Global Journal of Mathematical Sciences: Theory and Practical.
ISSN 0974-3200 Volume 3, Number 3 (2011), pp. 245-255

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# On The Stability of a Four Species: A Prey-Predator-Host-Commensal-Mutual-Syn-Eco-System-III (Predator Washed out State)

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**Abstract**

This paper deals with an investigation on a four species syn-ecological system (Predator Washed out State). The system comprises of a prey ($S_1$) , a predator ($S_2$) that survives upon $S_1$, two hosts $S_3$ and $S_4$ for which $S_1$, $S_2$ are commensal respectively i.e., $S_3$ and $S_4$ benefit $S_1$ and $S_2$ respectively, without getting effected either positively or adversely. Further $S_3$ and $S_4$ are mutual. The model equations of the system constitute a set of four first order non-linear ordinary differential coupled equations. In all, there are sixteen equilibrium points. Criteria for the asymptotic stability of one of the sixteen equilibrium points: the predator washed out state is established. The system would be stable if all the characteristic roots are negative, in case they are real, and have negative real parts, in case they are complex. The linearised equations for the perturbations over the equilibrium point are analyzed to establish the criteria for stability and the trajectories illustrated.

**Keywords:** Commensal, Eco-system, Equilibrium points, Host, Mutual, Prey, Predator, Stability, Trajectories.

## Introduction

Ecology is the scientific study of the relation of living organisms to each other and their surroundings. An ecosystem is a system whose members benefit from each other's participation via symbiotic relationships. Significant researches in the area of theoretical ecology had been initiated by Lotka [10] and by Volterra [16]. The general concepts of modeling have been presented in the treatises of Meyer [11], Kushing [7] and Kapur [5, 6].

K. Lakshminarayan and N.Ch. Pattabhi Ramacharyulu [8] studied the two species prey-predator ecological models incorporating a partial cover for the prey and alternate food for the predator. These authors have also analysed a prey-predator model with alternative food for the predator, harvesting of both the species [9]. The study on competitive eco-systems of two and three species with limited and unlimited resources was done by N.C. Srinivas [15]. R. Archana Reddy [1, 2] and B. Bhaskara Rama Sharma [3] investigated on interacting species with harvesting of both the species at constant rate and competitive eco-systems with time delay, employing analytical and numerical techniques. Further study on the stability of a Host – a flourishing commensal species pair with limited resources was done by N. Phani Kumar, N. Seshagiri Rao and N.Ch. Pattabhi Ramacharyulu [12]. The stability analysis of a four species eco-system with the interaction between $S_3$ and $S_4$ is neutralism was considered by B. Hari Prasad and N.Ch. Pattabhi Ramacharyulu [4]. Following this N. Shanker, K. Lakshminarayan and N.Ch. Pattabhi Ramacharyulu studied stability analysis of a four species eco-system with the interaction between $S_3$ and $S_4$ being mutual [13, 14].

The present investigation is on an analytical study of a four species ($S_1$, $S_2$, $S_3$, $S_4$) Prey-Predator-Host-Commensal-Mutual-Syn Eco-System. Fig.1 shows a Schematic Sketch of the system under investigation. In all sixteen equilibrium points are identified based on model equations and the stability analysis is carried out only for the predator washed out state.

