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On Some New Measures of Intutionstic Fuzzy Entropy and Directed Divergence

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Abstract

In the present paper, generalized measures of intutionistic fuzzy directed divergence with the proof of their validity are introduced .paticular case of corresponding directed divergence and symmetric divergence have also been discussed.

Keywords: Intutionistic Fuzzy Set, Entropy, Directed Divergence, Measures of Information.

Introduction

Uncertainty and fuzziness are the basic nature of human thinking and of many real world objectives.Fuzziness is found in our decision, in our language and in the way we process information. The main use of information is to remove uncewrtainity and fuzziness in fact we measure information supplied by the amount of probalistic uncertainity removed in experiment and the measure of uncertainity removed is also called as measure of information while measure of fuzziness is the measure of vagueness and ambiguity of uncertainties.

Shannon (1948) used “entropy” to measure uncertain degree of the randomness in a probability distribution.let X is a discrete random variable with pattern recognition P= $\left(P_{1,}P_2,P_3 \ldots\ldots P_n\right)$ in an experiment.the information contained in this experiment is given by

$$H(P) = -\sum_{i=1}^{n} p_i \log p_i$$

Which is known as Shannon entropy.

The concept of entropy has been widely used in different areas, e.g. communication theory,

Generalized measure of intutionstic fuzzy directed divergence corresponding to Havard & Charvat measure:-

Havard & Charvat (1967) defined the directed divergence measure of a probability distribution P=(P_1,P_2,…………,P_n) from another probability distribution Q=(q_1,q_2,…………,q_n)as

$$D^{\beta}(P:Q) = \frac{1}{\beta - 1}\sum_{i=1}^{n}\left(p_i^{\beta} q_i^{1-\beta} - 1\right);\ \beta > 0, \beta \neq 1\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots..(1)$$

Which is called generalized directed divergence of degree β.

The following measure of symmetric divergence was proposed by Kulbark (1959):

$$J^{\beta}(P:Q) = D^{\beta}(P:Q) + D^{\beta}(Q:P)$$

$$= \frac{1}{\beta - 1}\sum_{i=1}^{n}\left(p_i^{\beta} q_i^{1-\beta} + q_i^{\beta} p_i^{1-\beta} - 2\right)\ \ldots\ldots\ldots.(2)$$

Which is also called a distance measure of degree β.

Corresponding (1) and (2) we get following measure of intutionstic fuzzy directed divergence:

$$I^{\beta}(A:B) = \frac{1}{\beta - 1}\sum_{i=1}^{n}\left[\mu_A^{\beta}(x_i)\mu_B^{1-\beta}(x_i) + V_A^{\beta}(x_i)V_B^{1-\beta}(x_i) - 1\right]\ \ldots\ldots\ldots\ldots\ldots\ldots..(3)$$

And $J^{\beta}(A:B) = I^{\beta}(A:B) + I^{\beta}(B:A)$

Now we show that $I^{\beta}(A:B)$ is a valid measure of intutionstic fuzzy directed divergence.

$I^{\beta}(A:B)$ is defined in the range $0 \le \mu_A(x) + V_A(x) \le 1$ Further it is proved that $I^{\beta}(A:B) \ge 0$ for all $\beta \neq 1$ and $\beta > 0. I^{\beta}(A:B)$ is continuous function of $\mu_A(x)\ and$ $V_A(x)$. It is easy to see that $I^{\beta}(A:B) = 0$ when $\mu_A(x) = 0$ and $V_A(x) = 1$. $I^{\beta}(A:B)$ is increasing function of $\mu_A(x)$ in the range $0 \le \mu_A(x) \le 0.5$ and decreasing function of $V_A(x)$ in the range $0 \le V_A(x) \le 0.5. I^{\beta}(A:B)$ does not changed on changing $\mu_A(x)\ to\ V_A(x)$. To verify that $I^{\beta}(A:B)$ is convex function of $\mu_A(x)$, Let us consider$\mu_A(x) = s\ and\ V_A(x) = t$ then

$$\frac{1}{\beta - 1}\sum_{i=1}^{n}\left[\mu_A^{\beta}(x_i)\mu_B^{1-\beta}(x_i) + V_A^{\beta}(x_i)V_B^{1-\beta}(x_i) - 1\right]$$

$$\le \frac{1}{\beta - 1}\sum_{i=1}^{n}\left[s^{\beta}t^{1-\beta} + (1-s)^{\beta}(1-t)^{1-\beta} - 1\right]$$

