Global Journal of Mathematical Sciences: Theory and Practical.
ISSN 0974-3200 Volume 3, Number 3 (2011), pp. 217-234

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The Derivative of Three Dimensional Variables

Bhekuzulu Khumalo

Abstract

This paper is in some sense a continuation of the paper “The Concept of the Mathematical Infinity and Economics”, but it is more a correction of the papers “Revisiting the Rate of Change”, and “Revisiting the Derivative: Implications of Rate of Change Analysis”, both papers by Bhekuzulu Khumalo. The two papers are philosophically wrong though the mathematics in those papers is correct. The two papers are wrong in that they both wrote of the derivative derived by Leibniz and Bhaskara not realising that the derivative behaves differently depending on the dimension of the independent variables, as was established in the paper “The Concept of the Mathematical Infinity and Economics”. This mathematical philosophy paper established that there are infinite forms of the derivative it is wise to change what was termed the khumalo derivative, $f\Box_k(x)$ to derivative of three dimensional variable, $f\Box_3(x)$

The paper seeks to develop a series for the derivative in the third dimension, derivative for a three dimensional independent variable which is just $X_i - X_{i-1}$, the paper successfully does that, an achievement on its own.

Humans generally discuss the concept of continuous and discrete variable, they never really take into account the dimension of the variable, that has been a great omission.

Introduction

There are several categories of variables available for the discerning scientist. The basic different class of variables are what can be considered qualitative variables as well as quantitative variables. To quote David Lane, “*Variables can be quantitative or qualitative. (Qualitative variables are sometimes called "categorical variables.") Quantitative variables are measured on an ordinal, interval, or ratio scale; qualitative variables are measured on a nominal scale. If five-year old subjects were asked to name their favorite color, then the variable would be qualitative. If the time it took them to respond were measured, then the variable would be quantitative*.” The difference between the qualitative and quantitative is easy enough to understand.

When we look at quantitative variables, they too can be classed broadly into discrete variables and continuous variables. For the purposes of this paper we must go right back to basics and define things so that we build up a logical argument. A discrete variable are "*variables have measurements that are distinct, periodic, and unconnected between data points (e.g. the distance an athlete throws a discus) and are represented by bar graphs. Student test scores are discrete and cannot be broken into smaller units. Each student has an unique mark (bar). There is nothing to connect the students together so a line between them is meaningless.*" (Jim Reed). We will return to this definition, however from answer.com we get a simple definition of a discrete variable, "*variable that assumes only values in a discrete set, such as the integers.*" (www.answer.com). These might be definitions from high school, but remember one carries these definitions throughout their efforts as a scientist.

A continuous variable on the other hand are "*variables that have continuous intervals that are unbroken sequences (e.g. growth of a plant) are represented by line graphs. Heating time and water temperature represent variables which are continuous and can be broken into smaller units (i.e. seconds and ^{o}C). Connecting data points with a line illustrates the pattern that may have occurred between measurements.*" (Jim Reed). Answer.com again gives a more simple definition, "*In statistics, a variable, such as time and temperature, whose measurements do not fall into discrete classes, but take any value over a defined range.*" (www.answer.com). We have defined variables that we use in science and we have different probability functions for example for different types of variables.

These simple classifications however are not enough, they are not enough for the scientist who desires that their measurements be accurate, variables also have another quality, the quality of the dimensions of each variable. Take the act of tossing a coin, one can either end up with heads or tails, one can never end up with say 2.3 heads, or 2.99 tails, it is either heads or tails, and one in the long run can have a probability function for example that would suggest that after many throws the probability of either getting heads or tails is 0.5, or 1:2, or 50% depending on the mathematical language one enjoys. Clearly we are dealing with a discrete variable. In many instances the outcome given discrete variables is very much random, by random it is meant that the outcome does not depend on the last outcome, just like tossing a coin, because it is heads in the last throw, it does not mean it will be heads in the next throw, the next outcome is completely independent from the last throw.

We therefore must again define a very important concept, a random outcome and a predictable outcome. This is the core of mathematical modeling, to try and predict the next number after the last event, and this predictability depends first of all on the randomness of the outcome. Continuous variables are by their nature easier to predict as they are not random, the next outcome depends largely on the outcome of the last event. There is what can be considered some sort of orderliness, some sort of pattern. Take solid water, ice. We can heat this ice until it melts and becomes a liquid and continue heating it until it boils and becomes gaseous. The amount of heat applied will determine the state of the material water, 0 ^{o}C of heat will keep it ice, 100 ^{o}C of heat will turn it to a gas. It will of course not turn to gas immediately but will reach towards the boiling point and the time will depend on how much heat is applied. This is an example of a continuous variable, as the water boils, it depends on how much heat was applied before, it is not a random event, it is continuous and the state of the water depends on the amount of heat and time that the water has been receiving that heat. There is a clear cut dependent and indepependent variable, however the variables are continuous.

In the paragraph above we have determined an independent variable, the amount of time the water is under heat, and a dependent variable, the heat of the water as well as the state of the water.

Let us take a phenomenon occurring in a laboratory, after a stimulus this phenomenon multiplies itself say every hour, that is after 1 hour it is 1 unit large, after 2 hrs it is 4 units large, after 3 hours it is 9 units large, that is to say it grows in the manner defined as $Y = X^2$ where Y is the size and X the time. This of course is a continuous independent variable, the outcomes are not random, the scientist can predict with great accuracy the size of the phenomenon next week if it is not destroyed and conditions for its growth remain the same, obviously they will need a large enough laboratory.

