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Two Machines in Tandem with a Single Transport Facility in between and Setup Times for Jobs on Machines also Included

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Abstract

This paper considers the problem of two machines in tandem with a single transport agent in between and therefore considers transportation and returning times for the transport agent. We have also considered the time required for a job to setup on the machine. Hence, three types of times namely setup time, transportation time and returning time are under consideration. The procedure has been summarized as an algorithm. A numerical example is given to illustrate the procedure.

Keywords: Machines in tandem, setup time, transportation time, returning time.

Introduction

The idea of two production stages was given by Jackson (1954) [1] while studying a queuing system concerned with an industry in which the production of an item takes place in two distinct but successive stages. Jackson called such stages in tandem (or in series). Johnson(1954) [2] and Bellman(1956) [3] studied the problem of scheduling of n jobs on two machines arranged in tandem where time required to transport jobs from first machine to the second was assumed to be negligible. Maggu and Das(1980) [4] then introduced the concept of transportation time in going from one stage to the other but studied a system in which an infinite number of transport agents were available and no transport agent was required to return to stage1 from stage2. The concept was further modified by Khan, Maggu and Mudawi (1994) [5], as there may be cases where only a single transport agent is available. Here we study the case where only a single transport agent is available who, after delivering the items to

Machine2 has to come back to Machine1 for transporting the next item with the assumption that Machine1 starts processing the next item immediately after finishing the preceding one. Further, we have considered the setup times for all the jobs on both the machines as there may be cases where a machine needs a setup every time before processing an item. These cases may arise in different industries as in automobile sector casting parts of different diameters need machine setting every time for processing and hence a setup time is involved.

Two machines in tandem with a single transport agent in between and also considering setup time

Let us consider n items $\left(I_1, I_2, \ldots., I_n\right)$ being processed through two machines $\left(A \& B\right)$ in the order AB with an agent who transports an item processed at machine A to the machine B and returns back empty to A to transport the second item to B and so on until all the items were taken to B. Let item i to be processed on the two machines $A \& B$ requires setup times denoted by a_i and b_i respectively. Let t_i be the transportation time for item i to carry it from machine A to B; A_i & B_i are the service times on machines A and B respectively, and r_i is the returning time for agent from machine B to A after delivering item i at B. As the time by which the transport agent finishes with item $i-1$, the job of i^{th} item on machine A may or may not get finished, and the machine A after processing item $i-1$ immediately takes up item i for processing, the item i will wait for transport agent if it is not returned back by that time. The problem is to find an optimal schedule of items so as to minimize the total production time for completing all the items.

Theorem: An optimal sequence is obtained by sequencing the item $i-1, i, i+1$ such that $\min\left(a_i + A_i + t_i + R_{i-1}, b_{i+1} + B_{i+1} + t_{i+1} + R_i\right) < \min\left(a_{i+1} + A_{i+1} + t_{i+1} + R_i, b_i + B_i + t_i + R_{i-1}\right)$

where $R_{i-1} = \begin{cases} t_{i-1} + r_{i-1} - a_i - A_i & \text{if it is positive;} \\ 0 & \text{otherwise.} \end{cases}$

Proof: Let S and S' denote the sequences of items given by :

$$S = \left(I_1, I_2, \ldots., I_{i-1}, I_i, I_{i+1}.I_{i+2}, \ldots., I_n\right)$$
$$S' = \left(I'_1, I'_2, \ldots., I'_{i-1}, I'_{i+1}, I'_i.I'_{i+2}, \ldots., I'_n\right)$$

Let $\left(X_p, X'_p\right)$ and $\left(CX_p, C'X_p\right)$ be the processing time and completion time of any item p on machine $X\left(= A \text{ or } B\right)$ for the sequences $\left(S, S'\right)$ respectively. Let $\left(t_p, t'_p\right)$ denotes the transportation times of item p from machine A to machine B for the sequences $\left(S, S'\right)$. r_p is the returning time of the transport agent from machine B to machine A after delivering the p^{th} item at B. Let $\left(a_p, a'_p\right)$ and $\left(b_p, b'_p\right)$ be respectively the setup times of any item p on machines A & B for the sequences $\left(S, S'\right)$.