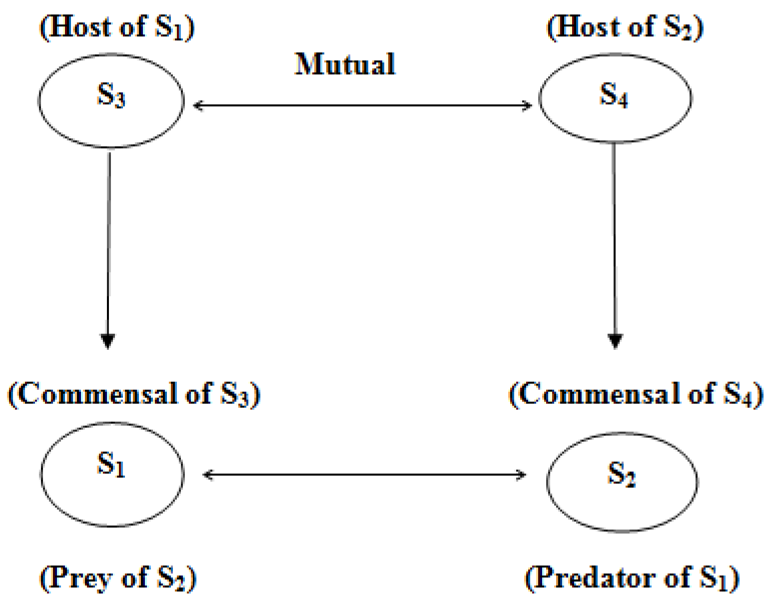


**Figure 1:** Schematic Sketch of the Syn Eco-System under investigation.

## Notation Adopted

$N_1$ (t) : The population of the prey species ($S_1$)

$N_2$ (t) : The population of the predator species ($S_2$)

$N_3$ (t) : The population of the host species ($S_3$) of the prey ($S_1$)

$N_4$ (t) : The population of the host ($S_4$) of the predator ($S_2$)

t : Time instant

$a_1, a_2, a_3, a_4$ : Natural growth rates of $S_1$, $S_2$, $S_3$, $S_4$

$a_{11}, a_{22}, a_{33}, a_{44}$ : Self inhibition coefficients of $S_1$, $S_2$, $S_3$, $S_4$

$a_{12}, a_{21}$ : Interaction (prey-predator) coefficients of $S_1$ due to $S_2$ and $S_2$ due to $S_1$

$a_{13}$ : Coefficient of commensalism of $S_3$ towards $S_1$

$a_{24}$ : Coefficient of commensalism of $S_4$ towards $S_2$

$a_{34}$ : Coefficient of mutualism of $S_4$ towards $S_3$

$a_{43}$ : Coefficient of mutualism of $S_3$ towards $S_4$

$K_i = \dfrac{a_i}{a_{ii}}$ : *Carrying* capacity of $S_i$, i=1,2,3,4

Further the variables $N_1$, $N_2$, $N_3$, and $N_4$ are non-negative and the model parameters $a_1$, $a_2$, $a_3$, $a_4$, $a_{11}$, $a_{22}$, $a_{33}$, $a_{44}$, $a_{12}$, $a_{21}$, $a_{13}$, $a_{24}$, $a_{34}$, $a_{43}$ are assumed to be non-negative constants.

## Basic Model Equations

The model equations for the growth rates of $S_1$, $S_2$, $S_3$, $S_4$ are

$$\frac{dN_1}{dt} = a_1 N_1 - a_{11} N_1^2 - a_{12} N_1 N_2 + a_{13} N_1 N_3 \tag{3.1}$$

$$\frac{dN_2}{dt} = a_2 N_2 - a_{22} N_2^2 + a_{21} N_1 N_2 + a_{24} N_2 N_4 \tag{3.2}$$

$$\frac{dN_3}{dt} = a_3 N_3 - a_{33} N_3^2 + a_{34} N_3 N_4 \tag{3.3}$$

$$\frac{dN_4}{dt} = a_4 N_4 - a_{44} N_4^2 + a_{43} N_3 N_4 \tag{3.4}$$

## Equilibrium states

The system under investigation has sixteen equilibrium states defined by

$$\frac{dN_i}{dt} = 0 \quad , \qquad i = 1, 2, 3, 4 \tag{4.1}$$

which are given in the following table.