We call this a continuous variable because at 1 hour we know the growth, and 1.6675hrs we know the growth and at 8.654329...hrs we know the growth. Because, at 1 hr, 1.6675 hrs, or 8.654329...hrs the phenomenon must exist because time can be split like that. We can even calculate the rate of growth by taking the differential. There is a quality of the independent variable that is rarely mentioned and not discussed though it has fundamental implications, that is why most disciplines emphasise getting the basics right. Time is not just a continuous variable, as is seen in the paragraph above, but time is also 1 dimensional. This has very important implications. In the above example, that we can say the phenomenon growth rate is given by the relationship $Y = X^2$ by its nature means it is not a random variable, it is not discrete, we can predict the amount in the future to a reasonable extent as compared to a random variable where prediction is impossible, tossing a coin be it heads or tails is absolutely up to chance.

According to definition we can have continuous random variables, the height of a person for example, but this continuity is limited, a person for example cannot be 5 meters tall, the quality of the continuous is such that between 0 and say 2 meters a human being can be of any height. Therefore continuous in statistics and mathematics merely means the divisibility of a variable. This makes it therefore, difficult for a mind to grasp thus far what this paper is enlightening, and it is this that has made scientists miss a very important quality of a variable, the dimension of the variable.

Above it was discussed a phenomenon, that once stimulated grew at a rate defined as $Y = X^2$. What if we now said that Y was the growth of economic output given labour X. The growth in output is given by the function $Y = X^2$. Now we have a complication, we have a discrete variable labour, according to definition, you cannot have 1.5 labourers, it has to be a real whole number the set of aleph 0, ($\aleph_0$). One can put a good argument that labour is a discrete variable. However we know that the more labourers we add the more output will increase, we have a function for this increase $Y = X^2$, therefore our function is continuous, not random, it has no limit, we therefore have the characteristics of discrete, continuous, non random all at the same time. Why is this? It is because labour is three dimensional, therefore a polynomial of the function $Y = X^2$ can represent a 1 dimensional independent variable as well as a three dimensional independent variable, in fact we can say it can represent variables of infinite dimensions as we shall see. It is this quality that has been missed.

This might seem a small quality but it has huge implications on say the derivative of the polynomial given the function $Y = \alpha X^n$. As the function $Y = \alpha X^n$ is by its nature taken as a continuous function, for whatever value X we will have a Y value we can for example graph the following function in a line graph and it will look like figure 1. The common logic around such a function is given to us by Mark C, Chu-Carroll, "*For example, in continuous math, given the equation $y=x^2$, we can say that at any x, y is changing at a rate of 2x.*". However this logic is flawed as we shall see and as amplified in the paper, "The Concept of the mathematical Infinity and Economics" by Bhekuzulu Khumalo. The logic is flawed because when we understand the concept of the mathematical infinity we understand it is not necessarily true that at any x, y is changing at a rate of 2x, some x can not exist, only whole x's can exist given the dimension of the x variable. However we can look at figure 1 and figure 2.

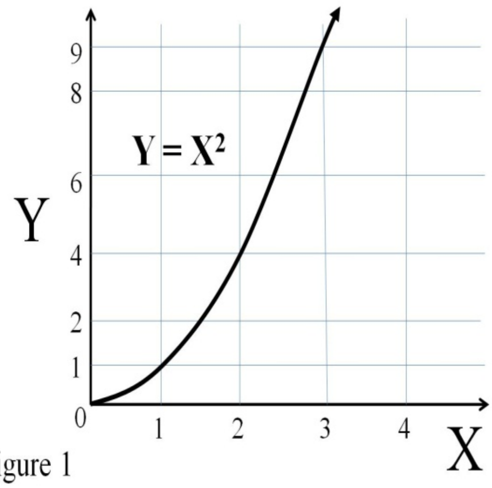

Figure 1

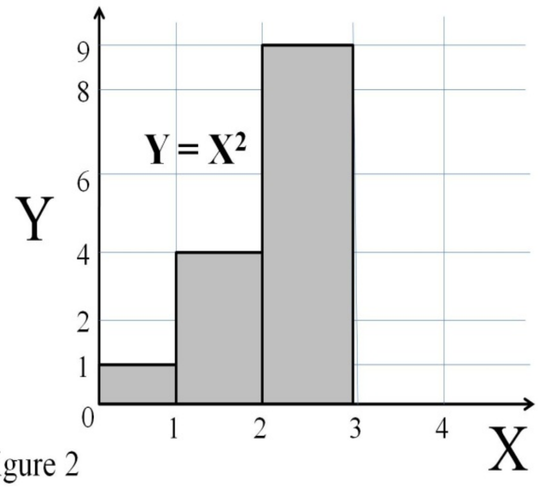

Figure 2

This concept was/ is amply discussed in the paper the "the concept of the mathematical infinity and economics" but it is worth mentioning here. For a one dimensional variable x, we use a line graph because the x is truly continuous in the way that has been understood thus far, one can get 1.245, 2.874528 or 3. Where as figure 2 draws the correct graph for a variable x that is three dimensional, but the function is still $Y = X^2$ however though continuous, we can not have 1.345, or 3.876, we must have a whole number 1,2,3,4,5... We have moved to another quality of a variable, the dimension of a variable. Figure 3 shows that there can be no tangent with bar graphs, the rate of change is fundamentally different when it comes to three dimensional x variables, the change is simply $X_i – X_{i-1}$. However for us scientists it is important to grasp the relationship, the function that defines this relationship $X_i – X_{i-1}$.