Note that we have defined $R_{p-1} = t_{p-1} + r_{p-1} - a_p - A_p \geq 0$. The completion time of p^{th} item on machine B is given by

$$CB_p = \max\left(CA_p + t_p + R_{p-1}, CB_{p-1}\right) + b_p + B_p \tag{1}$$

The sequence S is chosen if

$$CB_n < C'B_n \quad (2)$$

i.e., if

$$\max\left(CA_n + t_n + R_{n-1}, CB_{n-1}\right) + b_n + B_n < \max\left(C'A_n + t'_n + R'_{n-1}, C'B_{n-1}\right) + b'_n + B'_n$$

As $CA_n + t_n + R_{n-1} = \sum_{i=1}^{i=n} a_i + \sum_{i=1}^{i=n} A_i + t_n + R_{n-1} = C'A_n + t'_n + R'_{n-1}$, $b_n = b'_n$ and $B_n = B'_n$, result (2) will be true if:

$$CB_{n-1} < C'B_{n-1} \tag{3}$$

Proceeding in this way we get that inequality (2) is true if:

$$CB_p < C'B_p \quad \left(p = i+1, i+2, \ldots\ldots, n \text{ and } i = 1, 2, \ldots. n-1\right) \tag{4}$$

We now calculate the values of CB_{i+1} and $C'B_{i+1}$

$$\begin{aligned} CB_{i+1} &= \max\left\{CA_{i+1} + t_{i+1} + R_i, CB_i\right\} + b_{i+1} + B_{i+1} \\ &= \max\left\{CA_{i+1} + t_{i+1} + R_i, \max\left(CA_i + t_i + R_{i-1}, CB_{i-1}\right) + b_i + B_i\right\} + b_{i+1} + B_{i+1} \\ &= \max\{CA_{i+1} + t_{i+1} + R_i + b_{i+1} + B_{i+1}, CA_i + t_i + R_{i-1} + b_i + B_i + b_{i+1} + B_{i+1}, \\ &\qquad CB_{i-1} + b_i + B_i + b_{i+1} + B_{i+1}\} \end{aligned}$$

Now

$$\begin{aligned} CB_{i+1} = \max\{&CA_{i-1} + a_i + A_i + a_{i+1} + A_{i+1} + t_{i+1} + R_i + b_{i+1} + B_{i+1}, \\ &CA_{i-1} + a_i + A_i + t_i + R_{i-1} + b_i + B_i + b_{i+1} + B_{i+1}, \\ &CB_{i-1} + b_i + B_i + b_{i+1} + B_{i+1}\} \end{aligned} \tag{5}$$

Similarly

$$\begin{aligned} C'B_{i+1} = \max\{&C'A_{i-1} + a'_i + A'_i + a'_{i+1} + A'_{i+1} + t'_{i+1} + R'_i + b'_{i+1} + B'_{i+1}, \\ &C'A_{i-1} + a'_i + A'_i + t'_i + R'_{i-1} + b'_i + B'_i + b'_{i+1} + B'_{i+1}, \\ &C'B_{i-1} + b'_i + B'_i + b'_{i+1} + B'_{i+1}\} \end{aligned} \tag{6}$$

For the sequences S and S' it is obvious that

$$CA_{i-1} = C'A_{i-1}, CB_{i-1} = C'B_{i-1}$$
$$X_i = X'_{i+1}, (X = \text{a or b or } A \text{ or } B); t_i = t'_{i+1} \tag{7}$$
$$X_{i+1} = X'_i, (X = \text{a or b or } A \text{ or } B); t_{i+1} = t'_i$$
$$R_{i-1} = R'_i, R_i = R'_{i-1}.$$

Writing (4) for $p = i+1$ and using (7), we get

$$\begin{aligned} &\max\{CA_{i-1} + a_i + A_i + a_{i+1} + A_{i+1} + t_{i+1} + R_i + b_{i+1} + B_{i+1}, CA_{i-1} + a_i + A_i + \\ &\qquad t_i + R_{i-1} + b_i + B_i + b_{i+1} + B_{i+1}, CB_{i-1} + b_i + B_i + b_{i+1} + B_{i+1}\} \\ &< \max\{CA_{i-1} + a_{i+1} + A_{i+1} + a_i + A_i + t_i + R_{i-1} + b_i + B_i, CA_{i-1} + a_{i+1} + A_{i+1} + \\ &\qquad t_{i+1} + R_i + b_{i+1} + B_{i+1} + b_i + B_i, CB_{i-1} + b_{i+1} + B_{i+1} + b_i + B_i\} \end{aligned} \tag{8}$$

Subtracting $CB_{i-1} + b_i + B_i + b_{i+1} + B_{i+1}$ from both sides, the inequality (8) reduces to

$$\begin{aligned} &\max\{CA_{i-1} + a_i + A_i + a_{i+1} + A_{i+1} + t_{i+1} + R_i + b_{i+1} + B_{i+1}, \\ &\qquad CA_{i-1} + a_i + A_i + t_i + R_{i-1} + b_i + B_i + b_{i+1} + B_{i+1}\} \\ &< \max\{CA_{i-1} + a_{i+1} + A_{i+1} + a_i + A_i + t_i + R_{i-1} + b_i + B_i, \\ &\qquad CA_{i-1} + a_{i+1} + A_{i+1} + t_{i+1} + R_i + b_{i+1} + B_{i+1} + b_i + B_i\} \end{aligned} \tag{9}$$

