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Arithmetic ODD Decomposition of Extented Lobster

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Abstract

Let G = (V, E) be a simple connected graph with p vertices and q edges. If $G_1, G_2, \ldots, G_n$ are connected edge disjoint subgraphs of G with $E(G)=E(G_1)\cup E(G_2)\cup\ldots\cup E(G_n)$, then $(G_1, G_2, \ldots, G_n)$ is said to be a decomposition of G. A decomposition $(G_1, G_2, \ldots, G_n)$ of G is said to be continuous monotonic decomposition(CMD) if each G_i is connected and $|E(G_i)|=i$, for every i = 1, 2, 3, …,n. In this paper, we introduced the concept arithmetic odd Decomposition. A decomposition $(G_1, G_2, \ldots, G_n)$ of G is said to be a Arithmetic Decomposition or Linear decomposition if $|E(G_i)| = a+(i-1)d$, for every i=1, 2, 3, …, n and $a,d\in Z$. Clearly $q = \frac{n}{2}[2a + (n-1)d]$. If a=1 and d=1, then $q = \frac{n(n+1)}{2}$.That is, Arithmetic decomposition is a CMD. In this paper, we study the graphs when a=1 and d=2. If d=2, then $q = n^2$. That is, the number of edges of G is a perfect square. Also we obtained the bounds for n and diameter of Extended Lobster L_E.

Keywords: Decomposition of Graph, Continuous Monotonic Decomposition, Arithmetic Decomposition or Linear Decomposition, Arithmetic Odd Path Decomposition (OPD), Arithmetic Odd Star Decomposition (OSD).

AMS Subject Classification: 05C99.

Introduction

All basic terminologies from Graph Theory are used in this paper in the sense of

Harary [3]. By a graph we mean a finite, undirected graph without loops or multiple edges.

Definition 1.1: Let G = (V, E) be a simple connected graph with p vertices and q edges. If $G_1, G_2, \ldots, G_n$ are connected edge disjoint subgraphs of G with $E(G)=E(G_1) \cup E(G_2) \cup \ldots \cup E(G_n)$, then $(G_1, G_2, \ldots, G_n)$ is said to be a Decomposition of G.

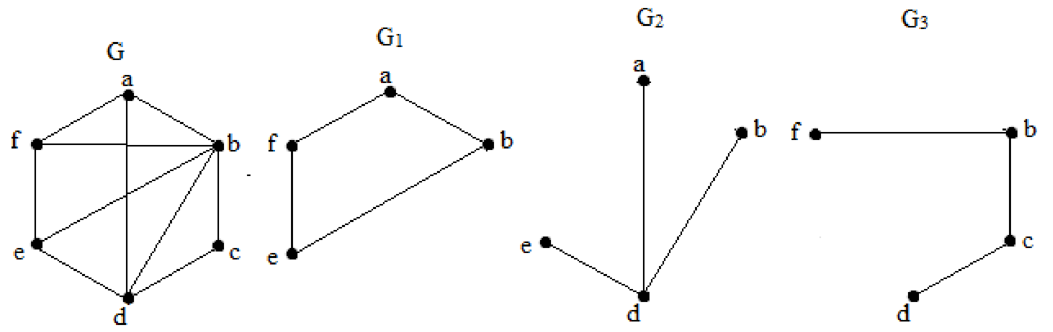

Figure (1): Decomposition (G_1, G_2, G_3) of G.

N.Gnanadhas and J.Paulraj Joseph discussed on Continuous Monotonic Decomposition (CMD) of graphs [4] and [5]. E.Ebin Raja Merly and N.Gnanadhas discussed Linear Path Decomposition or arithmetic odd path decomposition of Lobster [1] and Linear star decomposition or arithmetic odd star decomposition of Lobster [2]. This paper deals with Arithmetic odd Decomposition for a very particular class of unicyclic graph namely Extended Lobster denoted by L_E.

Definition 1.2: A Decomposition $(G_1, G_2, \ldots, G_n)$ of G is said to be Continuous Monotonic Decomposition (CMD) if $|E(G_i)|=i$, for every i=1, 2, 3, …,n. Clearly $q = \frac{n(n+1)}{2}$