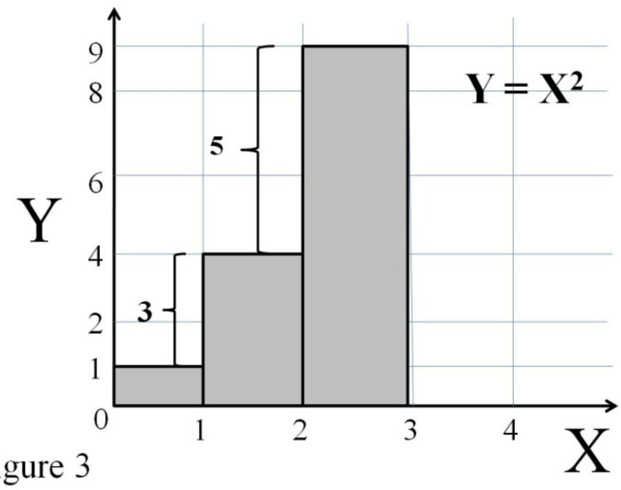

Figure 3

As we can see from figure 3 the because a 3 dimensional x can not have say 2.5, there can be no tangent, the changes are simply $X_i – X_{i-1}$, i.e. at X^3 the change is $3^2 – 2^2 = 9 – 4 = 5$. What would be the rate of change of adding one more unit to say 4, say we add one more labourer to get 4 labourers, that means we are moving from 3 units of labour to 4 units of labour, the rate of change will be $4^2 – 3^2 = 16 – 9 = 7$. Note however that using the traditional derivative created by Leibniz we would get 8, clearly different from 7. Why 8, because Leibniz defined the basic derivative as for a function $f(x) = \alpha X^n$, it follows that $f'(x) = n\alpha X^{n-1}$. Therefore for a three dimensional x variable we must modify the derivative created by Leibniz that has been in use for over 400 years.

From the logic developed in this paper as well as the logic developed in the paper "The concept of the Mathematical Infinity and Economics" it is clear that for one to understand the mathematics behind variables, they need to understand and not just take into account, the quality of whether a variable is discrete or continuous, they also need to take into account the dimensions of the variable, and in the later paper it was established that the general formula for the derivative was:

$$\text{Given } f(x) = \alpha X^n \tag{1}$$

Then:

$$f'_i(x) = n\alpha X^{n-1} - R_i \tag{2}$$

where R_i is the residual depending on the dimension of the x variable.

Hence R_1 is the residual for a one dimensional variable and

$R_1 = 0$ therefore with a one dimensional variable we have

$$f'_1(x) = n\alpha X^{n-1} \tag{3}$$

Note (3) is merely the derivative that we learn, that is taught. However a one dimensional variable can be broken up into infinite parts whereas a three dimensional variable can not. (2) refutes the conclusions that where drawn up from the paper's "Revisiting the Rate of Change" and "Revisiting the Derivative: Implications on the Rate of Change Analysis" by Bhekuzulu Khumalo but does not refute the mathematics. The conclusions where that the Leibniz derivative was wrong, this was because the author at the time did not realize it was right and the basis of all other subsequent derivatives depending on the dimension of the variable, the derivative created by Leibniz and Bhaskara is one dimensional and basis of all other derivatives. Remember when the variable has infinite dimensions $R_i = R_\infty = n\alpha X^{n-1}$ therefore $f'(x) = 0$. But that has been discussed in the paper "The Concept of the Mathematical Infinity and Economics".

For the rest of this paper we want to draw up the derivative for the three dimensional variable given a polynomial of the type $Y = \alpha X^n$. We will do each polynomial manually, the result must equal $X_i - X_{i-1}$.

The Derivative of a Three Dimensional Continuous variable Given the Polynomial $f(x) = \alpha X^n$ or $f(x) = X_i - X_{i-1}$

Given the polynomial function:

$$f(x) = \alpha X^n$$

The rate of change, the derivative for a three dimensional independent variable is:

$$f'_3(x) = n\alpha X^{n-1} - R_3 \tag{4}$$

For a three dimensional independent variable we use the khmalo derivative and it is given as:

$$f'_k(x) = n\alpha X^{n-1} - f'k(\alpha X^{n-1}) - R \tag{5}$$

where:

$$R_3 = f'(\alpha X^{n-1}) - R \tag{6}$$

And R is just a residual.

We shall do a step by step analysis so that the reader can understand, one can do these exercises alone simply by using Microsoft excel spread sheet or Google spreadsheets.

Let us take a function $f(x) = X$, i.e:

$\alpha = 1$

$n = 1$

To make our understanding easy let us say $f(x) = Y$, therefore our functions would be represented in the form $Y = \alpha X^n$.

Take table 1, it shows the manual method of deriving f'_k for a polynomial to the power of 1. For proper notation we shall now show the khumalo derivative as $f'_3(x)$, where the three represents that it is a derivative of a three dimensional variable.

Difference Between 'real difference' and differentiation of Y = X				
X	X = Y	diff	$f'(X) = 1$	%
0	0	n/a	0	N/A
1	1	1	1	100.00
2	2	1	1	100.00
3	3	1	1	100.00
4	4	1	1	100.00
5	5	1	1	100.00
6	6	1	1	100.00
7	7	1	1	100.00
8	8	1	1	100.00
9	9	1	1	100.00
10	10	1	1	100.00
11	11	1	1	100.00
12	12	1	1	100.00
13	13	1	1	100.00
14	14	1	1	100.00
15	15	1	1	100.00
Table 1				

In table 1 we see a column for X, we also see a column for Y represented as Y = X. The next column shows the difference between the Y values, for example when X = 12, Y = 12, and when X = 11 Y is 11 and the differences in the Y values is 1, infact Y grows in increments of 1, the change is always 1 so to say. Therefore when $Y = \alpha X^n$, and $\alpha = 1$ and $n = 1$ we have $Y = X$ and the rate of change for a three dimensional variable is equal to 1, or α. Therefore:

For a function $Y = \alpha X$ a linear function

$$f'_3(x) = \alpha \tag{7}$$

Note equation (7) is the same as for a derivative with one dimension, for a linear function, where $f(x) = \alpha X$ meaning that $f'(x) = \alpha$. Therefore with linear functions the derivative is the same, however one needs to just remember with a three dimensional independent variable, one can only have whole numbers, 1, 2 , 3 ,4 , 5 ..., one can not have 4.345 for example, that is a very important consideration.

