α-closed sets and α-generalized closed sets Mem. Fac. Sci. Kochi. Univ. Ser. A. Math. 15(1994), 51-63.

[17] M.E. Abd El-Monsef, A.A. El-Atik and M.M. El-Sharkasy some topologies induced by b-open sets, kyungpook Math. J. 45(2005) no.4, 539-547.

[18] [1]Ahmad Al-Omari and [2]Mohd. Salmi Md. Noorani On Generalized b-closed sets. Bull. Malays. Math. Sci. Soc(2) 32(1) (2009), 19-30.

[19] W.Dunham, $T_{1/2}$-spaces,Kyungpook Math.J.17(1977),161-169.

[20] C.Dorsett,Semi- $T_{1/2}$-spaces, Ann.Soc.Sci.Bruxells,92(1978),143

[21] P.Bhattacharyya and B.K.Lahiri,Semi generalized closed sets in topology,IndianJ.Math29(1987),375-382.

[22] J.Umehara and T.Noiri,Every topological space is Pre- $T_{1/2}$-spaces,Mem.Fac.Sci.Kochi.Univ.Ser.A(Math).17(1996),33-42.

[23] J.Dontchev,On generalizing semipreopen sets,Mem.Fac.Sci.Kochi Uni.Ser.A.Math.,16(1995),35-48.

[24] Generalized α-closed and α-generalized closed maps Indian J pure Appl.Math.29(1)(1998),37-49.

[25] Y.Gnananbal,On generalized preregular sets in topological spaces,Indian j Pure Appl.Math.28)3)(1997),351-360.

[26] H.Mahi,K.Balachandran and R.Devi,Remarks on semi generalized closed and generalized semiclosed sets,Kyungpook Math.J.,36(1996),155-163.