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Some New Transformation Techniques with Graceful Labeling

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Abstract

In this paper we give some new type of transformations based on component moving techniques using which we can obtain various types of graceful trees from a given graceful tree of certain type and as a consequence of this new graceful transformations, we show that all lobsters with each vertex excluding one end vertex of the central path attached to an even number of branches with equal number of odd, even and pendant vertices is graceful.

Keywords: Component moving transformations, transfer of 1st type, OD6TF, ED6TF, 2J6TF, 7TF

Definition 1.1
The notion of β-valuation was first introduced by Ringel, Kotzig-Rosa [5,16,17]
Let G be a graph with q edges. A one-one function f from the set of vertices $V(G)$ to the set $\{0,1,2,,............,q\}$ induces an edge labeling where each edge $\{u,v\}$ is assigned the label $\left|f(u)-f(v)\right|$. f is called a $\beta-valuation$ or a graceful labeling if the induced edge labels are distinct. Thus the set of all edge labels must be $\{1,2,..........q\}$.A graph which has a graceful labeling is called a graceful graph. In case of a tree as the number of vertices exceeds the number of edges by one, then a graceful labeling f is also onto and hence a bijection.

Definition 1.2
Transfer of 1st type
A transfer in which the labels of the transferred vertices constitute integers is called a transfer of the 1st type.

In [1, 4, 12, 14, 19, and 20] we find methods to form graceful trees from a given tree. In [2,6.7,8,9,10,11,15,18] we find some techniques to construct graceful trees by joining two or more graceful trees in certain manner.

In this paper we developed a various new types of transformations by using which one can construct graceful trees from a given graceful tree of certain type. Moreover using the transformation we show that the lobsters in which each vertex of the central path excluding x_0 attached to even number of branches with equal number of odd even & pendant branches of the lobster are graceful.

Definition 1.3
For an edge $e = \{u, v\}$ of a tree T, We define $u(T)$ as that connected component of $T - e$ which contains the vertex u. We say $u(T)$ is a component incident on the vertex x. If a and b are vertices of a Tree T. $u(T)$ is a component incident on a, and $b \notin u(T)$ then deleting the edg $\{a, u\}$ from T and making b and u adjacent is called the component moving transformation. Hence we say the component $u(T)$ has been transferred or moved from a to b. This is illustrated in figure –I

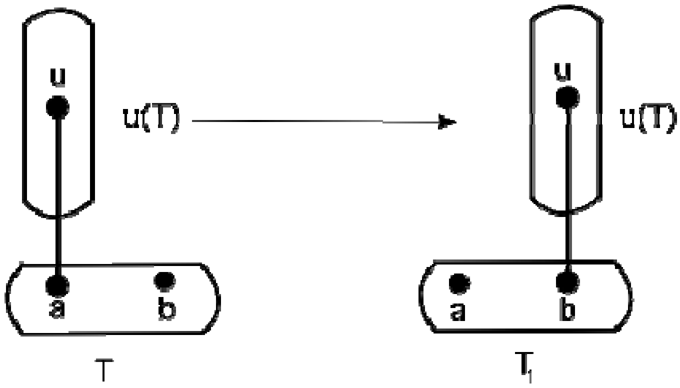


Figure 1: The tree T_1 obtained from T by moving the component $u(T)$ from the vertex a to the vertex b.

Notation (*) Let T be a tree and a and be two vertices of T. by $a \to b$ transfer we mean that some components from a have been moved to b. If we consider successive transfers $a \to b, b \to c, c \to d$,.......we simply write $a \to b \to c \to d$....... transfer. In a transfer $a_1 \to a_2 \to ....a_{n-1} \to a_n$ the vertices $a_1, a_2, ...a_{n-1}$ are called the vertices of transfer.

Lemma 1.1
Let f be a graceful labeling of a tree T; let a and b be two vertices of T: Let $u(T)$ and

$v(T)$ be two components incident on a where $b \notin u(T) \cup v(T)$. Then the following hold:

- If $f(u)+f(v)=f(a)+f(b)$ Then the tree T^* obtained from T by moving the components $u(T)$ and $v(T)$ from a to b is also graceful.
- If $2f(u)=f(a)+f(b)$ then the tree T^{**} obtained from T by moving the component $u(T)$ from a to be is also graceful.