The f'_3 gets more interesting however when we move up 1 degree on the polynomial. Let us move on to a polynomial with degree 2, such that we have a function $f(x) = X^2$. The relationship defined as X^2 is no longer a linear function.

Table 2 illustrates how we manually find the derivative of a three dimensional variable and the polynomial describing it is of two degrees.

Again when we look at table 2, we have a column for X, 1 – 15. We then have a column for Y and $Y = X^2$.

Difference Between 'real difference' and differentiation of $Y = X^2$					
X	$X^2 = Y$	diff	$f'(X) = 2X$	residual	%
0	0	n/a	0		N/A
1	1	1	2	1	200.00
2	4	3	4	1	133.33
3	9	5	6	1	120.00
4	16	7	8	1	114.29
5	25	9	10	1	111.11
6	36	11	12	1	109.09
7	49	13	14	1	107.69
8	64	15	16	1	106.67
9	81	17	18	1	105.88
10	100	19	20	1	105.26
11	121	21	22	1	104.76
12	144	23	24	1	104.35
13	169	25	26	1	104.00
14	196	27	28	1	103.70
15	225	29	30	1	103.45
20	400	39	40	1	102.56
30	900	59	60	1	101.69
50	2,500	99	100	1	101.01
Table 2					

The third column shows the difference between the Y values, for example when X = 10, Y = 100, and the difference is simply $f(10) - f(9) = 100 - 81 = 19$. Recalling equation (4), $f'_3(x) = n\alpha X^{n-1} - R_3$, therefore the next column is 2X, or $f'(x)$. The next column is the residual, and the residual is what is left over from the difference, and in each case it is 1 therefore, for $Y = X^2$, the derivative in the third dimension is $dy/dx = 2X - 1$. Therefore to generalize, given a function:

$$f(x) = \alpha X^2$$

$$f'_3(x) = 2\alpha X - \alpha \tag{8}$$

One of course can test this for themselves and they will find they will get the difference as described and illustrated in figure 3, that is the difference we want, because given a three dimensional variable we can not have a tangent, visualize figure 3.

In tables 2 and 3 there is a percentage column, this is not important in our calculations but is just there to show us the difference between Leibniz' derivative and the derivative for a third dimensional variable, the difference gets smaller with increases in the independent variable and with linear functions there is no difference at all.

The derivative of the derivative with a three dimensional independent variable becomes more complex as expected with the increase in the degree of the polynomial as we see when we get to $f(x) = \alpha X3$. Table 3 shows how we derive the $f'3(x)$ for such a function, again for ease of understanding we are using the function Y = X3. Again the first column is the X value, the second column is X3 = Y value, and the third column is the difference between the Y values, the difference merely being the Y value minus the previous Y value, this is so as that should be the value of the derivative given by the explanation and illustration for figures 1 -3.

				1	2	3	4	
X	$X^3 = Y$	diff	$f'(X) = 3X^2$	2X	$3X^2$ - 2X	residual	add 1	$f'_k(X)$
0	0	0	0	0	0	0		
1	1	1	3	2	1	0	1	1
2	8	7	12	4	8	1	2	7
3	27	19	27	6	21	2	3	19
4	64	37	48	8	40	3	4	37
5	125	61	75	10	65	4	5	61
6	216	91	108	12	96	5	6	91
7	343	127	147	14	133	6	7	127
8	512	169	192	16	176	7	8	169
9	729	217	243	18	225	8	9	217
10	1,000	271	300	20	280	9	10	271
11	1,331	331	363	22	341	10	11	331
12	1,728	397	432	24	408	11	12	397
13	2,197	469	507	26	481	12	13	469
14	2,744	547	588	28	560	13	14	547
15	3,375	631	675	30	645	14	15	631
Table 3								

Recalling equation (4), $f'_3(x) = n\alpha X^{n-1} - R_3$, and equation (5), $f'_k(x) = n\alpha X^{n-1} - f'k(\alpha X^{n-1}) - R$, we can attempt to derive $f'_3(x)$, recalling that $f'_3(x) = f'k(x)$, we use $f'_3(x)$ as it is the standard way to show the dimension of the variable, a 4 dimensional derivative would be for example $f'_4(x)$, but this paper only deals with three dimensional derivative, $f'_3(x)$.

Given equation (5), our next column is the Leibniz/ Bhaskara derivative, the basis of all other derivatives. The next column is $3X^2$. We know from equation 5 that we must subtract the derivative of the last αX^n, in this case X_2, and that is given to us by equation by equation (8), $f'_3(x) = 2\alpha X - \alpha$, for our purposes 2X -1, and we drop the 1, therefore we have a column for 2X. The next column is $3X^2 - 2X$. The next column is what is called the residual, and it is the difference between $3X^2 - 2X$ and the original difference in column 3.

If we look at the column residual we will see that the difference is merely X – 1, if we add 1 to that column we get X. Therefore the derivative of the third derivative given a function $Y = X^3$ is:

$$dy/\,dx_3 = 3X^2 - 2X - (X - 1)$$
$$dy/\,dx_3 = 3X^2 - 2X - X + 1$$
$$dy/\,dx_3 = 3X^2 - 3X + 1 \qquad (9)$$