**Table 1:** Equilibrium states.

| S.No. | Equilibrium States | Equilibrium Point |
|---|---|---|
| 1 | Fully washed out state | $\overline{N_1} = 0, \overline{N_2} = 0, \overline{N_3} = 0, \overline{N_4} = 0$ |
| 2 | Only the prey $S_1$ survives | $\overline{N_1} = \frac{a_1}{a_{11}}, \overline{N_2} = 0, \overline{N_3} = 0, \overline{N_4} = 0$ |
| 3 | Only the predator $S_2$ survives | $\overline{N_1} = 0, \overline{N_2} = \frac{a_2}{a_{22}}, \overline{N_3} = 0, \overline{N_4} = 0$ |
| 4 | Only the host ($S_3$) of $S_1$ survives | $\overline{N_1} = 0, \overline{N_2} = 0, \overline{N_3} = \frac{a_3}{a_{33}}, \overline{N_4} = 0$ |
| 5 | Only the host ($S_4$) of $S_2$ survives | $\overline{N_1} = 0, \overline{N_2} = 0, \overline{N_3} = 0, \overline{N_4} = \frac{a_4}{a_{44}}$ |
| 6 | Prey ($S_1$) and the predator ($S_2$) survives | $\overline{N_1} = \frac{a_1 a_{22} - a_2 a_{12}}{a_{11} a_{22} + a_{12} a_{21}}, \overline{N_2} = \frac{a_2 a_{11} + a_1 a_{21}}{a_{11} a_{22} + a_{12} a_{21}}, \overline{N_3} = 0, \overline{N_4} = 0$ |
| 7 | Predator ($S_2$) and the host ($S_4$) of $S_2$ washed out | $\overline{N_1} = \frac{a_1 a_{33} + a_3 a_{13}}{a_{11} a_{33}}, \overline{N_2} = 0, \overline{N_3} = \frac{a_3}{a_{33}}, \overline{N_4} = 0$ |
| 8 | Predator ($S_2$) and the host ($S_3$) of $S_1$ washed out | $\overline{N_1} = \frac{a_1}{a_{11}}, \overline{N_2} = 0, \overline{N_3} = 0, \overline{N_4} = \frac{a_4}{a_{44}}$ |
| 9 | Prey ($S_1$) and the host ($S_4$) of $S_2$ washed out | $\overline{N_1} = 0, \overline{N_2} = \frac{a_2}{a_{22}}, \overline{N_3} = \frac{a_3}{a_{33}}, \overline{N_4} = 0$ |
| 10 | Prey ($S_1$) and the host($S_3$) of $S_1$ washed out | $\overline{N_1} = 0, \overline{N_2} = \frac{a_2 a_{44} + a_4 a_{24}}{a_{22} a_{44}}, \overline{N_3} = 0, \overline{N_4} = \frac{a_4}{a_{44}}$ |
| 11 | Prey ($S_1$) and the predator ($S_2$) washed out | $\overline{N_1} = 0, \overline{N_2} = 0, \overline{N_3} = \frac{\alpha_2}{\alpha_1}, \overline{N_4} = \frac{\alpha_3}{\alpha_1}$<br>where $\alpha_1 = a_{33} a_{44} - a_{34} a_{43}$<br>$\alpha_2 = a_3 a_{44} + a_4 a_{34}$<br>$\alpha_3 = a_4 a_{33} + a_3 a_{43}$ |

