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Stability of a Viscoelastic (Maxwell) Fluid of Variable Density in an Inhomogeneous Magnetic Field through Porous Medium

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Abstract

This paper deals with the stability of a stratified layer of an incompressible, Maxwellian viscoelastic, infinitely conducting fluid in an inhomogeneous magnetic field. A variational principle is applied to characterize the problem and by using it a dispersion relation is derived and solved numerically. It is found that viscosity and rotation have stabilizing influence while the elasticity and permeability of porous medium have destabilizing influence on the unstable configuration.

Introduction

Since the pioneering work of RAYLEIGH (1883), the problem of the instability of a semi-infinite layer of a stratified fluid has been studied by several authors under various operative forces and a detailed account of these investigations has been given by CHANDRASEKHAR (1961) and DRAZIN and REID (1981), WOBIG (1972), SRIVASTAVA (1974), GUPTA and BHATIA (1991) have studied this problem under varying assumptions in all these studies the prevailing magnetic field is uniform. UBEROI and SEDLACK (1992) have investigated the Alfven waves in two-dimensional inhomogeneous magnetic field, the magnetic field varying along the perpendicular to the plane in which the magnetic field lines lie.

The instability of convective flow in hydrodynamics through porous medium has been studied by LAPWOOD (1948).Since then several researchers (e.g. RUDRAIAH and SRIMANI (1976), SHARMA (1985)) have studied the effects of permeability of the porous medium on the different instability problems in view of the importance of such studies in rocks and heavy oil recovery.

Since viscoelastic fluids play a significant role in industrial application. The studies of waves and stability in different viscoelastic fluid dynamical configuration has been carried out of several researchers (e.g. YADAV and RAY (1991), DANDAPAT and GUPTA (1997), DANDAPAT, HOLEMEDAL and ANDERSON (1994)).

ELSAYEED (1997) have investigated the electro-hydrodynamics instability of two superposed viscous streaming fluids through porous media. BHATIA and MATHUR (2003) have investigated the instability of viscoelastic fluid in a vertical magnetic field through porous medium and it is found that the magnetic field stabilizes the unstable configuration for wave number band k>k* in which the system is unstable in the absence of the magnetic field. It is also found that the viscosity, viscoelasticity and medium porosity have stabilizing influence while elasticity and medium permeability have destabilizing influence. Agrawal and Goel (1998) have studied the instability of viscoelastic fluid in a porous medium and their analysis reveals a destabilizing character of medium permeability and the shear velocity. BHATIA and STEINER (1973) have studied the problem of thermal instability of a Maxwellian viscoelastic fluid in the presence of rotation.

In this paper we have analyzed the stability of a viscoelastic Maxwellian fluid of variable density in an inhomogeneous magnetic field through porous medium.

Linearized Perturbation Equations

Consider the motion of incompressible, infinitely conducting visco – elastic Maxwell fluids in a porous medium. It is assumed that the system is immersed in a variable two dimensional horizontal magnetic field $\vec{H} = \left(H_x, H_y, 0\right)$. The relevant linearized perturbation equations are

$$\rho\left(1+\lambda\frac{\partial}{\partial t}\right)\frac{\partial\vec{u}}{\partial t} = \left(1+\lambda\frac{\partial}{\partial t}\right)\left[-\nabla\delta p + \vec{g}\delta\rho + \left(\nabla\times\vec{h}\right)\times\vec{H} + \left(\nabla\times\vec{H}\right)\times\vec{h}\right]$$

$$+\left[\nabla.\left(\mu\nabla\vec{u}\right) + \left(\nabla\mu.\nabla\right)\vec{u} - \frac{\mu}{k_1}\vec{u}\right] \qquad \ldots(1)$$

$$\frac{\partial\vec{h}}{\partial t} = \nabla\times\left(\vec{u}\times\vec{H}\right) \qquad \ldots(2)$$

$$\frac{\partial}{\partial t}(\delta\rho) + \left(\vec{u}.\nabla\right)\rho = 0 \qquad \ldots(3)$$

$$\nabla.\vec{u} = 0 \qquad \ldots(4)$$

$$\nabla.\vec{h} = 0 \qquad \ldots(5)$$

Where $\vec{u} = (u, v, w)$, $\vec{h} = \left(h_x, h_y, h_z\right)$, $\delta\rho$ and δp are respectively the perturbations in velocity $\vec{u}$, magnetic field $\vec{H}$, density ρ and pressure p of the fluid. Here μ is the coefficient of viscosity and $\vec{g}\,(0,0,g)$ is acceleration due to gravity. In equation (1), λ is the stress relaxation time and k_1 denotes the permeability of the porous medium.