Consider $f(s) = \frac{1}{\beta-1}\left[s^{\beta}t^{1-\beta} + (1-s)^{\beta}(1-t)^{1-\beta} - 1\right]$

Therefore $f'(s) = \frac{1}{\beta-1}\left[\beta s^{\beta-1}t^{1-\beta} + \beta(1-s)^{\beta-1}(-1)(1-t)^{1-\beta}\right]$

And $f''(s) = \frac{1}{\beta-1}\left[\beta(\beta-1)s^{\beta-2}t^{1-\beta} + \beta(\beta-1)(1-s)^{\beta-2}(1-t)^{1-\beta}\right] \geq 0$

Therefore $I^{\beta}(A{:}B)$ is convex function of $\mu_A(x_i)$ Hence $I^{\beta}(A{:}B)$ is valid measure of intutionistic fuzzy entropy.

Corresponding to Renyis measure of directed divergence:

$$D_{\propto}(P{:}Q) = \frac{1}{\propto -1}\log \sum_{i=1}^{n} p_i^{\propto} q_i^{1-\propto}$$

$\propto \neq 0, \propto > 0$ ....(4)

We define the measure of intutionistic fuzzy directed divergence

$$I_{\propto}(A{:}B) = \frac{1}{\propto -1}\log\sum_{i=1}^{n}[\mu_A^{\alpha}(x_i)\mu_B^{1-\alpha}(x_i) + V_A^{\alpha}(x_i)V_B^{1-\alpha}(x_i)] \quad (5)$$

Where $\propto \neq 0, \propto > 0$ and measure of intutionistic fuzzy symmetric divergence

$$J_{\propto}(A{:}B) = I_{\propto}(A{:}B) + J_{\propto}(B{:}A) \text{ ....(6)}$$

Now we show that $I_{\propto}(A{:}B)$ is a valid measure of intutionistic fuzzy directed divergence.

It is obvious that $I_{\propto}(A{:}B) \geq 0$ and is defined in the range $0 \leq \mu_A(x) + V_A(x) \leq 1$

$I_{\propto}(A{:}B)$ is continuous in this range.it is easy to see that if $\mu_A(x) = 0$ and $V_A(x) = 1$ than $I_{\propto}(A{:}B) = 0$.

$I_{\propto}(A{:}B)$ is increasing function of $\mu_A(x)$ in the range $0 \leq \mu_A(x) \leq 0.5$ and decreasing function of $V_A(x)$ in the range $0 \leq V_A(x) \leq 0.5$. $I_{\propto}(A{:}B)$ does not changed on changing $\mu_A(x)$ *to* $V_A(x)$. To verify that $I_{\propto}(A{:}B)$ is convex function of $\mu_A(x)$, Let us consider$\mu_A(x) = s$ *and* $V_A(x) = t$ then

$$\frac{1}{\propto -1}\log\sum_{i=1}^{n}[\mu_A^{\alpha}(x_i)\mu_B^{1-\alpha}(x_i) + V_A^{\alpha}(x_i)V_B^{1-\alpha}(x_i)] \leq \frac{1}{\propto -1}\log[s^{\propto}t^{1-\propto} + (1-s)^{\propto}(1-t)^{1-\propto}]$$

Let

$$f(s,t) = \frac{1}{\propto -1}\log[s^{\propto}t^{1-\propto} + (1-s)^{\propto}(1-t)^{1-\propto}]$$

$$f'(s,t) = \frac{1}{\propto -1}\,\frac{1}{s^{\propto}t^{1-\propto} + (1-s)^{\propto}(1-t)^{1-\propto}}[\propto s^{\propto-1}t^{1-\propto} - \propto (1-s)^{\propto-1}(1-t)^{1-\propto}]$$

And

$$f''(s,t) = \frac{1}{\propto -1}\left[\frac{1}{s^{\propto}t^{1-\propto} + (1-s)^{\propto}(1-t)^{1-\propto}}\{\propto(\propto -1)s^{\propto-2}t^{1-\propto} + \propto(\propto -1)(1-s)^{\propto-2}(1-t)^{1-\propto}\} - \frac{1}{[s^{\propto}t^{1-\propto} + (1-s)^{\propto}(1-t)^{1-\propto}]^2}(\propto s^{\propto-1}t^{1-\propto} - \propto(1-s)^{\propto-1}(1-t)^{1-\propto})^2\right]$$

Therefore $I_{\propto}(A:B)$ is convex function of $\mu_A(x_i)$ Hence $I_{\propto}(A:B)$ is valid measure of intutionistic fuzzy directed divergence.