| | | |
|---|---|---|
| 12 | Only the host ($S_4$) of $S_2$ washed out | $\overline{N_1}=\dfrac{\beta_2}{\beta_1},\overline{N_2}=\dfrac{\beta_3}{\beta_1},\overline{N_3}=\dfrac{a_3}{a_{33}},\overline{N_4}=0$ <br> *where* $\beta_1=a_{33}(a_{11}a_{22}+a_{12}a_{21})$ <br> $\beta_2=a_1a_{22}a_{33}+a_3a_{13}a_{22}-a_2a_{12}a_{33}$ <br> $\beta_3=a_2a_{11}a_{33}+a_1a_{21}a_{33}+a_3a_{13}a_{21}$ |
| 13 | Only the host($S_3$) of $S_1$ washed out | $\overline{N_1}=\dfrac{\theta_2}{\theta_1},\overline{N_2}=\dfrac{\theta_3}{\theta_1},\overline{N_3}=0,\overline{N_4}=\dfrac{a_4}{a_{44}}$ <br> *where* $\theta_1=a_{44}(a_{11}a_{22}+a_{12}a_{21})$ <br> $\theta_2=a_1a_{22}a_{44}-a_2a_{12}a_{44}-a_4a_{12}a_{24}$ <br> $\theta_3=a_2a_{11}a_{44}+a_4a_{11}a_{24}+a_1a_{21}a_{44}$ |
| 14* | Only the Predator ($S_2$)washed out | $\overline{N_1}=\dfrac{\psi}{a_{11}\alpha_1},\overline{N_2}=0,\overline{N_3}=\dfrac{\alpha_2}{\alpha_1},\overline{N_4}=\dfrac{\alpha_3}{\alpha_1}$ <br> *where* $\psi=a_1\alpha_1+a_{13}\alpha_3$ |
| 15 | Only the prey ($S_1$) washed out | $\overline{N_1}=0,\overline{N_2}=\dfrac{\delta}{a_{22}\alpha_1},\overline{N_3}=\dfrac{\alpha_2}{\alpha_1},\overline{N_4}=\dfrac{\alpha_3}{\alpha_1}$ <br> where $\delta=a_2\alpha_1+a_3a_{24}a_{43}+a_4a_{24}a_{33}$ |
| 16 | The co-existent state (or) Normal steady state | $\overline{N_1}=\dfrac{\sigma_2}{\sigma_1},\overline{N_2}=\dfrac{\sigma_3}{\sigma_1},\overline{N_3}=\dfrac{\alpha_2}{\alpha_1},\overline{N_4}=\dfrac{\alpha_3}{\alpha_1}$ <br> *where* $\sigma_1=(a_{11}a_{22}+a_{12}a_{21})\alpha_1$ <br> $\sigma_2=(-a_1a_{22}+a_2a_{12})\alpha_1+a_3(a_{12}a_{24}a_{43}-a_{13}a_{22}a_{44})$ <br> $+a_4(a_{12}a_{24}a_{33}-a_{13}a_{22}a_{34})$ <br> $\sigma_3=(a_1a_{21}+a_2a_{11})\alpha_1+a_3(a_{11}a_{24}a_{43}+a_{13}a_{21}a_{44})$ <br> $+a_4(a_{11}a_{24}a_{33}+a_{13}a_{21}a_{34})$ |

The stability analysis of the fully washed out state (in S.No.1), the stability analysis of prey and predator washed out states (in S.No's. 4, 5, and 11) has been carried by the present author [13, 14]. The present paper deals with the stability of only predator washed out state (marked *) of the above table. The stability of the other Equilibrium states will be presented in the forthcoming communications.

## Stability of the predator washed out equilibrium state (S.No. 14 in the above table)

To discuss the stability of equilibrium point $\overline{N_1}\neq 0,\overline{N_2}=0,\overline{N_3}\neq 0,\overline{N_4}\neq 0$

*Let us consider small deviations* $u_1(t),\ u_2(t),\ u_3(t),\ u_4(t)$ *from the steady state*

i.e., $N_i(t)=\overline{N}_i+u_i(t)$ , $i=1,2,3,4$ (5.1) where $u_i(t)$ is a small perturbation in the species $S_i$.

Substituting (5.1) in (3.1), (3.2), (3.3), (3.4) and neglecting products and higher *powers of* $u_1,\ u_2,\ u_3,\ u_4$ we get

$$\frac{du_1}{dt} = -a_{11}\bar{N}_1 u_1 - a_{12}\bar{N}_1 u_2 + a_{13}\bar{N}_1 u_3 \tag{5.2}$$

$$\frac{du_2}{dt} = \left(a_2 + a_{21}\overline{N_1} + a_{24}\overline{N_4}\right)u_2 \tag{5.3}$$

$$\frac{du_3}{dt} = -a_{33}\bar{N}_3 u_3 + a_{34}\bar{N}_3 u_4 \tag{5.4}$$

$$\frac{du_4}{dt} = a_{43}\bar{N}_4 u_3 - a_{44}\bar{N}_4 u_4 \tag{5.5}$$

the characteristic equation of this system is

$$(\lambda + a_{11}\bar{N}_1)\left[\lambda - \left(a_2 + a_{21}\overline{N_1} + a_{24}\overline{N_4}\right)\right]\left[\lambda^2 + \lambda\left(a_{33}\bar{N}_3 + a_{44}\bar{N}_4\right) + \left(a_{33}a_{44} - a_{34}a_{43}\right)\overline{N}_3\overline{N}_4\right] = 0 \tag{5.6}$$

one root $\lambda = a_2 + a_{21}\overline{N_1} + a_{24}\overline{N_4}$ of which is evidently positive. Hence the State is *Unstable*.