We can now generalize the function of the derivative with a third degree polynomial given $f(x) = \alpha X^3$ the derivative would be:

$$f'3(x) = 3\alpha X^2 - 3\alpha X + \alpha \tag{10}$$

When we get to a polynomial of degree 4 the process evolves even more, but we should be able to see an interesting pattern once we get there. Table 4 shows how we get to the derivative of a 3 dimensional variable given $Y = X^4$. Once again the first column is X, the second column is our Y value, $Y = X^4$, the next column is difference in the Y values, or increments in Y, this is the value of the derivative. But we need a function. We return to equation (5) $f'_k(x) = n\alpha X^{n-1} - f'k(\alpha X^{n-1}) - R = f'3(x)$, and we add a column for $n\alpha X^{n-1}$ and that is $4X^3$. We then must take out the effects of the derivative $f'3(x)$ of the last polynomial, hence take out effects of equation (9), $3X^2 - 3X + 1$. We therefore have a column for 3X2 and 3X and these are subtracted from $4X^3$ as indicated by equation (5)

				take out of $3X^2$	take out of 3X	**1st Residual $4X^3$-$3X^2$-3X**	Ist Residual - diff	take out 1	take out X = A
X	$X^4 = Y$	diff	$f'(X) = 4X^3$	$3X^2$	3X	**$4X^3$-$3X^2$-3X**			
0	0	0	0	0	0	0	0	-1	#DIV/0!
1	1	1	4	3	3	-2	-3	-4	-4
2	16	15	32	12	6	14	-1	-2	-1
3	81	65	108	27	9	72	7	6	2
4	256	175	256	48	12	196	21	20	5
5	625	369	500	75	15	410	41	40	8
6	1,296	671	864	108	18	738	67	66	11
7	2,401	1,105	1,372	147	21	1,204	99	98	14
8	4,096	1,695	2,048	192	24	1,832	137	136	17
9	6,561	2,465	2,916	243	27	2,646	181	180	20
10	10,000	3,439	4,000	300	30	3,670	231	230	23
11	14,641	4,641	5,324	363	33	4,928	287	286	26
12	20,736	6,095	6,912	432	36	6,444	349	348	29
13	28,561	7,825	8,788	507	39	8,242	417	416	32
14	38,416	9,855	10,976	588	42	10,346	491	490	35
15	50,625	12,209	13,500	675	45	12,780	571	570	38
Table 4									

At this point we thus have the expression

$$dy/dx_3 = 4X^3 - 3X^2 - 3X - R \tag{11}$$

where R is considered as the first residual in Table 4. With taking out 1 we find that what we have left over we can take out X and we and the final column can be expressed as $3X - 7$. Thus R can be expressed as:

$$R = (3X - 7)X + 1 \text{ simplified to}$$

$$R = 3X^2 - 7X + 1 \tag{12}$$

Hence our derivative for a polynomial of the 4^{th} degree would be:

$$dy/dx_3 = 4X^3 - 3X^2 - 3X - (3X^2 - 7X + 1) \text{ and this is equal to}$$

$$dy/dx_3 = 4X^3 - 3X^2 - 3X - 3X^2 + 7X - 1 \text{ simplified this is equal to}$$

$$dy/dx_3 = 4X^3 - 6X^2 + 4X - 1 \tag{13}$$

We can now generalize the function of the derivative of a three dimensional variable with a fourth degree polynomial given $f(x) = \alpha X^4$ the derivative would be:

$$f'_3(x) = 4\alpha X^3 - 6\alpha X^2 + 4\alpha X - \alpha \quad (14)$$

This paper will do one more derivative of a power, that ends up as a polynomial. Given a power of degree 5, we can also work out the derivative given $f(x) = X^5$, we can using the same techniques work out the derivative for a three dimensional variable. Again we just use a spreadsheet, in my case I used a Excel spreadsheet a product of Microsoft, but I am sure Google's spreadsheet, or lotus, or the free spreadsheets one can get over the internet.

We have table 5 to help us understand how we get the derivative for $f(x) = \alpha X^5$,as one can see the process is becoming more and more tedious. The first column of table 5 shows the X value, 1 – 15. The second column shows the column $Y = X^5$ and the third column shows the difference between the last y value. The fourth column shows the Leibniz derivative in this case simply $5X^4$.

X	$X^5 = Y$	diff	$f'(X) = 5X^4$	%	$-4X^3$	$+6X^2$	$-4X$	residual	2nd diff	add 1	take out X	take out 1	take out X	6X-4	$f'_3(X^5)$
0	0	n/a	0	N/A	0	0	0	0	#VALUE!	#VALUE!	#VALUE!	#VALUE!	#VALUE!	-4	0
1	1	1	5	500.00	4	6	4	3	2	3	3	2	2	2	1
2	32	31	80	258.06	32	24	8	64	33	34	17	16	8	8	31
3	243	211	405	191.94	108	54	12	339	128	129	43	42	14	14	211
4	1,024	781	1,280	163.89	256	96	16	1,104	323	324	81	80	20	20	781
5	3,125	2,101	3,125	148.74	500	150	20	2,755	654	655	131	130	26	26	2,101
6	7,776	4,651	6,480	139.32	864	216	24	5,808	1,157	1,158	193	192	32	32	4,651
7	16,807	9,031	12,005	132.93	1,372	294	28	10,899	1,868	1,869	267	266	38	38	9,031
8	32,768	15,961	20,480	128.31	2,048	384	32	18,784	2,823	2,824	353	352	44	44	15,961
9	59,049	26,281	32,805	124.82	2,916	486	36	30,339	4,058	4,059	451	450	50	50	26,281
10	100,000	40,951	50,000	122.10	4,000	600	40	46,560	5,609	5,610	561	560	56	56	40,951
11	161,051	61,051	73,205	119.91	5,324	726	44	68,563	7,512	7,513	683	682	62	62	61,051
12	248,832	87,781	103,680	118.11	6,912	864	48	97,584	9,803	9,804	817	816	68	68	87,781
13	371,293	122,461	142,805	116.61	8,788	1,014	52	134,979	12,518	12,519	963	962	74	74	122,461
14	537,824	166,531	192,080	115.34	10,976	1,176	56	182,224	15,693	15,694	1,121	1,120	80	80	166,531
15	759,375	221,551	253,125	114.25	13,500	1,350	60	240,915	19,364	19,365	1,291	1,290	86	86	221,551
Table 5															

