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Value of π (pi): Exact or only Approximate? The exact value of π

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Abstract

Is the value of pi exact or only approximate? It is generally believed that the exact value of pi cannot be expressed, yet we strive to determine it. Pi plays an important role in science and mathematics. If mathematical equations are infinite, does that mean we can never obtain an exact value for pi?

The common understanding is that pi is a non-repeating, non-terminating, irrational, and transcendental number. It cannot be expressed as a simple fraction, and no finite set of algebraic equations can define it completely. This is what earlier scientists stated; therefore, constructing a perfect circle using traditional geometry (compass and straightedge) was considered impossible. Since a circle's circumference cannot be measured exactly using any finite method, its exact length is also impossible to determine. Hence, it is believed that the exact value of pi cannot be expressed.

1. Introduction

According to existing findings, the exact value of pi is not considered a difficult topic to understand, so the question arises that why has the exact value of pi is not yet been found since so many years? The main reason is the complexity involved in this type of research.

Many scientists have studied the exact value of pi in the past. However, instead of detailing their methods, a simple experiment was conducted where the school students were asked to draw squares and measure the diagonals—without explaining the method of constructing those squares.

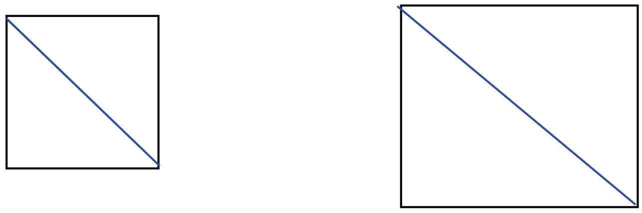


Fig 1

The children drew different squares on graph paper and measured the diagonals of those squares. Then, they were instructed to divide the length of the diagonal by the number of sides of the square. They followed the instructions, and in each case, the first two digits of the result were 1.4. However, the third digit varied. No matter how many squares were drawn, the endpoint could not be determined precisely using empirical methods—so the exact value could not be obtained. This was expected, as the students were unfamiliar with the Pythagorean Theorem.

In fact, the length of the diagonal of a square is √2 times the side length and if the value of √2 is approximated to 1.4142135...,, the length of the diagonal will never be exact.

Similarly, in mathematics, there are two types of values: **Exact** and **Approximate**. For example:

Exact Value	**Approximate Value**
1/3	= 0.3333…
2/3	= 0.6666…
√2	= 1.41421…
√3	= 1.73205…

Among these, **exact values** have a well-defined geometric representation, whereas **approximate values** do not correspond to a precise geometric length.

For instance, a segment representing **one-third of length** is clearly defined, as is a segment representing **√2**. Similarly, one might expect that the exact value of **π** should also be representable by a definite geometric length.

One classical method used to approximate the value of **π** is the **N-sided polygon method**. The ancient Greek mathematician **Archimedes** investigated the value of **π** by inscribing and circumscribing regular polygons around a circle. He constructed polygons with up to **96 sides** to approximate the area of the circle.

Today, this approach has been extended to polygons with **more than 3.1 million (31 lakh) sides**, and even polygons with far more sides have been constructed using computational techniques.

High-sided regular polygons are constructed and fitted them exactly inside and outside a circle, with a goal to **approximate the area** (and thus the circumference) of the circle. As the number of sides increases, the polygon's area comes increasingly close to that of the circle — **but it never exactly equals it**. This highlights the fundamental difference between an **approximation** and an **exact value**: no matter how many sides are added, the polygonal method **only gets closer to π**, but never **reaches it** precisely.