The equations (5.2), (5.3), (5.4), (5.5) yield the solutions

$$u_1 = -Me^{\alpha t} + GPe^{-\lambda_1 t} + HQe^{-\lambda_2 t} + [u_{10} + M - GP - HQ]e^{-a_{11}\overline{N_1}t} \tag{5.7}$$

$$u_2 = u_{20}e^{\alpha t} \tag{5.8}$$

$$u_3 = Pe^{-\lambda_1 t} + Qe^{-\lambda_2 t} \tag{5.9}$$

$$u_4 = Re^{-\lambda_1 t} + Se^{-\lambda_2 t} \tag{5.10}$$

where

$$P = \frac{-u_{30}s_1 + u_{30}a_{44}\overline{N_4} + u_{40}a_{34}\overline{N_3}}{\lambda_2 - \lambda_1},\quad Q = \frac{-u_{30}s_2 + u_{30}a_{44}\overline{N_4} + u_{40}a_{34}\overline{N_3}}{\lambda_1 - \lambda_2},$$

$$R = \frac{-u_{40}s_1 + u_{40}a_{33}\overline{N_3} + u_{30}a_{43}\overline{N_4}}{\lambda_2 - \lambda_1},\quad S = \frac{-u_{40}s_2 + u_{40}a_{33}\overline{N_3} + u_{40}a_{43}\overline{N_4}}{\lambda_1 - \lambda_2},$$

$$M = \frac{u_{20}a_{12}\overline{N_1}}{a_2 + \left(a_{11} + a_{21}\right)\overline{N_1} + a_{24}\overline{N_4}},\quad \alpha = (a_2 + a_{21}\overline{N_1} + a_{24}\overline{N_4})\,t,$$

$$G = \frac{a_{13}\overline{N_1}}{a_{11}\overline{N_1} - \lambda_1} \text{ and } H = \frac{a_{13}\overline{N_1}}{a_{11}\overline{N_1} - \lambda_2}$$

There would arise in all 576 cases depending upon the ordering of the magnitudes of *the growth rates* $a_1,\ a_2,\ a_3,\ a_4$ *and the initial values of the perturbations* $u_{10}(t),\ u_{20}(t),\ u_{30}(t),\ u_{40}(t)$ *of the* species $S_1$, $S_2$, $S_3$, and $S_4$. Of these 576 situations some typical variations are illustrated in figures 2 to 9 through respective solution curves that would facilitate to make some reasonable observations and the conclusions are presented here.

## Conclusions of the Perturbation Graphs

**Case A :** $a_{33}a_{44} > a_{34}a_{43}$

In this case both the roots are negative. Four typical variations are considered here under cases A1, A2, A3 and A4.

**Case A1: If** $u_{40} < u_{20} < u_{30} < u_{10}$ **and** $a_3 < a_4 < k_2 < k_1$

In this case the host ($S_3$) of the prey ($S_1$) has the least natural growth rate, and the host ($S_4$) of the predator ($S_2$) has the least initial population strength. The host ($S_3$) of the prey ($S_1$) initially dominates over the host ($S_4$) of the predator ($S_2$) till the time instant $t^*_{43} = \frac{1}{\lambda_1 - \lambda_2}\log\left(\frac{R-P}{Q-S}\right)$ and thereafter the dominance is reversed. Also the predator ($S_2$) dominates over the prey ($S_1$) till the time instant $t^*_{12}$ (obtained on solving equations 5.7 & 5.8) and thereafter the dominance is reversed as shown in Fig.2.