Having arrived at the one dimensional derivative created by Leibniz and Bhaskara, the foundation of all other derivatives, we take our guidance from equations (4) and (6) where:

(4) = $f'_3(x) = n\alpha X^{n-1} - R_3$ and

(6) = $R_3 = f'(\alpha X^{n-1}) - R$

Our objective is to work out R3 as we already know $n\alpha X^{n-1} = 5X^4$. Working out R_3, we already know $f'(\alpha X^{n-1})$ = equation (14) = $4\alpha X^3 - 6\alpha X^2 + 4\alpha X - \alpha$. Therefore we have

$dy/dx_3 = 5X^4 - (4X^3 - 6X^2 + 4X) - R$ we drop α in the solving process. This becomes:

$$dy/dx_3 = 5X^4 - 4X^3 + 6X^2 - 4X - R \quad (15)$$

This is clearly shown in Table 5 for columns 6, 7, and 8. After the column for -4X we have a column titled 'residual', this is R that we must solve for. The column next to residual is termed '2nd difference', it is merely the residual column minus the third column entitled difference, 'diff'. The next column we add 1, we know we add one because for the previous derivative we subtracted 1, when we add one we find we can neatly take out X. Having taken out X, that is to say divided by X, we find that if we take out 1 agian, taking out one meaning minus one, we have values that are defined by the function 6X – 4. We have thus solved for R in equation (6). To get to R we reverse all we have done as follows:

$R = [([(6X - 4)X] + 1)X] - 1$ simplification leads to
$R = (6X^2 - 4X + 1)X - 1$ further simplification leads to
$R = 6X^3 - 4X^2 + X - 1$

Taking guidance again from equations (4) and (6) and (15)

$dy/dx_3 = 5X^4 - 4X^3 + 6X^2 - 4X - (6X^3 - 4X^2 + X - 1)$
$dy/dx_3 = 5X^4 - 4X^3 + 6X^2 - 4X - 6X^3 + 4X^2 - X + 1$
$$dy/dx_3 = 5X^4 - 10X^3 + 10X^2 - 5X + 1 \quad (16)$$

We can now generalize the function of the derivative of a three dimensional variable with a fifth degree polynomial given $f(x) = \alpha X^5$ the derivative would be:

$$f'_3(x) = 5\alpha X^4 - 10\alpha X^3 + 10\alpha X^2 - 5\alpha X + \alpha \quad (17)$$

Finding the Pattern

Table 6 illustrates the differences between the derivative for a one dimensional independent variable like time, $f'(x)$, and the derivative for a three dimensional independent variable like labour, $f'_3(x)$.

Power	**Derivative $f'(x)$**	**Derivative $f'_3(x)$**
αX	α	α
αX^2	$2\alpha X$	$2\alpha X - \alpha$
αX^3	$3\alpha X^2$	$3\alpha X^2 - 3\alpha X + \alpha$
αX^4	$4\alpha X^3$	$4\alpha X^3 - 6\alpha X^2 + 4\alpha X - \alpha$
αX^5	$5\alpha X^4$	$5\alpha X^4 - 10\alpha X^3 + 10\alpha X^2 - 5\alpha X + \alpha$
Table 6		

From table 6 one can see that the derivatives are different but in percentage terms as the values get larger the difference is always decreasing, this derivative just adds more precision, because though decreasing in percentage terms the number difference is getting larger and larger.

As scientists, even if background is economics, we would like a series; we would not like to manually have to do every derivative in the third dimension of a power. Is there a pattern, because a pattern would be important in helping us be able to have derivatives without going through the tedious manual process as we have done above? Let us discuss table 7.

Power	Derivative $f'_3(x)$
αX	α
αX^2	$2\alpha X - \alpha$
αX^3	$3\alpha X^2 - 3\alpha X + \alpha$
αX^4	$4\alpha X^3 - 6\alpha X^2 + 4\alpha X - \alpha$
αX^5	$5\alpha X^4 - 10\alpha X^3 + 10\alpha X^2 - 5\alpha X + \alpha$
Table 7	

What we do see is some interesting patterns within patterns.

On the right hand side of the $f'3(x)$ column we see that α is always that except it changes in a deterministic manner from positive to negative and so on in a deterministic manner. So for a derivative, $f'_3(x)$ of αX^6, we can expect α to be negative.

We also not that X to the power of one also seems to rise in a deterministic manner of 1 unit, changing from positive to negative, hence we have $2\alpha X$, - $3\alpha X$, $4\alpha X$, - $5\alpha X$,...We would be correct to expect derivative of to have variable $6\alpha X$.

We also know that the first term in the resulting polynomial function is always the Leibniz derivative., always of the form $n\alpha X^{n-1}$ given a power of the type αX^n.

Therefore we need to know the middle parts. It is interesting that when we look at third dimensional derivative for αX^4 we find that the term immediately after the Leibniz derivative, $-6\alpha^2$ is merely the addition of the last two digits(in absolute terms to get to the coefficient) of the derivative for αX^3. This seems not to be a coincidence because when we look at the third dimensional derivative for αX5, we find that 10α2 is equal to adding the two previous digits (in absolute number terms) of the derivative of αX^4. This is more clearly shown in Table 8.

Power	Derivative $f'_3(x)$
αX	α
αX^2	$2\alpha X - \alpha$
αX^3	$3\alpha X^2 - 3\alpha X + \alpha$
αX^4	$4\alpha X^3 - 6\alpha X^2 + 4\alpha X - \alpha$
αX^5	$5\alpha X^4 - 10\alpha X^3 + 10\alpha X^2 - 5\alpha X + \alpha$
Table 8	

Table 8 illustrates what has been discussed above, from this information we should be able to predict the third dimensional derivative for say αX^6, αX^7... to infinity.

