Evaluation of Reliability of a Power Plant with the Aid of Boolean Function Expansion Algorithm

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Abstract

In this paper, investigation has been carried out for the evaluation of reliability behaviour of a power plant with the aid of Boolean function expansion algorithm. The complex system under consideration consists of four power generators in a power house. The object of the system is to supply power from a power house generated by generators to critical consumers fed from an output main switch and consequently, the reliability of the supply has been calculated by considering that failure times for various components i.e., cables, generators and main switch boards etc follow arbitrary distribution.

Introduction

Nowadays, reliability is far from an abstract concept and it ranks on the same level as the performance of an equipment. Moreover, evaluation of reliability is a basic requirement for all reliability studies. However, it is an open secret that reliability evaluation becomes more complicated when complexities increase in a system. Therefore, the derivation of a symbolic reliability expression in a simplified and compact form for a complex system is of the utmost importance.

The problem of ensuring the reliability of engineering systems is extremely complex and extends to all the stages of the service life of a system. Today, there exists a large number of problems which in designing marine power plants, for example, are still solved only on the basis of logical reasoning and experience and not with the aid of reliability calculations.

Keeping the above facts in view, we, therefore, consider a complex system consisting of four generators connected in parallel. The generators $G_1, G_2, G_3$ & $G_4$ are connected with two way main switches $MSB_1, MSB_2, MSB_3$ and $MSB_4$ respectively by perfectly reliable cables. A cable $C_5$ connects $MSB_1$, $MSB_2$, a cable $C_6$ connects
MSB₂, MSB₃ and a cable C₇ connect MSB₃, MSB₄. Further, cables C₁, C₂, C₃ & C₄ connect the two way main switch MSB₁ to output main two way switch MSB₂ to output main switch OPMS₅, two way switch MSB₃ to output main switch OPMS₅ and two way switch MSB₄ to output main switch OPMS₅ respectively. Thus, the complex system under consideration is nothing but a power plant. The object of the system is to supply power generated by G₁, G₂, G₃ & G₄ to critical consumers and consequently the reliability of the power supply fed from OPMS₅ has been estimated with the aid of Boolean Function Expansion Algorithm by considering that failure times for various components of the system follow arbitrary time distribution.

**Assumptions**

1. The reliabilities of all constituent components of the system are known in advance.
2. The state of all components is statistically independent.
3. The state of each component and of the whole system is either good (operating) or bad (failed).
4. There is no standby or switched redundancy.
5. The failure times for all the components are arbitrary.
6. There is no repair facility.
7. The system can fail i.e. The supply of power can fail only if
   i. All the four generators fail.
   ii. At least one component (switch or cable) in the routes of the power supply fails.
Notations

\( x_1, x_2, x_3, x_4 \) States of generators G1, G2, G3 & G4
\( x_5, x_6, x_7, x_8 \) States of MSB1, MSB2, MSB3 & MSB4
\( x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16} \) States of the cables C1, C2, C3, C4, C5, C6 & C7
\( x_{17} \) State of OPMS4
\( x_k' \) Negation of \( x_k \) (k=1-16)
\( \land \) Conjunction
\( \lor \) Disjunction

\( x_i \) \( i \) in bad state
\( 1 \) in good state (i=1-16)

Pr (f=1) The probability of the successful operation of the function f.

Formulation of Mathematical Model

By using Boolean Function Technique, the conditions of capability for the successful operation of the complex system in terms of logical matrix are expressed as

\[
f(x_1, x_2, \ldots, x_{16}) = \begin{array}{cccccccc}
  x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 \\
  x_9 & x_{10} & x_{11} & x_{12} & x_{13} & x_{14} & x_{15} & x_{16}
\end{array}
\]

(1)
Solution of the Model

By the application of algebra of logic equation (1) may be written as

\[ f(x_1, x_2, \ldots, x_{16}) = x_{13} \land g(x_1, x_2, x_3, \ldots, x_{12}, x_{14}, x_{15}, x_{16}) \]  

(2)

where

\[ g(x_1, x_2, \ldots, x_{12}, x_{14}, \ldots, x_{16}) = \]

\[
\begin{array}{c|cc|cccc|cccc|cr}
  & x_1 & x_5 & x_9 & x_6 & x_{10} & x_{14} & x_6 & x_7 & x_{11} & x_{14} & x_{15} & x_{16} \\
  x_2 & x_6 & x_{10} & x_5 & x_9 & x_{14} & x_7 & x_{11} & x_{15} & x_7 & x_8 & x_{12} & x_{15} & x_{16} & x_3 & x_7 & x_{11} & x_{16} & x_4 & x_8 & x_{12} & x_6 & x_5 & x_6 & x_7 & x_9 & x_{14} & x_{15} & x_{16} & x_7 & x_8 & x_{12} & x_{16} \\
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  x_7 & x_8 & x_{12} & x_{15} & x_{16} \\
\end{array}
\]