We assume that the magnetic field $\vec{H}$ is stratified along the vertical i.e. $H_x = H_0(Z)$ and $H_y = H_1(z)$.

We assume that all the perturbed quantities very with y, z and t as

$$F(z)\exp.\,(ik_y y + nt) \qquad \ldots(6)$$

Where k_y is the wave number along y-axis and n (may be complex) denotes the rate at which the system disturbs the equilibrium.

Using (6) in equations (1) to (5) and eliminating some of the variables we finally get an equation in terms of w

$$(1+n\lambda)\Big[n\{\rho k_y^2 w - D(\rho Dw)\} - \frac{g}{n} k_y^2 (D\rho) w - \frac{k_y^2}{n}\{2H_1 DH_1 DW$$
$$+ H_1^2 (D^2 - k_y^2) w\ \}\ \Big] + \Big[\mu (D^2 - k_y^2)^2 w + 2(D\mu)(D^2 - k_y^2) Dw$$
$$+ D^2\mu (D^2 + k_y^2) w + \frac{1}{k_1} D\mu\, Dw + \frac{\mu}{k_1}(D^2 - k_y^2) w\ \Big] = 0 \qquad \ldots(7)$$

Where $D = \dfrac{d}{dz}$

We assume that the system is confined between two planes z = 0 and z =d. The normal component of velocity must vanish at these boundaries, we must therefore have

$$w = 0 \text{ at } z = 0 \text{ and } z = d \qquad \ldots(8)$$

Also either

$$Dw = 0 \text{ or } D^2 w = 0 \text{ at } z = 0 \text{ and } z = d \qquad \ldots(9)$$

According as the boundaries are rigid or free.

Variational Principle

Let us assume that the solutions w_i and w_j of equation (7) belong to the characteristic values n_i and n_j respectively, Multiplying equation (7) for i, by w_j and integrating across the vertical extent L of the system, we get after performing integration by parts once or repeatedly and using the boundary conditions (8) and (9) and setting i = j.

$$(1+n\lambda)\left[\begin{array}{l} n^2\int_0^d \rho\left\{k_y^2 w^2+(Dw)^2\right\}dz - gk_y^2\int_0^d (D\rho)w^2 dz \\ +k_y^2\int_0^d \left\{H_1^2(Dw)^2+k_y^2H_1^2w^2\right\}dz \end{array}\right]$$

$$+n\left[\int_0^d \mu\left\{\left(D^2+k_y^2\right)^2 w^2+4k_y^2(Dw)^2\frac{1}{k_1}\left((Dw)^2+k_y^2w^2\right)\right\}dz\right]=0 \quad \ldots(10)$$

By proceeding along the usual lines, we can show that to the first order in small quantities $\delta n = 0$, equation (10), therefore provides the basis for obtaining the solutions through approximately of the stability of the stratified fluid layer in an inhomogeneous magnetic field.

Exponentially Stratified Fluid Layer

To obtain the solution of the stability problem of a layer in which the density ρ of the fluid, varies exponentially along the vertical, i.e.

$$\rho(z) = \rho_0 \exp(\beta z) \quad \ldots(11)$$

where ρ_0 and β are constants.

The magnetic field H_1 and viscosity μ are also assumed to very exponentially so that

$$\mu(z) = \mu_0 \exp(\beta z) \quad \ldots(12)$$

$$H_1^2(z) = H_m^2 \exp(\beta z) \quad \ldots(13)$$

where μ_0 and H_m^2 are constants.