Particular cases

$\lim_{\propto\to 1} I_{\alpha}(A:B) = I(A:B)$ and $\lim_{\propto\to 1} J_{\alpha}(A:B) = J(A:B)$

Where $I(A:B)$ and $J(A:B)$ are intutionistic fuzzy directed divergence intutionistic fuzzy symmetry divergence.

Let B=A_{IF} the most intutionistic fuzzy set i.e. $\mu_B(x_i) = 0.5$ *and* $V_B(x_i) = 0.5\ \forall\ i$

Then

$$I_{\propto}(A:A_{IF}) = \frac{1}{\propto-1}\sum_{i=1}^{n}\log[\mu_A^{\propto}(x_i)(0.5)^{1-\propto} + V_A^{\propto}(x_i)(0.5)^{1-\propto}]$$

$$= \frac{1}{\propto -1}\left[\sum_{i=1}^{n}\log(0.5)^{1-\propto} + \sum_{i=1}^{n}\log\{\mu_A^{\propto}(x_i) + V_A^{\propto}(x_i)\}\right]$$

$$= \frac{1}{\propto -1}\sum_{i=1}^{n}(\propto -1)\log 2 + \frac{1}{\propto -1}\sum_{i=1}^{n}\log\{\mu_A^{\propto}(x_i) + V_A^{\propto}(x_i)\}$$

$$= n\log 2 - \frac{1}{\propto-1}\sum_{i=1}^{n}\log\{\mu_A^{\propto}(x_i) + V_A^{\propto}(x_i)\}$$

Thus $I_{\propto}(A:A_{IF}) = n\log 2 - \frac{1}{\propto-1}\sum_{i=1}^{n}\log\{\mu_A^{\propto}(x_i) + V_A^{\propto}(x_i)\}$

$= n\log 2\ -$(Entropy of the intutionistic fuzzy set)

Intutionistic fuzzy entropy corresponding to Sharma and Mittals measure

Sharma and mittals (1975) characterized non additive entropy of discrete probability distribution given by

$$H_{\alpha}^{\beta}(p) = \frac{1}{2^{1-\beta} - 1}\left[\left(\sum_{i=1}^{n} {p_i}^{\alpha}\right)^{\frac{\beta-1}{\alpha-1}} - 1\right]$$

Where $\propto \neq 1, \propto > 0, \beta > 0, \beta \neq 1$

Corresponding measure of instutionistic fuzzy entropy is given by

$$H_{\alpha}^{\beta}(A) = \frac{1}{2^{1-\beta} - 1}\left[\sum_{i=1}^{n}\left(\mu_A^{\alpha}(x_i) + V_A^{\alpha}(x_i)\right)^{\beta-1/\alpha-1} - 1\right]$$

Where $\propto \neq 1, \propto > 0, \beta > 0, \beta \neq 1$

Next, we prove $H_\alpha^\beta(A)$ is valid measure $H_\alpha^\beta(A)$ is defined in the range $0 \leq \mu_A(x) + V_A(x) \leq 1$ $H_\alpha^\beta(A)$ is continuous in the range.

$H_\alpha^\beta(A)$ is zero when $\mu_A(x) = 0$ and $V_A(x)$=1.

$H_\alpha^\beta(A)$ is increasing function of $\mu_A(x)$ in the range $0 \leq \mu_A(x) \leq 0.5$ and decreasing function of $V_A(x)$ in the range $0 \leq \mu_A(x) \leq 0.5$. $H_\alpha^\beta(A)$ does not change on changing $\mu_A(x)$ to $V_A(x)$.

Also, $H_\alpha^\beta(A)$ is concave function of $\mu_A(x)$.

Let $\mu_A(x_i) = s, V_A(x_i) = t$

$$f(s,t) = \frac{1}{2^{1-\beta} - 1}\left[(s^\alpha + t^\alpha)^{\frac{\beta-1}{\alpha-1}} - 1\right]$$

$$f'(s,t) = \frac{1}{2^{1-\beta} - 1}\frac{\beta - 1}{\alpha - 1}\,(s^\alpha + t^\alpha)^{\frac{\beta-\alpha}{\alpha-1}}\,\alpha s^{\alpha-1}$$

$$f''(s,t) = \frac{\alpha}{2^{1-\beta} - 1}\frac{\beta - 1}{\alpha - 1}\left[\frac{\beta - 1}{\alpha - 1}(s^\propto + t^\propto)^{\frac{\beta-2\alpha+1}{\alpha-1}}\,s^{\alpha-1} + (s^\alpha + t^\alpha)^{\frac{\beta-\alpha}{\alpha-1}}\,(\alpha - 1)s^{\alpha-2}\right]$$

Intutionistic fuzzy directed divergence corresponding to Sharma & Mittals measure.