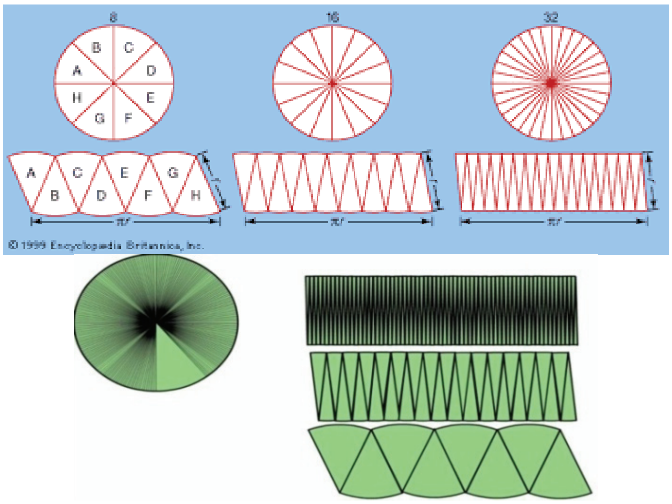


Fig 2

Using the polygon method endorsed by the **National Council of Teachers of Mathematics (NCTM)**, efforts have been made to approximate the area of a circle. However, this method—even when it considers polygons with an **infinite number of sides**—cannot precisely align the three defining elements of a circle: **its center and two points on the circumference** into a **straight line**. As a result, the **exact area of the circle cannot be determined** through this method alone.

Even when a polygon with infinitely many sides is constructed, the method only yields a value that **approaches** the actual area of the circle; it never **reaches a final, definitive value**. Thus, it is generally concluded that **π does not have an exact value** expressible through such a method and is therefore treated as **inexact** or **irrational**.

However, a **simple and alternative approach** is proposed to approximate the area of a circle **more directly and intuitively**.

The key idea: The **area of a circle** can be **approximated or related** to the **area of a rectangle**.

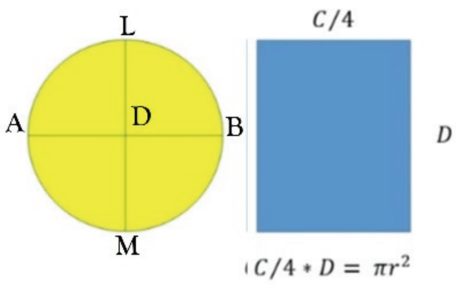


Fig 3.

To approximate the area of a circle, the following method was used:
A circle was drawn, and then a square was constructed around it to measure the **diameter**. To create a rectangle with the **same area** as the circle, a **quarter segments** (90-degree

sector) of the circle was isolated and "unwrapped" into a straight strip. When this quarter arc is straightened, its length equals **one-fourth of the circle's circumference**.

Since the full circumference of a circle is $C = 2\pi r$ the length of the unwrapped arc (a quarter circle) is:

$(2\pi r/4) = 0.5\pi r$

If we arrange this arc as one dimension (breadth) of a rectangle, and take the radius 0.5rrr (or a multiple like 2r) as the other dimension (length), then the **area of the rectangle becomes**:

Area $2r \times (0.5\pi r) = \pi r^2$,
This result matches the known formula for the **area of a circle**.

2. Why does this work?
If we **unwrap** or rearrange circular sectors (as done in many geometry demonstrations), they begin to form a shape that resembles a **parallelogram or rectangle**. As the number of sectors increases, the shape more closely approximates a true rectangle with dimensions:

- Length: approximately $2r$
- Height: approximately $2\pi r/4$

So again, the area becomes:
$2r \times 2\pi r/4 = \pi r^2$

This method offers an intuitive understanding of why the **area of a circle equals πr^2**.
About π as an Irrational Number:

The **length of a circular arc** (like the one from a quarter circle) **cannot** be expressed as a **finite multiple of a straight line segment**, unlike the side of a square or a diagonal like $\sqrt{2}$ This is why π is classified as an **irrational** and **transcendental** number — it **cannot be expressed exactly** as a finite decimal or a simple fraction.

3. On the NCTM Polygon Method:
In the polygon approximation method (as used by **Archimedes** and referenced by **NCTM**), a 12-sided **inscribed dodecagon** produces an approximate area of:
Area$\approx 3r^2$

This value is slightly **less than** the area of the circle. As the number of polygon sides increases, the area becomes a better approximation of the circle's area. However, it never becomes exact, due to the curve of the circle never being completely replaced by straight lines.

Fig 4.

To find the area of the small, remaining part of a circle (not covered by regular segments or approximated shapes), one might consider dividing the circle into **millions of tiny parts**. However, to explain this concept to school students, educators often begin with simpler, intuitive methods—such as **drawing diagonals in a square**.