Suppose that the boundary surfaces are both rigid. Then in order that w = Dw = 0 at the boundaries, let us take,

$$w(z) = A(1+\cos lz) \quad \ldots(14)$$

Where $l = \dfrac{2\pi s}{d}$, s being an integer.

Substituting the trial solution (14) in equation (10), and writing

$$x = \frac{k_y}{1},\ a = \frac{\beta}{l}, \nu_0 = \frac{\mu_0}{\rho_0}\ and\ V_A^2 = \frac{H_m^2}{\rho_0} \quad \ldots(15)$$

where ν_0 is coefficient of kinematic viscosity is constant throughout and V_A is Alfven velocity is also constant throughout.

To non-dimensionalized the dispersion relation in terms of $\sqrt{g\beta}$, we have written

$$n* = \frac{n}{\sqrt{g\beta}}, \lambda* = \sqrt{g\beta}, k_1^* = k_1 1^2, \; \nu_0^* = \frac{\nu_0 1^2}{\sqrt{g\beta}} \; and \, V_A^{*2} = \frac{V_A^2 1^2}{\sqrt{g\beta}} \quad \ldots(16)$$

we obtain the dispersion relation as

$$n^{*3}\lambda* + n^{*2}\left[1 - \frac{6i\lambda* x\Omega*}{\left(a^2+3x^2+1\right)}\right]$$
$$+ n*\left[\lambda* V_A^{*2}x^2 - \frac{\nu_0^*}{k_1^*} - \lambda*\frac{3x^2}{\left(a^2+3x^2+1\right)} + \nu_0^* x^2 \frac{\left(10a^2+3x^2+4\right)}{\left(a^2+5x^2+1\right)}\right]$$
$$+\left[V_A^{*2}x^2 - \frac{5x^2}{\left(2a^2+5x^2+1\right)}\right] = 0 \quad \ldots(17)$$

Discussion

The dispersion relation (17) has been solved numerically for β > 0 (unstable configuration) for different values of the physical parameters involved.

The calculations are presented in fig. (1), (2), (3) and (4) where we have plotted the growth rate n^* against wave number k_y for several values of parameters characterizing elasticity, viscosity, medium permeability and rotation.

It is seen from fig. (1), that as elasticity increases growth rate also increases for the same k_y, showing thereby destabilizing influence. From fig.(2), it is seen that growth rate decreases as viscosity increases for the same k_y, showing thereby stabilizing influence. From fig. (3), it is seen that as medium permeability decreases the growth rate also decreases for the same k_y, showing thereby destabilizing influence. The effect of rotation is given in fig. (4) from which it is seen that as growth rate increases rotation decreases for the same k_y, showing thereby stabilizing influence of rotation.