(3)

Let us take \( x_{15} \) and break the complex event into incompatible events as follows:

\[ g(x_1, x_2, \ldots, x_{12}, x_{14}, x_{15}, x_{16}) = x_{15}' y_0 \lor x_{15} y_1 \]  

(4)

where

\[
\begin{array}{c|cc|cccc|cccc|cr}
  & x_1 & x_5 & x_9 & x_6 & x_{10} & x_{14} & x_2 & x_6 & x_{10} & x_5 & x_9 & x_{14} & x_3 & x_7 & x_{11} & x_8 & x_{12} & x_{16} & x_4 & x_8 & x_{12} & x_7 & x_{11} & x_{16} \\
  y_0 = & \end{array}
\]

(5)
In $y_0$, ten arguments, $(x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{14}, x_{16})$ entering into equation (5) twice, therefore, any of them may be taken to perform the expansion. Let us take $x_{16}$ and break the complex event into incompatible events as follows.

$$y_0 = x'_{16} \ y_{00} \lor \ x_{16} \ y_{01}$$

where

$$y_{00} = \begin{bmatrix} x_1 & x_5 & x_9 \\ x_2 & x_6 & x_{10} \\ x_3 & x_7 & x_{11} \\ x_8 & x_{12} \\ x_4 & x_5 & x_{10} & x_{14} \\ x_2 & x_6 & x_{10} \\ x_3 & x_7 & x_{11} \\ x_8 & x_{12} \\ x_4 & x_5 & x_{10} & x_{14} \\ x_2 & x_6 & x_{10} \\ x_3 & x_7 & x_{11} \\ x_8 & x_{12} \end{bmatrix}$$

$$y_{01} = \begin{bmatrix} x_1 & x_5 & x_9 \\ x_2 & x_6 & x_{10} \\ x_3 & x_7 & x_{11} \\ x_4 & x_8 & x_{12} \\ x_7 & x_{11} \end{bmatrix}$$

(6)
Now, expand the function $y_{00}$ by argument $x_{14}$ as follows.

$$y_{00} = x'_{14}y_{000} \lor x_{14}y_{001},$$

where

$$y_{000} = \begin{vmatrix}
  x_1 & x_5 & x_9 \\
  x_2 & x_6 & x_{10} \\
  x_3 & x_7 & x_{11} \\
  x_4 & x_8 & x_{12}
\end{vmatrix}$$

$$y_{001} = \begin{vmatrix}
  x_1 & x_5 & x_9 & x_6 & x_{10} \\
  x_2 & x_6 & x_{10} & x_5 & x_9 \\
  x_3 & x_7 & x_{11} \\
  x_4 & x_8 & x_{12}
\end{vmatrix}$$

(9)

(10)

Since all the letters occur is equation (9) only once, it implies that $y_{000}$ is non-iterated.

Now, expand the function $y_{001}$ by argument $x_9$ (say) as follows.

$$y_{001} = x'_9y_{0010} \lor x_9y_{0011},$$

where

$$y_{0010} = \begin{vmatrix}
  x_1 & x_5 & x_6 & x_{10} \\
  x_2 & x_6 & x_{10} & x_5 & x_9 \\
  x_3 & x_7 & x_{11} \\
  x_4 & x_8 & x_{12}
\end{vmatrix}$$

$$y_{0011} = \begin{vmatrix}
  x_1 & x_5 & x_6 & x_{10} & x_6 & x_{10} \\
  x_2 & x_6 & x_{10} & x_5 & x_9 \\
  x_3 & x_7 & x_{11} \\
  x_4 & x_8 & x_{12}
\end{vmatrix}$$

(11)

(12)

Now expand $y_{0010}$ by argument $x_{10}$ (say) as follows.

$$y_{0010} = x'_{10}y_{00100} \lor x_{10}y_{00101},$$

where

$$y_{00100} = \begin{vmatrix}
  x_3 & x_7 & x_{11} \\
  x_4 & x_8 & x_{12}
\end{vmatrix}$$

(13)
Evaluation of Reliability of a Power Plant

\[ y_{00101} = \begin{bmatrix} x_1 & x_5 & x_6 \\ x_2 & x_6 \\ x_3 & x_7 & x_{11} \\ x_4 & x_8 & x_{12} \end{bmatrix} \]  \quad (14)

Since all the letters occur in \( y_{00100} \) only once, it implies that \( y_{00100} \) is non-iterated.