Lemma 1.2
In a graceful labeling f of a of a graceful tree T, let $a, a-1, a-2, \ldots\ldots, a-p_1, b, b+1, b+2, \ldots\ldots b+p_2$
$(respectively,\ a, a+1, a+2, \ldots\ldots, a+r_1, b, b-1, b-2, \ldots\ldots b+r_2)$

Be some vertex labels. Let the vertex a be attached to a set A of vertices (or components) having labels $n, n+1, n+2, \ldots\ldots, n+p$ (different from the above vertex labels) in f and satisfying either $(n+i+1)+(n+p-i)=a+b$ or $(n+i)+(n+p-1-i)=a+b$, $0 \le i \le \left[\dfrac{p-1}{2}\right]$. Then the following hold.

- By making a transfer $a \to b$ of the first type, we can keep from A and odd number of elements at a and more the rest to b such that the resultant tree then formed will be graceful.
- By making a sequence of transfer of the first type $a \to b \to a-1 \to b+1 \to a-2 \to b+2 \to, \ldots\ldots z$
  $(respectivley, a \to b \to a+1 \to b-1 \to a+2 \to b-2 \to \ldots\ldots \to z)$

Where $z=a-p_1\ or\, b+p_2 (respectively, z=a+r_1\ or\, b-r_2)$ an odd number of elements from A can be kept at each vertex of the transfer, such that the resultant tree then formed will be graceful.

Notation 2.1
For the transfer $J_j : a_1^{(j)} \to a_2^{(j)} \to a_3^{(j)} \to \ldots\ldots\ldots \to a_{kj}^{(j)}, 1 \le j \le n$ by the notation "$R : T_1 \to T_2 \to T_3 \to \ldots\ldots\ldots T_n$" we mean the transfer r

Theorem 2.1
Let the tree T, the vertices $a-i, 0 \le i \le p_1$ and $b+j, 0 \le j \le p_2$ (respectively $a+i, 0 \le i \le r_1$ and $b-j, 0 \le j \le r_2$), the set A and the properties satisfied by A be the same as in Lemma 2.1.Consider the transfer
$a \to b \to a-1 \to b+1 \to, \ldots\ldots \to z$
$(respectivley, a \to b \to a+1 \to b-1 \to \ldots\ldots \to z)$,

With $z=a-p_1\, or\, b+p_2$ $(respectivley, a+r_1 or b-r_2)$,such that R is partitioned as

$R: T_1 \to T_2 \to T_3 \to ....T_n$,where each $T_i, 1 \le i \le n$ is either a transfer of the first type or any of the desired transfers. Construct a tree T^* from T by making the transfer R part wise i.e. first the transfer T_1,then T_2 and so on and keeping components(vertices) from A at the vertices of the transfer $T_i, 1 \le i \le n$, in exactly the same manners in Lemma1.2 of T_i is a transfer of first or Lemma 2.1 of T_i is OD6TF,ED6TF,2J6TF or7TF respectively then T^* is graceful.

Proof

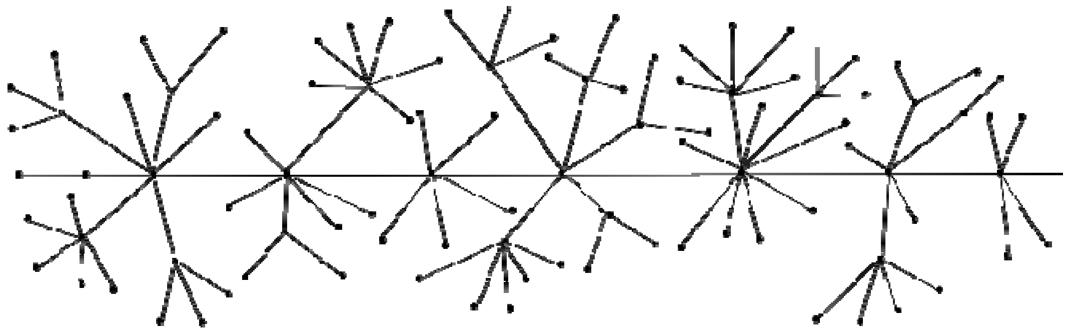
On Lobsters, is either a transfer of first type or any of the desired transfers. Construct a tree T^* from T by making the transfer R part wise, i.e. first the transfer T_1,then T_2 and so on and keeping components(vectors) from A at the vertices of tranfer T_i, $1 \le i \le n$,in exactly the same manners in Lemma 1.2 of T_i

We represent a lobster with diameter at least five in the following manner.