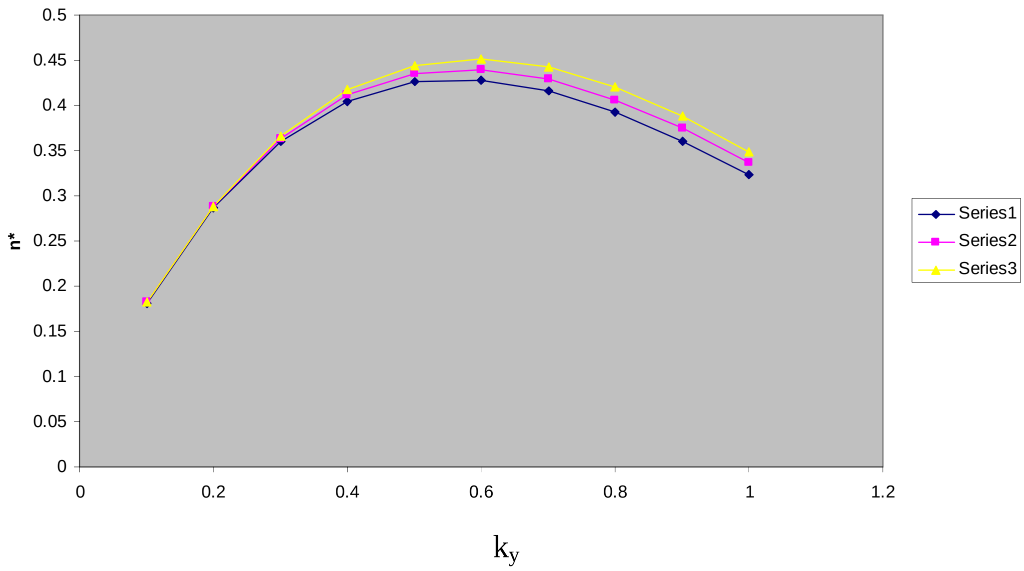

Figure 1: Plot of growth rate n* against the wave number k_y for $\lambda^* = 0.6, 0.8$ and 1.0 when a = 0.2, $V_A{}^* = 0.5$, $\nu_0{}^* = 0.8$, $k_1{}^* = 10$ and $\Omega^* = 1.0$ Series 1, 2 and 3 stands for $\lambda^* = 0.6, 0.8$ and 1.0 respectively

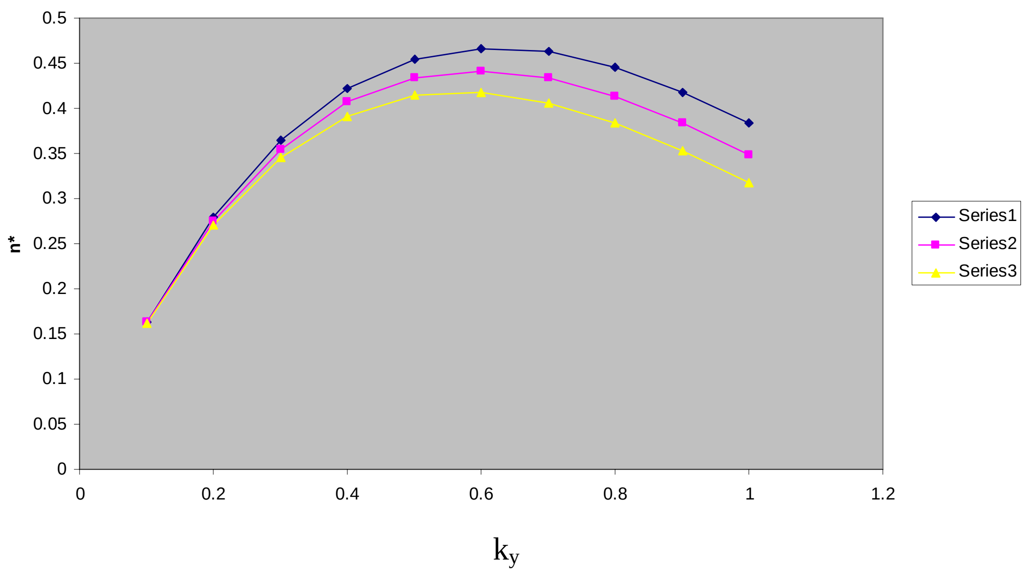

Figure 2: Plot of growth rate n* against the wave number k_y for $\nu_0{}^* = 0.6, 0.7$ and 0.8 when a = 0.2, $V_A{}^* = 0.5$, $\lambda^* = 0.6$, $k_1{}^* = 20$ and Ω =1.0 Series 1, 2 and 3 stands for $\nu_0{}^* = 0.6.7$ and 0.8 respectively