What is $f'_3(x)$ of αX^6, we still need to work them out from previous information. We can do this from information contained in $f'_3(x)$ for αX^5. We know the first term must be the Leibniz derivative of αX^6 that is equal to $6\alpha X^5$.

From the logic built up explained for Table 8. The next term must be $-15\alpha X^4$, from adding 10 and 5. The next term should be $20\alpha X^3$, from adding 10 and 10. The next term should be $-15\alpha^2$, from adding 10 and 5. The next term should be $6\alpha X$ and last term $-\alpha$.

The third dimensional derivative for $\alpha X^6 = f'_3(X^6) =$

$$6\alpha X^5 - 15\alpha X^4 + 20\alpha X^3 - 15\alpha X^2 + 6\alpha X - \alpha \tag{18}$$

Because of the deterministic nature of most mathematics once you find a law it should work all the time, from equation 18 we should be able to find the third dimensional derivative for αX^7. We know the first term must always be the Leibniz derivative, the Leibniz derivative for αX7 is equal to 7αX6.

From the logic built up for table 8 that was used to derive equation (18), the next term should be $-21\alpha X^5$, adding 6 and 15 from equation 18. The next term should be $35\alpha X^4$, adding 20 and 15 from equation (18). The next term should be $-35\alpha X^3$ adding what should be clear the absolute numbers to get the coefficient in this case 15 and 20 from equation (18). The next term should be $21\alpha X^2$. The next term is -7α and the last term of course α.

The third dimensional derivative of $\alpha X^7 = f'_3(X^7) =$

$$7\alpha X^6 - 21\alpha X^5 + 35\alpha X^4 - 35\alpha X^3 + 21\alpha X^2 - 7\alpha + \alpha \tag{19}$$

One can verify equations (18) and (19) with a simple spreadsheet as illustrated in table 9. Table 9 shows two simple functions, $Y = X^6$ and $Y = X^7$. Say X is labor, we must use a three dimensional formula for the derivative, this formula at its most simple logic is equal to $X_i - X_{i-1}$, and from table 9 one can see that the difference is equal to the derivative.

X	$X^6 = Y$	diff	$6X^5 - 15X^4 + 20X^3 - 15X^2 + 6X - 1$ $f'_3(X^6)$	$X^7 = Y$	diff	$7X^6 - 21X^5 + 35X^4 - 35X^3 + 21X^2 - 7X + 1$ $f'_3(X^7)$
1	1	1	1	1	1	1
2	64	63	63	128	127	127
3	729	665	665	2,187	2,059	2,059
4	4,096	3,367	3,367	16,384	14,197	14,197
5	15,625	11,529	11,529	78,125	61,741	61,741
6	46,656	31,031	31,031	279,936	201,811	201,811
7	117,649	70,993	70,993	823,543	543,607	543,607
8	262,144	144,495	144,495	2,097,152	1,273,609	1,273,609
9	531,441	269,297	269,297	4,782,969	2,685,817	2,685,817
10	1,000,000	468,559	468,559	10,000,000	5,217,031	5,217,031
11	1,771,561	771,561	771,561	19,487,171	9,487,171	9,487,171
12	2,985,984	1,214,423	1,214,423	35,831,808	16,344,637	16,344,637
13	4,826,809	1,840,825	1,840,825	62,748,517	26,916,709	26,916,709
14	7,529,536	2,702,727	2,702,727	105,413,504	42,664,987	42,664,987
15	11,390,625	3,861,089	3,861,089	170,859,375	65,445,871	65,445,871

Table 9

Having this information and technique one does no longer need to do heavy manual mathematics to get to the derivative as we only had to do that to establish a pattern Table 10 shows the derivatives in third dimensions.

Power	Derivative $f'(x)$	Derivative $f'_3(x)$
αX	α	α
αX^2	$2\alpha X$	$2\alpha X - \alpha$
αX^3	$3\alpha X^2$	$3\alpha X^2 - 3\alpha X + \alpha$
αX^4	$4\alpha X^3$	$4\alpha X^3 - 6\alpha X^2 + 4\alpha X - \alpha$
αX^5	$5\alpha X^4$	$5\alpha X^4 - 10\alpha X^3 + 10\alpha X^2 - 5\alpha X + \alpha$
αX^6	$6\alpha X^5$	$6\alpha X^5 - 15\alpha X^4 + 20\alpha X^3 - 15\alpha X^2 + 6\alpha X - \alpha$
αX^7	$7\alpha X^6$	$7\alpha X^6 - 21\alpha X^5 + 35\alpha X^4 - 35\alpha X^3 + 21\alpha X^2 - 7\alpha X + \alpha$
αX^8	$8\alpha X^7$	$8\alpha X^7 - 28\alpha X^6 + 56\alpha X^5 - 70\alpha X^4 + 56\alpha X^3 - 28\alpha X^2 + 8\alpha X - \alpha$
αX^9	$9\alpha X^8$	$9\alpha X^8 - 36\alpha X^7 + 84\alpha X^6 - 126\alpha X^5 + 126\alpha X^4 - 84\alpha X^3 + 36\alpha X^2 - 9\alpha X + \alpha$
αX^{10}	$10\alpha X^9$	$10\alpha X^9 - 45\alpha X^8 + 120\alpha X^7 - 210\alpha X^6 +252\alpha X^5 - 210\alpha X^4 + 120\alpha X^3 - 45\alpha X^2 + 10\alpha X - \alpha$

Table 10

The Series for the derivative for third dimensional derivative/ $X_i - X_{i-1}$

Given a function $f(x) = \alpha X^{n-1}$, the derivative for a three dimensional variable would be given as:

$$f'_3(x) = (n-1)\alpha X^{n-2} - C_1\alpha X^{n-3} + C_2\alpha X^{n-4} - C_3\alpha X^{n-5} + C_4\alpha X^{n-6} - C_5\alpha X^{n-7} + C_6\alpha X^{n-8} - C_7\alpha X^{n-9} + \ldots \pm C_{n-1}\alpha X \pm C_n\alpha \quad (20)$$

where

$C_n-1 = n-1$

C_n is 1. If n-1 is even C_n is negative and if n-1 is odd C_n is positive.