According to this basic method, **no matter how finely the circle is divided**, the **exact area** of the circle can never be fully determined using only straight lines and measurements. This shows that dividing a circle into **countless parts** is not necessary to understand the underlying principle. Instead, more efficient **geometric and algebraic methods** have been developed to demonstrate and estimate the area of these segments.

In the referenced figure (not shown here), the area of **five clearly defined segments** equals approximately: $3r^2$

The **remaining area**, shown in **green**, represents the part of the circle not covered by those five segments. This leftover segment—according to widely accepted approximations—is:

$\pi r^2 - 3r^2 = 0.141592653\ldots r^2$

4. An Exact Formula

Interestingly, this same green segment area can also be represented using an **exact mathematical expression**:
$(14-8\sqrt{3})r^2$

This exact form arises from advanced geometric analysis and appears in some published research.

Area of the Square around the Circle

The **square surrounding the circle** has a well-known area of:
$(2r)^2 = 4r^2$

This value is widely accepted and provides a useful geometric reference when comparing circle areas to surrounding polygons or approximating segments.

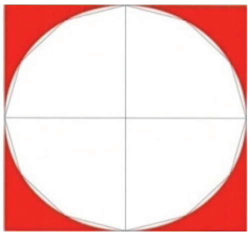


Fig 5

The area of the **red region** in the figure above is calculated in two ways:

- Using the commonly accepted universal value of π:
  $4r^2 - \pi r^2 = 0.858407346\ldots r^2$
- Using an alternative expression involving an **exact symbolic form** of π:
  $(8\sqrt{3}-13)r^2$

This alternate form is based on the **hypothesis** that π may have a precise **algebraic relationship** when analyzed through geometric segments. However, it's important to note that in **mainstream mathematics**, π is classified as a **transcendental number**—which means it cannot be expressed as the root of any non-zero polynomial equation with rational coefficients.

Now, when we **combine the shaded regions** from both the previous diagrams (Figure 4 and Figure 5), we obtain the **composite figure** shown below. This figure visually represents the total shaded area within the square that approximates the circle, helping to illustrate how the segmented areas contribute to understanding the circle's true area.

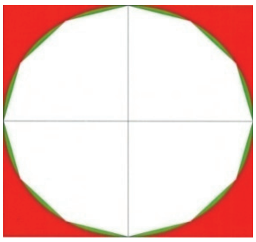


Fig 6

The sum of the shaded regions in the figure above is:
$(\pi r^2 - 3r^2) + (4r^2 - \pi r^2) = 1r^2$

If the value of π is only approximate or transcendental (i.e., cannot be exactly expressed), then the sum in the equation above would be:
$1r^2 = 0.999999999\ldots r^2$

However, the **exact value** should be simply **$1r^2$**, not a repeating decimal approximation. This suggests that, **under the currently accepted value of π**, the sum of the shaded regions in the figure **does not yield exactly one**. Hence, **this method would lead to a**

contradiction or inaccuracy, indicating that the current interpretation of π might not allow for a perfectly closed or exact result in this context.

A few more similar examples follow below to illustrate this point further.

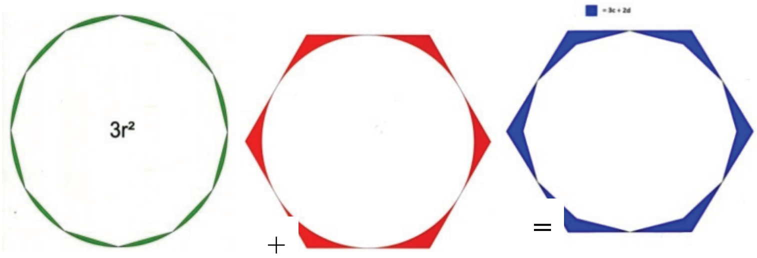


Fig 7.