**Case A2: If** $u_{30} < u_{10} < u_{40} < u_{20}$ **and** $k_2 < a_3 < k_1 < a_4$

In this case the predator ($S_2$) has the least natural growth rate, and the host ($S_3$) of the prey ($S_1$) has the least initial population strength as shown in Fig.3.

**Case A3: If** $u_{20} < u_{40} < u_{10} < u_{30}$ **and** $a_4 < k_1 < a_3 < k_2$

In this case the host ($S_4$) of the predator ($S_2$) has the least natural growth rate, and the predator ($S_2$) has the least initial population strength. The host ($S_4$) of the predator ($S_2$) initially dominates over the predator ($S_2$) till the time instant $t^*_{24}$ (obtained on solving equations 5.8 & 5.10) and thereafter the dominance is reversed. Also the prey ($S_1$) initially dominates over the predator ($S_2$) till the time instant $t^*_{21}$ (obtained on solving equations 5.7 & 5.8) and thereafter the dominance is reversed as shown in Fig.4.

**Case A4: If** $u_{10} < u_{30} < u_{20} < u_{40}$ **and** $k_1 < k_2 < a_4 < a_3$

In this case the prey ($S_1$) has the least natural growth rate and also it has the least initial population strength. The host ($S_4$) of the predator ($S_2$) initially dominates over the predator ($S_2$) till the time instant $t^*_{24}$ (obtained on solving equations 5.8 & 5.10) and thereafter the dominance is reversed. Also the host ($S_4$) of the predator ($S_2$) initially dominates over the host ($S_3$) of the prey ($S_1$) till the time instant $t^*_{34} = \frac{1}{(\lambda_1 - \lambda_2)} \log\left(\frac{R-P}{Q-S}\right)$ and thereafter the dominance is reversed as shown in Fig.5.

**Case B :** $a_{33}a_{44} < a_{34}a_{43}$

In this case one root is positive and the other root is negative. Four typical variations are considered here under cases B1, B2, B3 and B4.

**Case B1: If** $u_{30} < u_{40} < u_{20} < u_{10}$ **and** $a_4 < k_1 < a_3 < k_2$

In this case the host ($S_4$) of the predator ($S_2$) has the least natural growth rate, and the host ($S_3$) of the prey ($S_1$) has the least initial population strength. The prey ($S_1$) initially dominates over the predator ($S_2$), host ($S_4$) of the predator ($S_2$) and host ($S_3$) of the prey ($S_1$) till the time instant $t^*_{21}$ , $t^*_{41}$ and $t^*_{31}$ (obtained on solving equations 5.7 & 5.8, 5.7 & 5.10 and 5.7 & 5.9 respectively) and thereafter the dominance is reversed as shown in Fig.6.

**Case B2: If** $u_{20} < u_{10} < u_{30} < u_{40}$ **and** $a_3 < a_4 < k_2 < k_1$

In this case the host ($S_3$) of the prey ($S_1$) has the least natural growth rate, and the predator ($S_2$) has the least initial population strength. The prey ($S_1$) initially dominates over the predator ($S_2$) till the time instant $t^*_{21}$ (obtained on solving equations 5.7 & 5.8) and thereafter the dominance is reversed. Also the host ($S_4$) of the predator ($S_2$) initially dominates over the predator ($S_2$) till the time instant $t^*_{24}$ (obtained on solving equations 5.8 & 5.10) and thereafter the dominance is reversed. Also the host ($S_3$) of the prey ($S_1$) initially dominates over the predator ($S_2$) and prey ($S_1$) till the time instant $t^*_{23}$ , $t^*_{13}$ (obtained on solving equations 5.8 & 5.9 and 5.7 & 5.9 respectively) and thereafter the dominance is reversed as shown in Fig.7.