Sharma & Mittals (1977) also studied the following generalized measure of directed divergence

$$I_\alpha^\beta(P:Q) = \frac{1}{1 - 2^{1-\beta}}\left[\left(\sum_{i=1}^{n} p_i^\alpha q_i^{1-\alpha}\right)^{\frac{\beta-1}{\alpha-1}} - 1\right]$$

Where $\propto \neq 1, \propto > 0, \beta > 0, \beta \neq 1$

Corresponding measure of instutionistic fuzzy directed divergence is

$$I_\alpha^\beta(A:B) = \left[\left(\sum_{i=1}^{n} \mu_A^\alpha(x_i)\mu_B^{1-\alpha}(x_i)\right)^{\frac{\beta-1}{\alpha-1}} + \left(\sum_{i=1}^{n} V_A^\alpha(x_i)V_B^{1-\alpha}(x_i)\right)^{\frac{\beta-1}{\alpha-1}} - 1\right]$$

Where $\propto \neq 1, \propto > 0, \beta > 0, \beta \neq 1$

Now we define following measure of symmetric instutionistic fuzzy directed divergence

$$I_\alpha^\beta(A:B) = I_\alpha^\beta(A:B) + I_\alpha^\beta(B:A)$$

Next, we prove $I_\alpha^\beta(A:B)$ is valid measure it is obvious that $I_\alpha^\beta(A:B)$ is defined in the range $0 \leq \mu_A(x) + V_A(x) \leq 1$ and is continuous in the range.$I_\alpha^\beta(A:B)$ is zero when $\mu_A(x) = 0$ and $V_A(x)$=1.$I_\alpha^\beta(A:B)$ is increasing function of $\mu_A(x)$ in the range $0 \leq \mu_A(x) \leq 0.5$ and decreasing function of $V_A(x)$ in the range $0 \leq V_A(x) \leq 0.5$. it does not change on changing $\mu_A(x)$ to $V_A(x)$.

Next we show that $I_\alpha^\beta(A:B)$ is convex function of $\mu_A(x)$ for this we consider $s=\mu_A(x)$ and $t=\mu_B(x)$ then

$$I_\alpha^\beta(A:B)=\frac{1}{2^{\beta-1}-1}\left[\left(\sum_{i=1}^n \mu_A^\alpha(x_i)\mu_B^{1-\alpha}(x_i)\right)^{\frac{\beta-1}{\alpha-1}}+\left(\sum_{i=1}^n V_A^\alpha(x_i)V_B^{1-\alpha}(x_i)\right)^{\frac{\beta-1}{\alpha-1}}-1\right]$$

$$\leq \frac{1}{2^{\beta-1}-1}\left[\sum_{i=1}^n (s^\alpha t^{1-\alpha})^{\frac{\beta-1}{\alpha-1}}+\sum_{i=1}^n ((1-s)^\alpha(1-t)^{1-\alpha})^{\frac{\beta-1}{\alpha-1}}-1\right]$$

Consider

$$f(s,t)=\frac{1}{2^{\beta-1}-1}\left[(s^\alpha t^{1-\alpha})^{\frac{\beta-1}{\alpha-1}}+((1-s)^\alpha(1-t)^{1-\alpha})^{\frac{\beta-1}{\alpha-1}}-1\right]$$

$$f'(s,t)=\frac{1}{2^{\beta-1}-1}\left[\left(\frac{\beta-1}{\alpha-1}\right)(s^\alpha t^{1-\alpha})^{\frac{\beta-\alpha}{\alpha-1}}\alpha s^{\alpha-1}t^{1-\alpha}\right.$$
$$\left.+\left(\frac{\beta-1}{\alpha-1}\right)((1-s)^\alpha(1-t)^{1-\alpha})^{\frac{\beta-\alpha}{\alpha-1}}\alpha\,(-1)(1-s)^{\alpha-1}(1-t)^{1-\alpha}\right]$$