Lemma

If L is a lobster with diameter at least five then there exists a unique path $H = x_0, x_1, x_2, ...x_m$ in L such that besides the adjacencies in H,

- x_0 and x_m are adjacent to the centers of at least one star $k_{1,s}$ where $s \ge 1$
- Each $x_{i,\ 1 \le i \le m-1,}$ is at most adjacent to the centers of some stars $k_{1,s}$ where $s \ge$



Definition

We call the path H the above lemma the central path of L. We use H to denote the central path of a lobster with diameter at list five.Take $x_i \in V(H)$. If x_i is adjacent to the center of some $k_{1,s}$ then $k_{1,s}$ will be called an even branch of s if s is nonzero even, and odd branch if S is odd, and pendant branch if S=0. Therefore, according to above lemma, the branch incident on a vertex in the central path of a lobster can be divided into three types i.e. even, odd and pendant defined as above, further more, whenever we say x_i, for some $0 \le i \le m$, is attached to an even number of branches we mean a "non-zero" even number of branches unless otherwise stated. Fig-1 is a lobster with diameter greater than or equal to five whose adjacent path is $H = x_0, x_1, x_2, ...x_6$

Notation

A combination y branches incident on any x_i, $0 \le i \le m_1$ can be represented by a triple (x, y, z), where x, y and z represent the number of odd, even and pendant branches, respectively, incident an x_i Here we use the symbols e, o, and o^* to represent a non-zero even-number an odd number, and an odd number greater than or equal to 3 respectively.

For example: (e,0,o) means an even number of odd branches, no even branch and an odd number of pendant branches. If in a triple e or o appears more than once then it does not mean that the corresponding branches are equal in number, for example (e,e,o) does not mean that the number of odd branches i.e. equal to the number of even branches. In the lobster appears in fig-1, the vertex x_0 is attached to combination (0,e,e), x_1 is attached to (0,e,e), x_2 is attached to (0,0,o), x_3 is attached to (e,o,0), x_4 is attached to (0,0,0), x_5 is attached to (o,0,o), and x_6 is attached to (e,e,e).

Results

Definition

Let G be a labeled tree with vertices a,b,a-1,b+1,a-2,b+2,a-3,b+3 or (a,b,a+1,b-1,a+2,b-2,a+3,b-3) where a and b are distinct integers

OD6TF

A sequence of transfer a$\to$b+1$\to$a-1$\to$b$\to$a-2$\to$b+2$\to$a-3 (respectively, a$\to$b-1 $\to$a+1$\to$b$\to$a+2$\to$b-2$\to$a+3) is of first types called odd dominated six transfer of first type or (OD6TF) from a$\to$a-3 (respectively, a$\to$a+3)

ED6TF

A sequence of transfer a$\to$b+1$\to$a-2$\to$b$\to$a-1$\to$b+2$\to$a-3 (respectively, a$\to$b-1 $\to$a+2$\to$b$\to$a+1$\to$b-2$\to$a+3) of first type is called an even dominated six transfer of first type of (ED6TF) from a$\to$a-3 (respectively, a$\to$a+3)

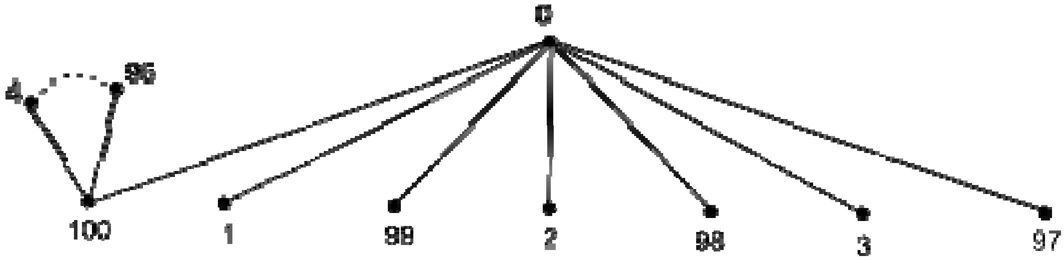
2J6TF

A sequence of transfers a$\to$b+1$\to$a-2$\to$b+2$\to$a-3 (respectively, a$\to$b-1$\to$a+1$\to$ b-2$\to$a+3) of first type is called a two jump six transfer of 1^{st} type or (2J6TF) from a $\to$a-3 (respectively, a$\to$a+3)