**Case B3: If** $u_{40} < u_{20} < u_{10} < u_{30}$ **and** $k_1 < k_2 < a_4 < a_3$

In this case the prey ($S_1$) has the least natural growth rate and the host ($S_4$) of the predator ($S_2$) has the least initial population strength. The prey ($S_1$) initially dominates over the predator ($S_2$) till the time instant $t^*_{21}$ (obtained on solving equations 5.7 & 5.8) and thereafter the dominance is reversed. Also the host ($S_3$) of the prey ($S_1$) initially dominates over the predator ($S_2$) till the time instant $t^*_{23}$ (obtained on solving equations 5.8 & 5.9) and thereafter the dominance is reversed. Also the prey ($S_1$) initially dominates over the host ($S_4$) of the predator ($S_2$) till the time instant $t^*_{41}$ (obtained on solving equations 5.7 & 5.10) and thereafter the dominance is reversed. Also the host ($S_3$) of the prey ($S_1$) initially dominates over the host ($S_4$) of the predator

($S_2$) till the time instant $t^*_{43} = \frac{1}{\lambda_1 - \lambda_2} \log\left(\frac{R-P}{Q-S}\right)$ and thereafter the dominance is reversed as shown in Fig.8.

**Case B4: If** $u_{10} < u_{30} < u_{40} < u_{20}$ **and** $k_2 < a_3 < k_1 < a_4$
In this case the predator ($S_2$) has the least natural growth rate, and the prey ($S_1$) has the least initial population strength. The host ($S_3$) of the prey ($S_1$) initially dominates over the prey ($S_1$) till the time instant $t^*_{13}$ (obtained on solving equations 5.7 & 5.9) and thereafter the dominance is reversed. Also the prey ($S_1$) initially dominates over the host ($S_4$) of the predator ($S_2$) till the time instant $t^*_{42}$ (obtained on solving equations 5.8 & 5.10) and thereafter the dominance is reversed as shown in Fig.9.

## Trajectories of Perturbations

*The trajectories in $u_1 - u_2$, $u_2 - u_3$, $u_2 - u_4$ planes are*

$$u_1 = -M\left(\frac{u_2}{u_{20}}\right) + GP\left(\frac{u_2}{u_{20}}\right)^{\frac{-\lambda_1}{\alpha}} + HQe^{\frac{-\lambda_2}{\alpha}} + \left(u_{10} + M - GP - HQ\right)\left(\frac{u_2}{u_{20}}\right)^{\frac{-\alpha_{11}\overline{N_1}}{\alpha}} ,$$

$$u_3 = P\left(\frac{u_2}{u_{20}}\right)^{\frac{-\lambda_1}{\alpha}} + Q\left(\frac{u_2}{u_{20}}\right)^{\frac{-\lambda_2}{\alpha}} \text{ and } u_4 = R\left(\frac{u_2}{u_{20}}\right)^{\frac{-\lambda_1}{\alpha}} + S\left(\frac{u_2}{u_{20}}\right)^{\frac{-\lambda_2}{\alpha}} \text{ respectively.}$$

Similarly the trajectories in $u_1 - u_3$ , $u_1 - u_4$ and $u_3 - u_4$ can be found.