Or

$$f'(s,t)=\frac{1}{2^{\beta-1}-1}\left(\frac{\beta-1}{\alpha-1}\right)\alpha\left[(s^\alpha t^{1-\alpha})^{\frac{\beta-\alpha}{\alpha-1}}s^{\alpha-1}t^{1-\alpha}\right.$$
$$\left.+((1-s)^\alpha(1-t)^{1-\alpha})^{\frac{\beta-\alpha}{\alpha-1}}(-1)(1-s)^{\alpha-1}(1-t)^{1-\alpha}\right]$$

$$f''(s,t)=\frac{1}{2^{\beta-1}-1}\left(\frac{\beta-1}{\alpha-1}\right)\alpha\left[\left(\frac{\beta-\alpha}{\alpha-1}\right)(s^\alpha t^{1-\alpha})^{\frac{\beta-2\alpha+1}{\alpha-1}}s^{\alpha-1}t^{1-\alpha}\alpha s^{\alpha-1}\right.$$
$$+(s^\alpha t^{1-\alpha})^{\frac{\beta-\alpha}{\alpha-1}}(\alpha-1)s^{\alpha-2}t^{1-\alpha}$$
$$+\left(\frac{\beta-\alpha}{\alpha-1}\right)((1-s)^\alpha(1-t)^{1-\alpha})^{\frac{\beta-2\alpha+1}{\alpha-1}}\alpha(1-s)^{\alpha-1}(1-t)^{1-\alpha}(1-s)^{\alpha-1}(1-t)^{1-\alpha}$$
$$\left.+((1-s)^\alpha(1-t)^{1-\alpha})^{\frac{\beta-\alpha}{\alpha-1}}(\alpha-1)(1-s)^{\alpha-2}(1-t)^{1-\alpha}\right]$$

We have $f''(s,t)\geq 0$.

So $I_\alpha^\beta(A:B)$ is convex function of $\mu_A(x)$ therefore $I_\alpha^\beta(A:B)$ is valid measure of intutonistic fuzzy diverted divergence.

Particular cases

$\lim_{\beta\to 1} I_\alpha^\beta(A:B)=I_\alpha(A:B)$ and $\lim_{\beta\to 1} J_\alpha^\beta(A:B)=J_\alpha(A:B)$

Let B=A_{IFS}, the most fuzzy set i.e. $\mu_B(x_i)=0.5$ and $V_B(x_i)=0.5\ \forall\ x_i$ then

$$I_\alpha^\beta(A:A_{IFS})=\frac{1}{2^\beta-1}\left[\left(\sum_{i=1}^n \mu_A^\alpha(x_i)(0.5)^{1-\alpha}\right)^{\frac{\beta-1}{\alpha-1}}+\left(\sum_{i=1}^n \mu_A^\alpha(x_i)(0.5)^{1-\alpha}\right)^{\frac{\beta-1}{\alpha-1}}-1\right]$$

$$=\frac{1}{2^{\beta}-1}\frac{1}{2^{1-\beta}}\sum_{i=1}^{n}\left[\left(\mu_A^{\alpha}(x_i)+V_A^{\alpha}(x_i)\right)^{\frac{\beta-1}{\alpha-1}}-2^{1-\beta}\right]$$
$$=\frac{1}{1-2^{1-\beta}}\sum_{i=1}^{n}\left[\left(\mu_A^{\alpha}(x_i)+V_A^{\alpha}(x_i)\right)^{\frac{\beta-1}{\alpha-1}}-2^{1-\beta}\right]$$
$$=\frac{-n2^{1-\beta}}{1-2^{1-\beta}}+\frac{1}{1-2^{1-\beta}}\sum_{i=1}^{n}\left[\left(\mu_A^{\alpha}(x_i)+V_A^{\alpha}(x_i)\right)^{\frac{\beta-1}{\alpha-1}}-2^{1-\beta}\right]+\frac{n}{1-2^{1-\beta}}$$
$$=\text{n-}H_{\alpha}^{\beta}(A)$$

Thus $I_{\alpha}^{\beta}(A: A_{\beta})$=n – (Entropy of intutionistic fuzzy set)

Conclusion

We have proposed some new measures of intutionistic fuzzy directed divergence measures and proved their validity. Total ambiguity measures and fuzzy information improvement measures have also been introduced. Further comparative investigations for the amount of total ambiguity in different measures suggested for different pairs of fuzzy sets with different possible values of α and β can be computationally made and similar investigation can be done for the corresponding fuzzy information improvement measures suggested.

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