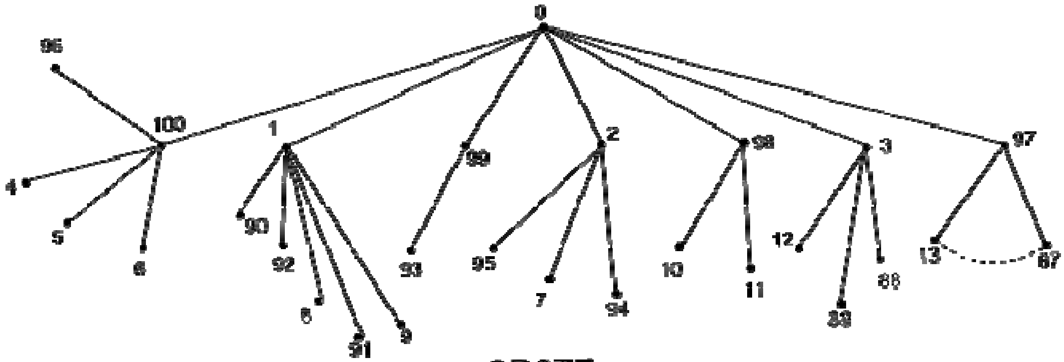
7TF

A sequence of transfer of first type a$\to$b+2$\to$a-2$\to$b$\to$a-1$\to$b+2$\to$a-3 (respectively, a$\to$b-2$\to$a+2$\to$b$\to$a+1$\to$b-2$\to$a+3$\to$b-3) is called a seven transfer of 1^{st} type or 7TF from a$\to$b+3 (respectively, a$\to$b-3)