## Graphs of the Perturbation

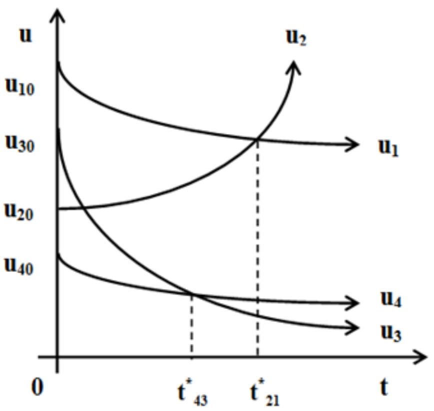

Fig.2

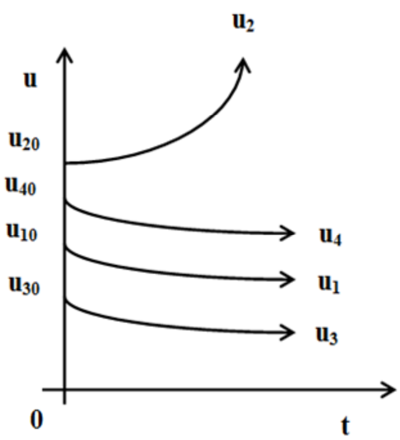

Fig.3

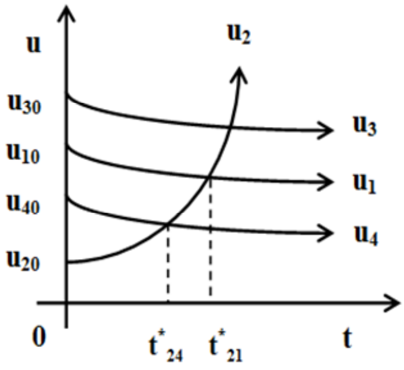


Fig. 4

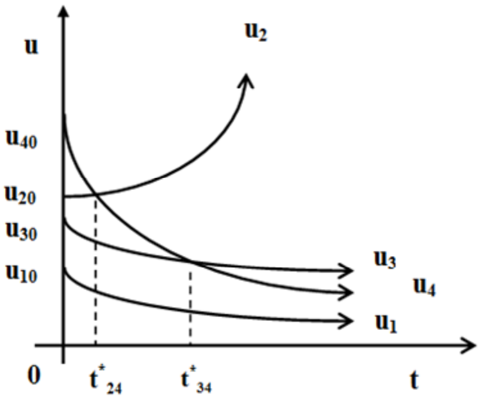


Fig. 5

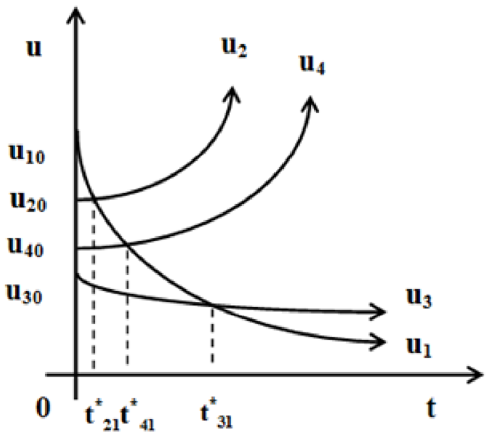


Fig. 6

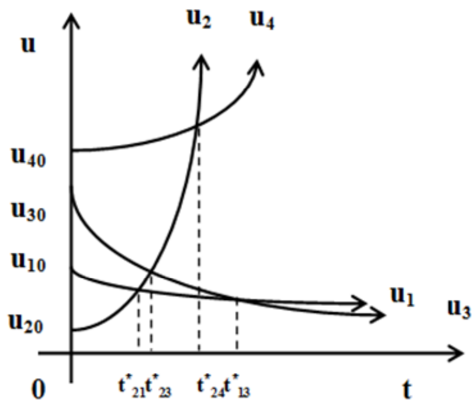


Fig.7

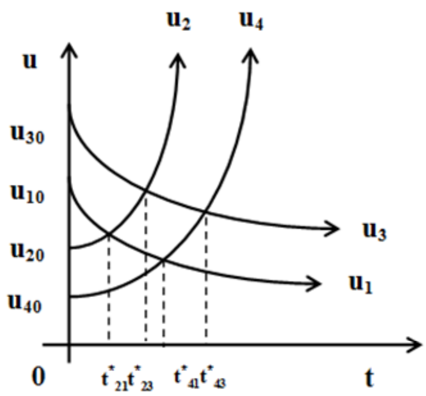


Fig.8

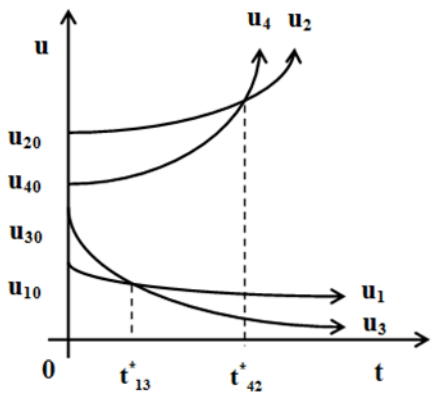


Fig.9

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