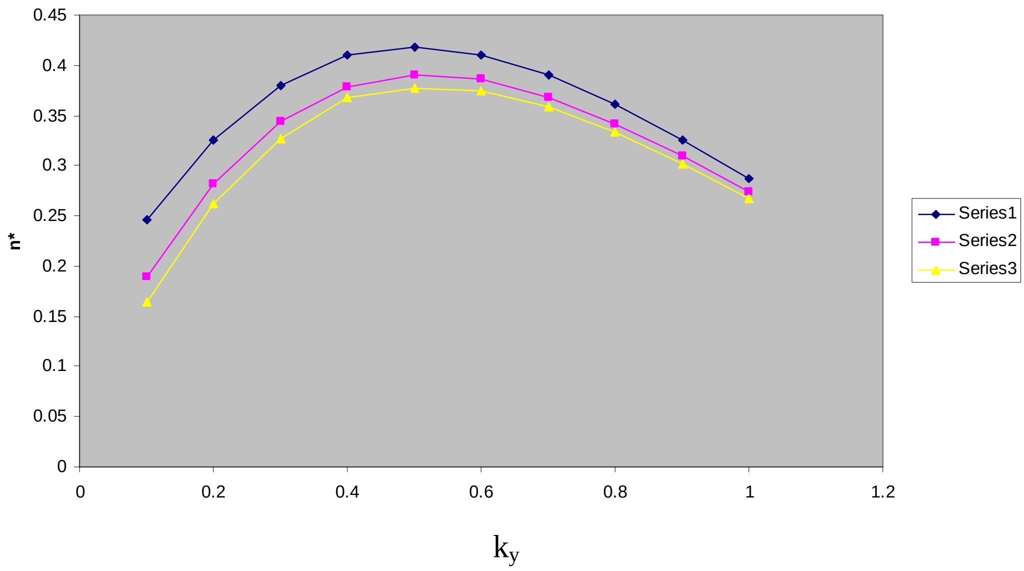

Figure 3: Plot of growth rate n* against the wave number k_y for $k_1^* = 5$, 10 and 20 when $a = 0.2$, $V_A^* = 0.5$, $\lambda^* = 0.6$, $\nu_0^* = 1.0$ and $\Omega = 1.0$. Series 1, 2 and 3 stands for $k_1^* = 5$, 10 and 20 respectively

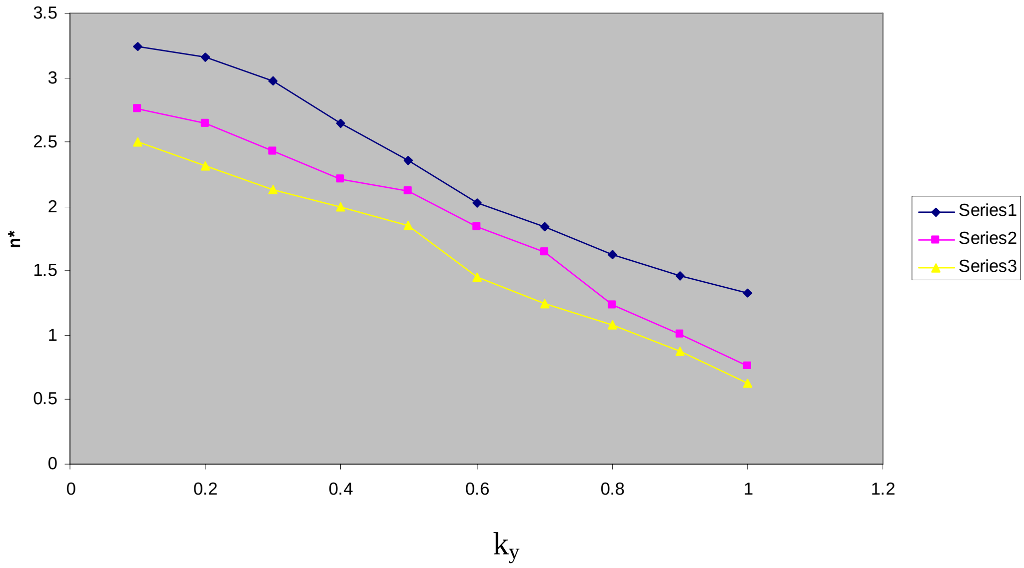

Figure 4: Plot of growth rate n* against the wave number k_y for $\Omega^* = 0.2$, 0.4 and 0.6 when $a = 0.2$, $V_A^* = 0.5$, $\lambda^* = 0.6$, $\nu_0^* = 1.0$ and $k_1^* = 10$ Series 1, 2 and 3 stands for $\Omega^* = 0.2$, 0.4 and 0.6 respectively

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