Now, expand the function \( y_{00101} \) by the argument \( x_6 \) (say) as follows.

\[ y_{001010} = x_6 y_{001010} \lor x_6 y_{001011} \text{ where} \]

\[ y_{001010} = \begin{bmatrix} x_3 & x_7 & x_{11} \\ x_4 & x_8 & x_{12} \\ x_1 & x_5 \end{bmatrix} \quad (15) \]

\[ y_{001011} = \begin{bmatrix} x_2 & x_6 & x_5 \\ x_3 & x_7 & x_{11} \\ x_4 & x_8 & x_{12} \end{bmatrix} \quad (16) \]

Since all the letters occur in equation (15) & (16) only once, it implies that \( y_{001010} \) and \( y_{001011} \) are non-iterated.

Now, we expand the function \( y_{0011} \) by argument \( x_{10} \) (say) as follows.

\[ y_{0011} = x_{10} y_{00110} \lor x_{10} y_{00111} \text{ where} \]

\[ y_{00110} = \begin{bmatrix} x_2 & x_6 & x_5 \\ x_3 & x_7 & x_{11} \\ x_4 & x_8 & x_{12} \\ x_1 & x_5 & x_6 \end{bmatrix} \quad (17) \]

\[ y_{00111} = \begin{bmatrix} x_2 & x_6 & x_5 \\ x_3 & x_7 & x_{11} \\ x_4 & x_8 & x_{12} \end{bmatrix} \quad (18) \]

Since all the letters occur in equation (17) only once, it implies that \( y_{00110} \) is non-iterated.

Now, we expand the function \( y_{0011} \) by argument \( x_6 \) (say) as follows.

\[ y_{0011} = x_6 y_{001110} \lor x_6 y_{001111} \text{ where} \]

\[ y_{00110} = \begin{bmatrix} x_3 & x_7 & x_{11} \\ x_4 & x_8 & x_{12} \end{bmatrix} \text{ which is non-iterated} \quad (19) \]
Since all the letters occur in equation (19) only once, it implies that $y_{001110}$ is non-iterated.

Proceeding in this way, finally it is observed that all the functions are non-iterated and also not subjected to further transformations.

So making use of equations, we get

$$g(x_1, x_2, \ldots, x_{12}, x_{14}, x_{15}, x_{16}) =$$
### Evaluation of Reliability of a Power Plant

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Now using Baye’s formula, the probability of successful operation of the function $g$ is given by
\[ Pr(g = 1) = \sum_{i=1}^{94} Pr(K_i) Pr(g \mid K_i) \]  
(22)

If \( R_i \) is the reliability of the component of the complex system corresponding to state \( x_i \) and \( Q_i \) is the corresponding unreliability, then from equation (22) finally, the probability of the successful operation (i.e., reliability) of the complex system is given by

\[ R_s = Pr(f = 1) = Pr(x_{13}).Pr(g = 1) \]

**Particular Cases**

If reliability of each component of the complex system is \( R \), then

\[ R_s = 4R^4 - 12R^5 + 17R^6 + 16R^7 + 10R^9 - 34R^{10} + 36R^{11} - 17R^{12} - 3R^{13} + 10R^{14} - 8R^{15} + 2R^{16}. \]

When failure rates follow Weibull distribution, let failure rate of each component of the complex system be \( \lambda \), then reliability of the system at an instant \( t \) is given by

\[ R_s(t) = 4e^{-4\lambda t} - 12e^{-5\lambda t} + 17e^{-6\lambda t} - 20e^{-7\lambda t} + 16e^{-8\lambda t} + 10e^{-9\lambda t} - 34e^{-10\lambda t} + 36e^{-11\lambda t} - 17e^{-12\lambda t} - 3e^{-13\lambda t} + 10e^{-14\lambda t} - 8e^{-15\lambda t} + 2e^{-16\lambda t}. \]

where \( \alpha \) is a positive parameter.

When failure rates follow Exponential distribution, exponential distribution is a particular case of Weibull distribution for \( \alpha = 1 \). The reliability of the complex system in this case at an instant \( t \) is

\[ R_s(t) = 4e^{-4\lambda t} - 12e^{-5\lambda t} + 17e^{-6\lambda t} - 20e^{-7\lambda t} + 16e^{-8\lambda t} + 10e^{-9\lambda t} - 34e^{-10\lambda t} + 36e^{-11\lambda t} - 17e^{-12\lambda t} - 3e^{-13\lambda t} + 10e^{-14\lambda t} - 8e^{-15\lambda t} + 2e^{-16\lambda t}. \]

\[ \lambda = 0.1, \ \alpha = 2. \]
Conclusion
This graph computes the variation of reliability with respect to time, when failure follows Exponential and Weibull distributions. A critical examination of Reliability Vs Time graph shows that the reliability of the complex system decreases approximately at a uniform rate in case of Exponential distribution, whereas it decreases very rapidly when failure follows Weibull distribution.

References