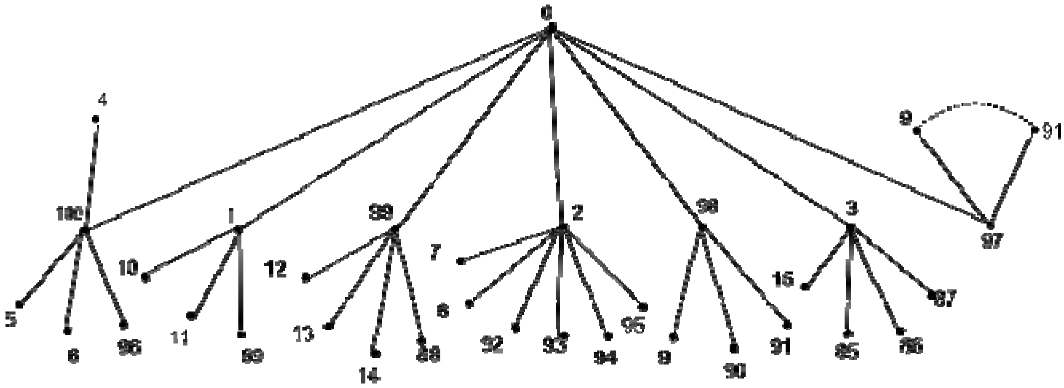
Example 1



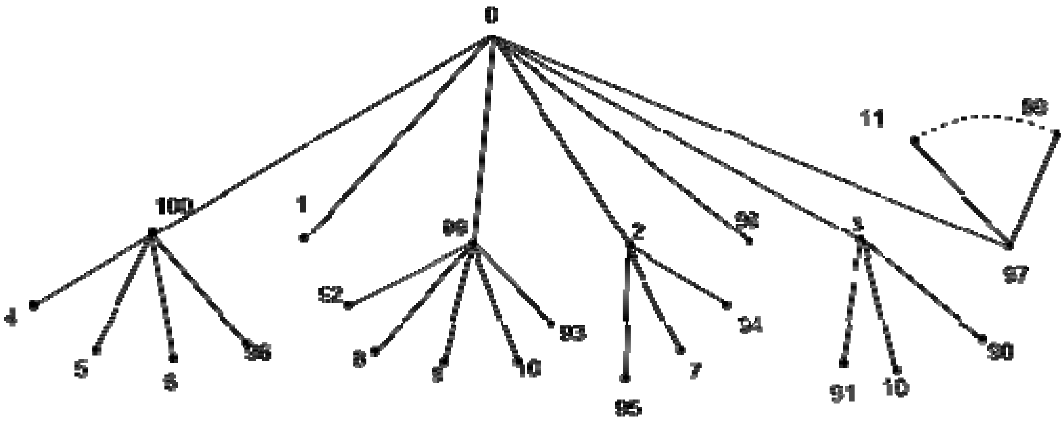
Initial Tree



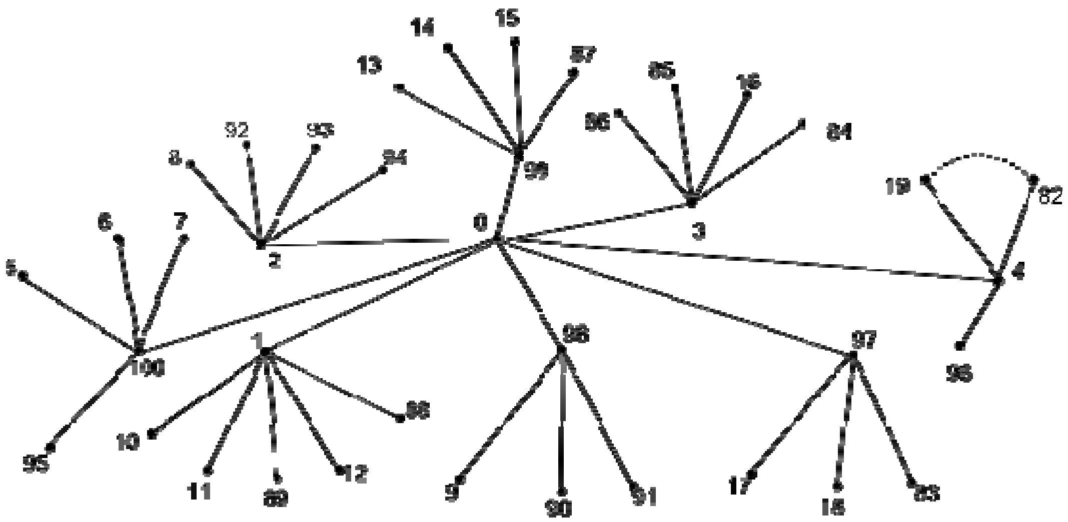
OD6TF



ED6TF



2J6TF



7TF

Lemma 2.1

In a graceful labeling f of a tree T, let $a, a-1, a-2, ..., a-p_1, b, b+1, b+2, ..., b+p_2$ (respectively $a, a+1, a+2, ..., a+r_1, b, b-1, b-2, ..., b-r_2$ be some vertex labels, let the vertex a be attached to a set A of vertices (or components) having label $a, n+1, n+2, ..., n+p$ (different from the above vertex labels) in f. If the vertex labels in A satisfy either $(n+i+1)+(n+p-i)=a+b$ or $(n+i)+n+p-1-i=a+b$ $0 \le i \le \left[\dfrac{p+1}{2}\right]$, then the following hold.

- By making an OD6TH from $a \to a-3$ (respectively from $a \to a+3$) we can keep an even number of vertices at each of the vertices label a and a-2 (respectively a and a+2, and odd number of vertices at each of the vertices $b, b+1, b+2,$ *and* $a-1$ (respectively $b, b-1, b-2,$ *and* $a+1$) and the resultant tree thus formed is graceful.

- By carrying an ED6TF from $a \to a-3$ (respectively from $a \to a+3$) we can keep an even number of vertices at each of the vertices label $a, a-1, b+1$ *and* $b+2$ (*respectively* $a, a+1, b-1, b-2$) and an odd number of vertices at b *and* $a-2$ (*respectively* b *and* $a+2$) and the resultant tree thus formed is graceful.
- By carrying a 2J6TF from $a \to a-3$ (respectively from $a \to a+3$) we can keep an even number of vertices at a *and* $b+2$ (*respectively* a *and* $b-2$) and an odd number of vertices at $a-1$ *and* $b+1$ (*respectively* $a+1$ *and* $b-1$ and no vertex at b *and* $a-2$ (b *and* $a+2$) and the resultant tree thus formed is graceful.
- By carrying a 7TF from $a \to b+4$ (respectively from $a \to b-4$) we can keep an odd number of vertices at each of the vertex $b, a-2$ *and* $a-3$ (*respectively* $b, a+2$ *and* $a+3$) and an even number of vertices at $a, a-1, b+1, b+2$ (*respectively* $a, a+1, b-1, b-2$ and the resultant tree thus formed is graceful.

Proof (a)

Here each transfer will be a transfer of the first type

Let us first consider the transfer $a \to b+1$ (*respectively* $a \to b-1$)

Here we observe that $(n+2+i)+(n+p-i) = a+b+1,\ 0 \le i \le \left[\frac{p+1}{2}\right]$.