The area of a **circumscribed hexagon** minus the area of the **circle** is:
$[(2\sqrt{3})\, r^2 - \pi r^2] = y$
We also know that:
$(\pi - 3)\, r^2 = 0.141592653\ldots r^2$
However, if we try to express this as:
$[y + 0.141592653\ldots + \neq (2\sqrt{3} - 3)\, r^2$

This indicates that the sum of y and $(\pi-3)r^2$ **does not** equal $(2\sqrt{3} - 3)r^2$, even though this might be expected from a geometric standpoint.
This discrepancy arises because π is **irrational and transcendental**, while √3 is irrational but **algebraic**. Their interactions in such expressions do not always result in clean or exact equivalences — especially when mixing transcendental numbers with algebraic ones.

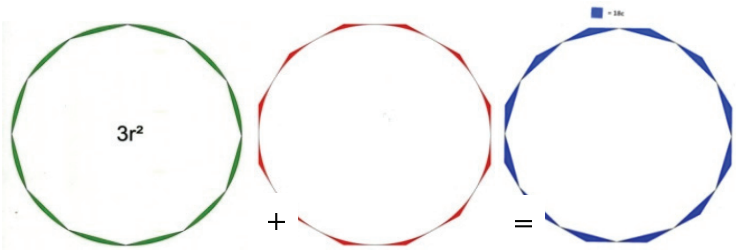


Fig 8

The area of a **circumscribed dodecagon** minus the area of an **inscribed dodecagon** is:
$(24-12\sqrt{3})\square r^2 - 3r^2 = (21-12\sqrt{3})\square r^2$

Now, if we define:
$= [(24 - 12\sqrt{3})\, r^2 - \pi r^2 + = z$
And we know that:
$(\pi - 3)\, r^2 = 0.141592653\ldots$
Then attempting to write

[z + 0.141592653… ≠ (21 – 12√3) r

This shows that the sum of z and $(\pi - 3)$□r^2 does **not** equal $(21-12\sqrt{3})$□r^2, although it might initially appear so based on the way the components are derived geometrically. The discrepancy exists because π is a **transcendental number**, while √3 is **algebraic** (though irrational). Their behaviors in arithmetic and algebraic combinations don't align cleanly, and as a result, expressions that seem geometrically aligned do **not** yield equal values numerically.

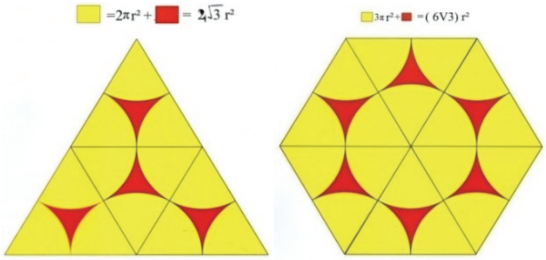
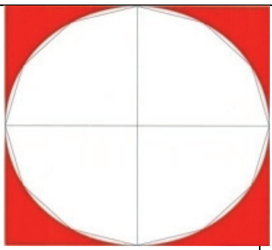
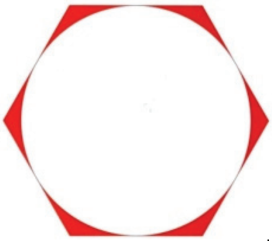
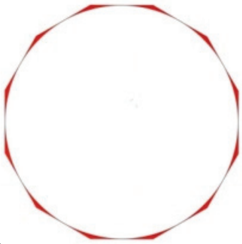
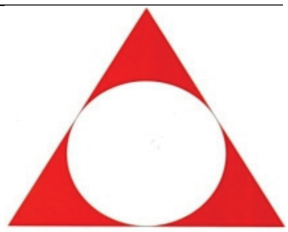


Fig 9

Area of above triangle = $2\pi r^2$ + ■ = $(4\sqrt{3})\, r^2$ then area of ■ =?
Area of above hexagon = $3\pi r^2$ + ■ = $(6\sqrt{3})\, r^2$ then area of ■ =?