Further subtracting $CA_{i-1} + a_i + A_i + a_{i+1} + A_{i+1} + t_i + t_{i+1} + R_{i-1} + R_i + b_i + B_i + b_{i+1} + B_{i+1}$ from each side, we get

$$\begin{aligned} &\max\{-t_i - R_{i-1} - b_i - B_i, -a_{i+1} - A_{i+1} - t_{i+1} - R_i\} \\ &< \max\{-t_{i+1} - R_i - b_{i+1} - B_{i+1}, -a_i - A_i - t_i - R_{i-1}\} \end{aligned} \tag{10}$$

$$\begin{aligned} &\min\{a_i + A_i + t_i + R_{i-1}, t_{i+1} + R_i + b_{i+1} + B_{i+1}\} \\ &< \min\{a_{i+1} + A_{i+1} + t_{i+1} + R_i, b_i + B_i + t_i + R_{i-1}\} \end{aligned} \tag{11}$$

Algorithm: Our problem can be represented in tableau form as follows:

Item i	a_i	Machine A (A_i)	t_i	r_i	b_i	Machine B (B_i)
1	a_1	A_1	t_1	r_1	b_1	B_1
2	a_2	A_2	t_2	r_2	b_2	B_2
.	.	.	.	.	.	.
.	.	.	.	.	.	.
n	a_n	A_n	t_n	r_n	b_n	B_n

Where a_i, b_i; A_i, B_i are respectively the setup and service times on A & B respectively. t_i is the transportation time for item i in transporting from machine A to B and r_i is the returning time of the transport agent from machine B to A after delivering the i^{th} item.

The result of Theorem 1 gives the following procedure for an optimal sequence:

1. Assuming the two fictitious machines $G \& H$ in place of A & B respectively with the service times G_i and H_i where

$G_i = a_i + A_i + t_i + R_{i-1}$, $H_i = b_i + B_i + t_i + R_{i-1}$

2. Applying Johnson's (1954) rule to the fictitious machine times $G_i \& H_i$ constructed in step 1, we obtain the optimal sequence.

Example: Let a machine tandem queuing problem be given in the following tableau form:

Item i	a_i	Machine $A(A_i)$	t_i	r_i	R_{i-1}	b_i	Machine $B(B_i)$
1	7	7	4	2	-	2	9
2	2	6	3	4	0	3	6
3	5	4	4	3	0	4	7
4	3	9	7	6	0	1	3
5	1	8	5	5	4	6	6

Solution: Let G and H be two fictitious machines representing A and B respectively. Let G_i and H_i be the service times of G and H respectively. Then our reduced problem is:

Item i	Machine G $(G_i = a_i + A_i + t_i + R_{i-1})$	Machine H $(H_i = b_i + B_i + t_i + R_{i-1})$
1	18	15
2	11	12
3	13	15
4	19	11
5	18	21

By Johnson's rule to the above reduced times, the optimal sequence is 2,3,5,1,4. The minimum total production time is calculated as follows:

Y_{i-1} represents the time at which the transport agent returns to machine A to take the next item.

Z_i represents the time at which the transport agent reaches to machine B to leave an item.

$Y_{i-1} = CA_i + t_i + r_i \quad Z_i = CA_i + t_i$

Item i	a_i	Machine A in-out	t_i	r_i	Y_{i-1} -	Z_i	b_i	Machine B in-out	Idle Time A B
2	2	2-8	3	4	15	11	3	14-20	0 11
3	5	13-17	4	3	24	21	4	25-32	0 01
5	1	18-26	5	5	36	31	6	38-44	0 00
1	7	33-40	4	2	46	44	2	46-55	0 00
4	3	43-52	7	6		59	1	60-63	11 04

The total processing time of all the items through the system (i.e., total production time) is 63 hrs. Idle time (it excludes setup times for machines) for machine A is 11 hrs., for B it is 16 hrs. & for the agent is 20 hrs. So machine A is busy 82.54% of time, B is busy 74.60% of time and the transport agent is busy 68.25% of time.

References

[1] Jackson, R.R.P., 1954, "Queuing systems with phase-type service," O.R. Quart., 5, pp. 109-120.

[2] Johnson, S.M.,1954, "Optimal two-and-three stage production scheduling," Naval. Rs. Log. Quart., 1, pp. 61-68.

[3] Bellman, R., 1956, "Mathematical aspects of scheduling theory," J. Soc. Indus. Appl. Math., 4, pp. 168-205.

[4] Maggu, P.L. and Das, G., "On 2 × n sequencing problem with transportation times of jobs," PAMS, 12 (1-2), pp. 1-6.

[5] Khan, S.U., Maggu, P.L. and Mudawi, M.H., 1994, "Two machines in tandem with a single transport agent in between-A heuristic approach," PAMS, 40, pp. 11-21.

[6] Aziz, R.C. and Singh, T.P., 1991, "On Heuristic principle of Dominance for solution of Scheduling with transportation time for a job given," Journal of Indian Society of Statistics and Operations Research, 7, pp. 15-19.