Therefore at a we keep the vertices n, n+1 and the pairs $n+2+i, n+p-i$, $i = 0,1,2.........., k-1 \quad k \ge 1$

The vertices which are transferred to the vertex $b-1$ will be of the form $\{n+2+k, n+3+k........., n+p-k\}$ with

$$(n+2+k+i)+(n+p-k-i) = a+b+1 \qquad \forall i$$

Now, consider the transfer $b+1 \to a-1$

Excluding the vertex $n+p-k$, the remaining vertices can be paired whose sum $= a+b$

i.e. we have $(n+2+k+i)+(n+p-k-1-i) = a+b$ *for each* i

Therefore we keep an odd number of vertices at $b+1$, namely $n+p-k, n+2+k+i, n+p-k-i-1$ for desired i, and transfer the rest to the vertex $a-1$ for $i = 0,1,2..........., k_1-1, \quad k_1 \ge 1$

The set of vertices which are transferred to the vertex $a-1$ will be of the form,

$\{n+2+k_1, n+3+k_1, ...n+p-1-k_1\}$

With the condition that $(n+2+k_1+i)+(n+p-1-k_1-i) = a+b$

Consider the transformation $a-1 \to b$ excluding the vertex $n+p-k_1-1$

remaining vertices can be odd number of vertices at $a-1$ namely $n+p-k_1-1, n+2+k_1+i, n+p-k_1-2-i$ for some desired i and transfer the rest to the vertex b

The set of vertices which are moved to the vertex b will be of the form $n+2+k_1, n+2+k_2+1, \ldots\ldots\ldots\ldots\ldots\ldots n+p-2-k_2$. Consider the transfer of $b \to a-2$ excluding the vertex $n+p-k_2$ the remaining vertices can be paired whose sum is $a+b-2$. Therefore at b we keep an odd number of vertices namely $n+p-2-k_2, n+2+k_2+i, n+p-3-k_2-i$ for $i=0,1,2 \ldots\ldots\ldots\ldots k_2-1, \quad k_2 \geq 1$ and transfer the rest to $a-2$.

The set of vertex which are transferred to the vertex $a-2$ is of the form $\{n+2+k_3, n+3+k_3, \ldots\ldots\ldots, n+p-3-k_3\}$

Next consider the transfer $a-2 \to b+2$.

Excluding the vertices $n+2+k_3$ *and* $n+3+k_3, n+4+k_3$ the remaining vertices can be paired whose sum equal to $a+b$.

We keep an even number of vertices at $a-2$, namely $n+2+k_3, n+3+k_3, n+4+k_3+i, n+p-3-k_3-i,$ for

$i=0,1,2, \ldots\ldots\ldots k_4-1, \qquad k_4 \geq 1$

at $a-2$ and transfer the rest to the vertex $b+2$.

The set of vertices which are moved to the vertex $b+2$ is $\{n+4+k_4, n+5+k_4, \ldots\ldots\ldots\ldots\ldots n+p-3-k_4\}$

Excluding the vertex $n+p-3-k_4$ the remaining vertices can be paired when sum $= a+b-1$

So we carry out the transfer $b+2 \to a-3$ keeping the vertices $b+2 \to a-3$ keeping the vertices $n+p-3-k_4, n+4+k_4+i, n+p-4-k_4-i$ for some desired i and transfer the rest to the vertex $a-3$. So in this way we carried out a sequencing transfer keeping desired number of vertices at each vertex of the transfer.

Proof (b)

Here each transfer will be a transfer of the first type

Let us first consider the transfer $a \to b+1 (respectivdy\, a \to b-1)$

Here we observe that $(n+2+i)+(n+p-i) = a+b+1,\ 0 \leq i \leq \left[\dfrac{p+1}{2}\right]$.

Therefore at a we keep the vertices n, n+1 and the pairs $n+2+i, n+p-i$, $i=0,1,2 \ldots\ldots\ldots, k-1 \quad k \geq 1$

The vertices which are transferred to the vertex $b-1$ will be of the form $\{n+2+k, n+3+k \ldots\ldots\ldots, n+p-k\}$ with

$$(n+2+k+i)+(n+p-k-i) = a+b+1 \qquad \forall i$$

Now, consider the transfer $b+1 \to a-2$

Excluding the vertices, $n+p-k, n+p-k-1$, the remaining vertices can be

paired whose sum $= a+b-1$
i.e. we have $(n+2+k+i)+(n+p-k-2-i)=a+b-1$ *for each* i

Therefore we keep an even number of vertices at $b+1$, namely $n+p-k, n+p-k-1, n+2+k+i, n+p-k-2-i$ for $i=0,1,2..........., k_1-1, \quad k_1 \geq 1$

And transfer the rest vertices to the vertex $a-2$

The set of vertices which are transferred to the vertex $a-2$ will be of the form,

$\{n+2+k_1, n+2+k_1+1, ...n+p-k_1-2\}$

With the condition that $(n+2+k_1+i)+(n+p-k_1-3-i)=a+b-2$

Consider the transformation $a-2 \to b$ excluding the vertex $n+p-k_1-2$ remaining vertices can be even number of vertices at $a-2$, namely $n+p-k_1-2, n+2+k_1+i, n+p-k_1-3-i, ...$ for $i=0,1,2,...............k_2-1, \quad k_2 \geq 1$ and transfer the rest to the vertex b