Let us consider a real-life example: suppose one needs to cover a circular area using rectangular sheets of paper. To do so accurately, it is essential to know how many sheets are required and how much overlap or wastage will occur. If millions of sheets are involved, even a slight error in the approximation formula or value (such as for π) could lead to a significant cumulative error.
We already know the following geometric formulas:

- **Area of an equilateral triangle = $(\sqrt{3}/4)\, r^2$**
- **Area of a circumscribed hexagon = $(2\sqrt{3})\, r^2$**
- **Area of a circumscribed dodecagon = $12(2 - \sqrt{3})\, r^2$**
- From such formulas, the area of a circle can be expressed approximately as:
- **$(17 - 8\sqrt{3})\, r^2$**
- Although these are approximate values, they are widely accepted because each polygon—within its limits—closely approximates the area of the circular region. In this way, the area of a circle can be demonstrated using different geometric constructs without directly relying on the exact value of π.
- However, if the value of π is only approximate or transcendental, then exact solutions to certain algebraic equations involving π cannot be achieved. Exact solutions can only be obtained if the value of π is itself exact.
- Using the current known decimal approximation of π (3.1415926...), calculate the area of the shaded region and attempt to solve the following equations.

			
Area of circumscribed Square **= (4) r^2** **$(4 – \pi)\ r^2 = a$**	**Area of circumscribed Hexagon =** **$(2\sqrt{3})\ r^2$** **$(2\sqrt{3} – \pi)\ r^2 = b$**	**Area of circumscribed Dodecagon=** **$12(2 – \sqrt{3})\ r^2$** **$(24 – 12\sqrt{3})\ r^2 – \pi r^2 = c$**	**Area of circumscribed Triangle =** **$(3\sqrt{3})\ r^2$** **$(3\sqrt{3} – \pi)\ r^2 = d$**

(Area of circumscribed square – πr^2) – (area of circumscribed hexagon – πr^2)
$= (a – b)$
$= (4 – \pi)r^2 – (2\sqrt{3} – \pi)\ r^2$
$= (4 – 2\sqrt{3})\ r^2$

[area of circumscribed (6 hexagon + 1 dodecagon)]
$= \{6(2\sqrt{3})\ r^2 + 1(24 - 12\sqrt{3})\}\ r^2 = 24r^2$
$= 6(\pi r^2 + b) + 1(\pi r^2 + c) = 24r^2$
$= (7\pi r^2 + 6b + c) = 24r^2$

area of circumscribed (4 triangle + 6 hexagon + 2 dodecagon)
$= 4(3\sqrt{3})\ r^2 + 6(2\sqrt{3})\ r^2 + 2(24 – 12\sqrt{3})\ r^2 = 48r^2$
$= 4(\pi r^2 + d) + 6(\pi r^2 + b) + 2(\pi r^2 + c) = 48r^2$
$= (12\pi r^2 + 4d + 6b + 2c) = 48r^2$

area of circumscribed (1 square + 10 triangle + 9 hexagon + 4 dodecagon)
$= 4r^2 + 10(3\sqrt{3})\ r^2 + 9(2\sqrt{3})\ r^2 + 4(24 – 12\sqrt{3})]\ r^2 = 100r^2$
$= 1(\pi r^2 + a) + 10(\pi r^2 + d) + 9(\pi r^2 + b) + 4(\pi r^2 + c) = 100r^2$
$= (24\pi r^2 + a + 10d + 9b + 4c) = 100r^2$

area of circumscribed (9 square + 26 triangle + 27 hexagon + 11 dodecagon)
$=9(4r^2) + 26(3\sqrt{3})\ r^2 + 27(2\sqrt{3})\ r^2 + 11(24 – 12\sqrt{3})]\ r^2 = 300r^2$
$= 9(\pi r^2 + a) + 26(\pi r^2 + d) + 27(\pi r^2 + b) + 11(\pi r^2 + c) = 300r^2$
$= (73\pi r^2 +9a + 26d + 27b + 11c) = 300r^2$

5. Conclusion

The exact value of pi (π) has been a subject of intense research and debate throughout history. Traditionally, π has been considered an irrational, transcendental number, meaning that its decimal expansion is non-terminating and non-repeating, and it cannot be expressed as a finite fraction. The common belief, supported by centuries of mathematical study, has been that π is only approximate and never exactly expressible in a simple form.