Given a second function, $f(x) = \alpha X^n$ then the derivative for the three dimensional variable would be given as:

$$f'_3(x) = n\alpha X^{n-1} - [(n-1) + |C_1|]\alpha X^{n-2} + [|C_1| + |C_2|]\alpha X^{n-3} - [|C_2| + |C_3|]\alpha X^{n-4} + [|C_3| + |C_4|]\alpha X^{n-5} - [|C_4| + |C_5|]\alpha X^{n-6} + [|C_5| + |C_6|]\alpha X^{n-7} - [|C_6| + |C_7|]\alpha X^{n-8} + [|C_7| + |C_8|]\alpha X^{n-9} - \ldots \pm [|C_{n-1}| + |C_n|]\alpha X \pm [|C_n| + 0]\alpha \quad (21)$$

Some Examples

Take the function

$$f(x) = \alpha X \quad (22)$$

Given equation 8 we need to know function for α and it is zero

Therefore the function for $X_i - X_{i-1}$ = given $f(x) = \alpha X$ is:

$$f'_3(x) = \alpha$$

Take the function

$$f(x) = \alpha X^2 \tag{23}$$

We would like to find out the formula that defines $f\square_3(x) / X_i - X_{i-1}$, given equation (23)

From equation (21) we know we must have:

$f\square_3(x) = 2\alpha X - [(n-1) + |(C_1)| =$

$2\alpha X - [1 + |0|]\alpha X^{n-2}$ therefore

$2\alpha X - \alpha X^0$

$f\square_3(x) = 2\alpha X - \alpha$

(22) = (8)

Take the function

$$f(x) = \alpha X^3 \tag{23}$$

We would like to find out the formula that defines $f\square_3(x)/ X_i - X_{i-1}$ given equation (23)

From equation (21) we know we must have:

$f\square_3(x) = 3\alpha X^2 - [(n-1) + |C_1|]\ \alpha X^{n-2} + [|C_1| + |C_2|]\alpha X^{n-3}$

$f\square_3(x) = 3\alpha X^2 - [2 + |1|]\alpha X^1 + [|1| + |0|]\alpha X^0$

$f\square_3(x) = 3\alpha X2 - 3\alpha X + \alpha$

(24) = (10)

Take the function

$$f(x) = \alpha X^4 \tag{25}$$

We would like to find out the formula that defines $f\square_3(x)/ X_i - X_{i-1}$ given equation (25)

From equation (21) we know we must have:

$f\square_3(x) = 4\alpha X^3 - [(n-1) + |C1|]\alpha X^{n-2} + [|C1| + |C2|]\alpha X^{n-3} - [|C2| + |C3|]\alpha X^{n-4} =$

$f\square_3(x) = 4\alpha X^3 - [3 + |-3|]\alpha X^2 - [|-3| + |1|]\alpha X^1 - [|1| + |0|]\alpha X^0 =$

$f\square_3(x) = 4\alpha X^3 - 6\alpha X^2 + 4\alpha X^1 - 1\alpha X^0 =$

$f\square_3(x) = 4\alpha X^3 - 6\alpha X^2 + 4\alpha X - \alpha$

(26) = (14)

Properties of $f'_3(x)$

$$\sum_{i=1}^{n} Ci = 1$$

Therefore $\sum_{i=1}^{n} Ci - C_n = 0$ or 2. 0 if n is odd and 2 if n is positive.

The first C variable is the same as the last C variable taking the last α out. The second C variable is the same as the second last C variable. The third C variable is the same as the third last C variable and so on not minding the negative or positive signs. This can be easier illustrated by looking at a real derivative. Take for example the derivative of a third dimensional variable defined as $f(x) = \alpha X10$, then:

$$f'3(x) = 10\alpha X^9 - 45\alpha X^8 + 120\alpha X^7 - 210\alpha X^6 + 252\alpha X^5$$
$$- 210\alpha X^4 + 120\alpha X^3 - 45\alpha X^2 + 10\alpha X - \alpha$$

taking out the last value, α, one can see for themselves that the first digit is equal to the last digit both are 10. The second digit is equal to the second last digit both are 45. The third digit is equal to the third last C both are 120 and so on. When the first digit is even then signs are the same, when first digit is odd the signs are opposite. There is what we can symmetry with the C coefficients.

Conclusions

One can only hope that the concepts above are appreciated and that a new and vast array into the calculus has opened up, we should be able to do the same with all types of functions if we put our mind to it, polynomials, powers, trigonometric, rational, exponential, and logarithmic, given enough time and resources. The integrals should also be affected. One can be sure that when physicists for example discuss four and five dimensions, they miss understanding stems as a result from the mathematics not been clear, hopefully this paper has shed light on the possibilities of trans – dimensional mathematics, even if it remains just theoretic and finds a use 50 years in the future. As a fellow seeker of knowledge one can only hope you enjoyed reading this paper.

Special Thanks

I would like to thank very much Guido Travaglini without him I would not have found the series that defines $f\square_3(x)$ or $f\square_k(x)$ as we called it at the beginning, he insisted there must be a series whilst I insisted you need a computer, alas he was right and the series has been presented to you, I would have given up on finding a series and left it for somebody else or another generation, but the series is there and what beautiful symmetry it has. And special thanks to you the reader, without you, why bother seek new knowledge.

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