The set of vertices which are moved to the vertex b will be of the form $n+2+k_2, n+2+k_2+1, ...n+p-3-k_2$. Consider the transfer of $b \to a-1$ excluding the vertex $n+2+k_2$ the remaining vertices can be paired whose sum is $a+b-1$.i.e. $(n+3+k_2+i)+(n+p-k_2-3-i)=a+b-1$ Therefore at b we keep an even number of vertices namely $n+2+k_2, n+3+k_2, ...n+p-k_2-3-i$ for $i=0,1,2..................k_3-1, \quad k_3 \geq 1$ and transfer the rest to $a-1$.

The set of vertices which are transferred to the vertex $a-1$ is of the form $\{n+3+k_3, n+4+k_3, ..., n+p-3-k_3\}$

Next consider the transfer $a-1 \to b+2$.

Excluding the vertices $n+3+k_3$ *and* $n+4+k_3$ the remaining vertices can be paired whose sum equal to $a+b+1$.i.e. $(n+5+k_3+i)+(n+p-3-k_3-i)=a+b+1$ w

We keep an odd number of vertices at $a-1$, namely $n+3+k_3, n+4+k_3, n+5+k_3+i, n+p-3-k_3-i$ for $i=0,1,2,...........k_4-1, \quad k_4 \geq 1$ at $a-1$ and transfer the rest to the vertex $b+2$.

The set of vertices which are moved to the vertex $b+2$ is $\{n+5+k_4, n+6+k_4, ...n+p-3-k_4\}$

Excluding the vertex $n+p-3-k_4, n+p-4-k_4, n+5+k_4+i, n+p-5-k_4-i$,

For $i=0,1,2............k_5-1, \quad k_5 \geq 1$ the remaining vertices can be paired when sum $=a+b-1$

So we carry out the transfer $b+2 \to a-3$ keeping the vertices $b+2 \to a-3$ keeping the vertices $n+p-3-k_4, n+4+k_4+i, n+p-4-k_4-i$ for some desired i and transfer the rest to the vertex $a-3$.So in this way we carried out a sequencing transfer keeping desired number of vertices at each vertex of the transfer.

By lemma * the resultant tree thus obtained is graceful.

Observation

After the transfer $a \to b$ of the first type in Lemma 1.2 the set A_1 of the components of A which are transferred to b is of the form $A_1 = \{n+r, n+r+1,....n+r_1\}$ with $(n+r+i)+(n+r_1-i)=a+b$ for $0 \le i \le \left[\frac{r_1-r-1}{2}\right]$

(a) $(n+r+i)+(n+r_1-1-i)=a+b-1$ and

(b) $(n+r+1+i)+(n+r_1-i)=a+b+1$

Therefore, next if we make a transfer $b \to a-1(respectively\ b \to a+1)$, then the Set A_1 and the vertices b and $a-1$ (respectively, b and a+1) Satisfy the properties of the set A and the vertices a and b respectively of

Lemma1.2

1.2 The vertices of the sequence of the transfer we deal in this paper have the property “P: for any three consecutive vertices u, v, w of the sequence of the transfer we have w=u$\neq$1”. Because of the property “P” we can use Lemma 1.2 and part (a) of this observation, repeatedly and keep an odd number of components at each vertex of the transfer.

Lemma 2.2

Let the tree T, the vertices $a-i, 0 \le i \le p_1$ and $b+j, 0 \le j \le p_2$ (respectively, $a+i, 0 \le i \le r_1$ and $b-j, 0 \le j \le r_2$,the set A be the same as Lemma 2.1. Consider the transfer T_1 from $a \to u$,where T_1 may be a transfer of first type or any transfer of type

OD6TH,ED6TF, 2J6TH, 7TF and u is the vertex b if T_1 is of first type $a-3$ if T_1 is a

OD6TH,ED6TF or 2J6TH and is the vertex $b+3$ if T_1 is a 7TF. Let $A_1 \subset$ A be the set of vertices transferred to u after the transfer T_1 and v be the vertex appears next to u in the sequence $(a, b, a-1, b+1, a-2, b+2,...)$ (respectively $a, b, a+1, b-1, a+2, b-2,...)$

Then the set A_1 and verices u and v satisfy the properties of the set A and vertices a and b respectively, of Lemma 1.2 or 2.1.