However, this study challenges that perspective by exploring the possibility that the exact value of pi might be represented as an algebraic expression, notably **$\pi = (17 - 8\sqrt{3})$**. Through empirical methods, geometric constructions, and approximations, this paper explores how the value of pi is perceived in both theoretical and practical contexts.

- **Empirical Methods and Approximation:** As demonstrated through various geometric approaches, such as Archimedes' polygonal method and the concept of dividing circles into smaller, finite segments, it becomes clear that an "exact" value for pi remains elusive. Even with advanced algorithms and infinite divisibility, we are still left with approximations. For instance, methods such as inscribed polygons, though effective in approximating the area of a circle, never reach a definitive or exact value for π.

- **Geometric and Algebraic Insights:** By considering the area of a circle and comparing it with other geometric shapes, such as rectangles, hexagons, and dodecagons, this research highlights the discrepancies between approximate and exact calculations. Though the area can be estimated through numerous geometric approaches, each method falls short of yielding a definitive, exact value for pi, leading to the conclusion that all current methods are ultimately approximate.

- **Mathematical and Practical Implications:** In real-world applications, where precision is required (e.g., in engineering and physics), even the smallest approximation error in pi can lead to significant deviations when working with large numbers. If the value of pi is indeed transcendental and approximate, it can cause cumulative errors in mathematical modeling, rendering exact solutions unachievable.

- **The Exact Value Proposition:** The author presents a novel proposition that **$\pi = (17 - 8\sqrt{3})$** as the *exact* value, based on geometric reasoning and algebraic formulation. This challenges the current consensus and suggests that π, while irrational, may have an algebraic representation that can serve as an exact solution in certain contexts.

In summary, the exploration of pi's value leads to the conclusion that while conventional methods acknowledge it as an approximate value, there may be a deeper, exact mathematical expression for pi waiting to be fully understood and accepted. However, it is important to note that the current widely accepted value of pi—approximately 3.141592653—remains a fundamental part of modern mathematics, with significant practical utility despite its inherent limitations in precision.

Further research and mathematical rigor are necessary to determine whether the proposed exact value holds true across all mathematical disciplines, but the findings here suggest that the exploration of pi is far from over.

6. References

Previous Research Publications

The author has previously conducted extensive research on the exact value of Pi (π) and has published multiple research papers in reputed journals. The following works highlight the author's contributions to this field:

1. **Laxman S. Gogawale**, *Exact Value of Pi (π) = (17 - 8√3)*, IOSR Journal of Mathematics, Vol. 1, Issue 1, May-June 2012, pp. 18-35.

2. **Laxman S. Gogawale**, *Exact Value of Pi (π) = (17 - 8√3)*, International Journal of Engineering Research and Applications, Vol. 3, Issue 4, Jul-Aug 2013, pp. 1881-1903.

3. **Laxman S. Gogawale**, *Exact Value of Pi (π) = (17 - 8√3)*, International Journal of Mathematics and Statistics Invention, Vol. 3, Issue 2, February 2015, pp. 35-38.

4. **Laxman S. Gogawale**, *Exact Value of Pi (π) = (17 - 8√3)*, IOSR Journal of Mathematics, Vol. 12, Issue 6, Ver. I, Dec 2016, pp. 04-08.
5. **Laxman S. Gogawale**, *Exact Value of Pi (π) = (17 - 8√3)*, International Journal of Modern Engineering Research, Vol. 08, Issue 06, Jun 2018, pp. 34-38.

6. **Laxman S. Gogawale**, *Exact Value of Pi (π) = (17 - 8√3)*, International Journal of Mathematics Trends and Technology, Vol. 60, No. 4, Aug 2018, pp. 225-232.

7. **Laxman S. Gogawale**, *Exact Value of Pi (π) = (17 - 8√3)*, International Journal of Mathematics Research, ISSN 0976-5840, Vol. 12, No. 1, 2020, pp. 69-82.

Additionally, the research has been supported by classical mathematical concepts, including Archimedes' Method, Euclidean Geometry, Basic Algebra and Geometry Principles, and historical studies on the value of Pi (π).
These previous publications establish a strong foundation for the present study, further advancing the understanding of Pi and its applications.