Proof of Lemma 2.2

If T_1 is the transfer of the first type then the Lemma holds by observation 1.1(a),since each derived transfer is a sequence of transfer of first type, the set A_1 consists of consecutive integers, say A_1={ $n+s, n+s+1,...n+s+i$ }.If T_1 is a OD6TF,ED6TF or2J6TF, u is the vertex a-3(respectively,a+3). One notices from the proof of Lemma 2.1 (a),(b),(c),the set A_1 satisfies the condition

$(n+s+i)+(n+s_1-i)=a+b-1(respectively, a+b+1)$

Where $0 \le i \le \left\lceil \frac{s_1-s-1}{2} \right\rceil$.By re-pairing the elements of A_1,we get, for $0 \le i \le \left\lceil \frac{s_1-s-1}{2} \right\rceil$, $(n+s+i+1)+(n+s_1-1)=a+b=u+v(respectively,$ $(n+s+i)+(n+s_1-i-1)=a+b=u+v)$

If T_1 is a 7TF then u is the vertex $b+3(respectively, b-3)$. from the proof of the Lemma 2.1(d), we have $(n+s+i)+(n+s_1-i)=a+b, where 0 \le i \le \left\lceil \frac{s_1-s-1}{2} \right\rceil$.

By re-pairing the elements of A_1, we get for $0 \le i \le \left\lceil \frac{s_1-s-1}{2} \right\rceil$, $(n+s+i)+(n+s_1-i-1)=a+b-1=u+v(respectively, (n+s+i)+(n+s_1-i)=a+b+1=u+v)$
Hence the result.

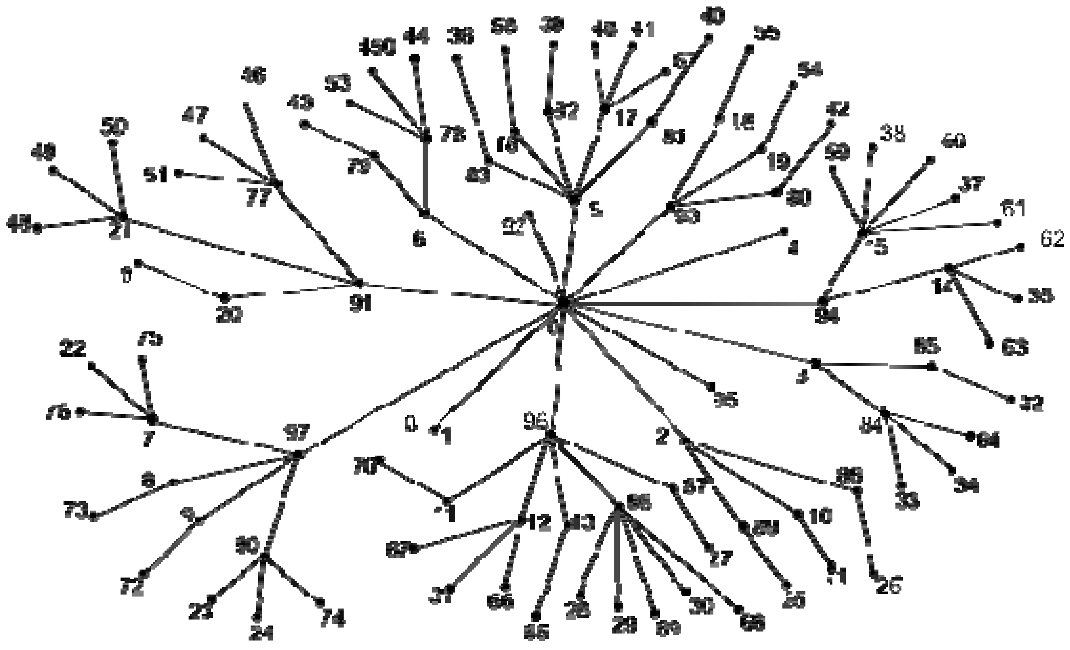
Application

1.1 In this paper we give conditions under which the transfers can be used repeatedly so as to obtain different type of graceful trees.

As an application of this graceful transformation we give graceful labeling to a class of lobsters with the following characteristic features. Each vertex $x_i, 1 < i < n$ of the central path in attached to even number of branches with equal number of odd, even and pendant branches.

The non pendant branches incident on the central path are either all odd branches or even branches.

1.2 The number of vertices in the central path is $[1]_6$ *or* $6K+1$ where k is a positive integer
For each the integer the vertices x_{6r+1} *and* x_{6r+6}, x_{6r+3} *and* x_{6r+4} are attached to odd no of branches, and x_{6r+2} *and* x_{6r+5} are adjacent to no branch.



DIAMETER-6

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