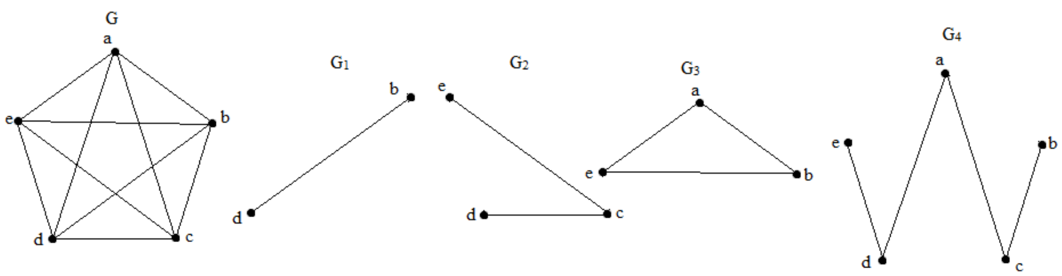

Figure (2): Continuous Monotonic Decomposition (G_1, G_2, G_3, G_4) of G

Definition 1.3: A decomposition (G_1, G_2, …, G_n) of G is said to be an Arithmetic decomposition or Linear decomposition if $|E(G_i)| = a+ (i-1)\, d$, for every i=1, 2, 3…, n, and a, d∈Z. Clearly $q = \frac{n}{2}[2a + (n-1)d]$

If a=1 and d=1, then $q = \frac{n(n+1)}{2}$. That is, Arithmetic decomposition is a CMD. If a=1 and d = 2 then, $q = n^2$. That is, the number of edges of G is a perfect square. Since the number of edges of G is a perfect square, q is the sum first n odd numbers 1, 3, 5, …, (2n-1). Thus we call the Arithmetic Decomposition with a = 1 and d = 2 as Arithmetic Odd Decomposition (AOD). Since the number of edges of each subgraph of G is odd, we denote the AOD as (G_1, G_3, G_5, …, $G_{(2n-1)}$).

Example 1.4: For the graph G in figure (3), (G_1, G_3, G_5, G_7) is an AOD.

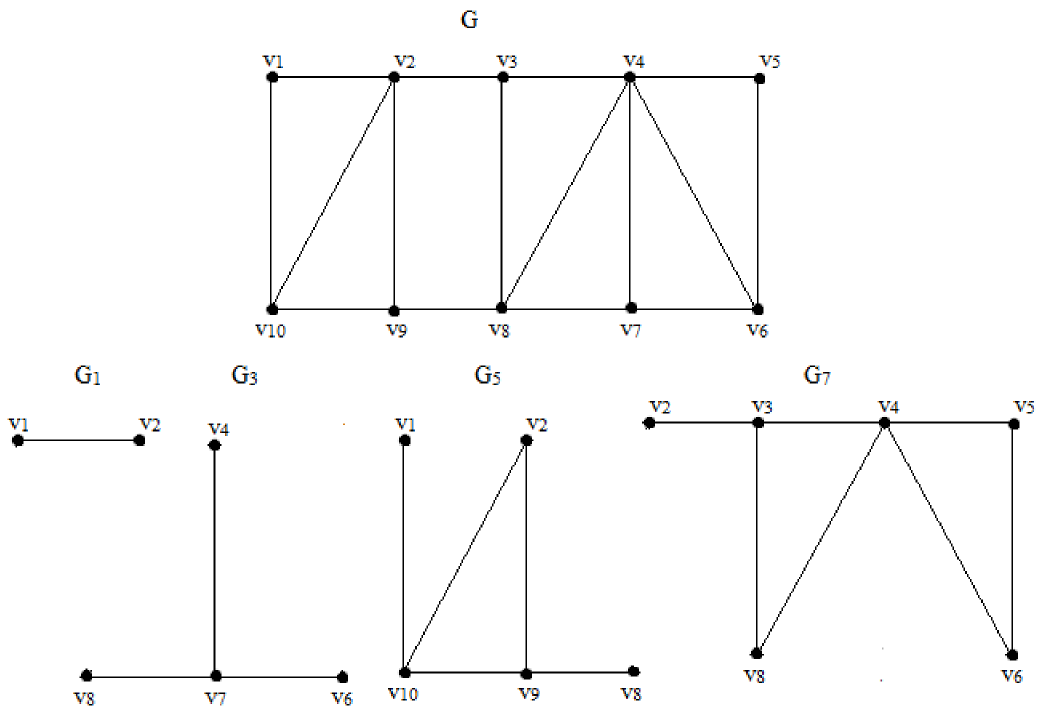

Figure (3)

Some definitions will be helpful here.

Definition 1.5: Unicyclic graph is a connected graph containing exactly one cycle.

Definition 1.6: An Arithmetic odd decomposition (G_1, G_3, G_5, …, G_{2n-1}) in which each G_i is a path P_i with i edges is said to be an Arithmetic odd Path Decomposition or simply odd path decomposition(OPD)

Definition 1.7: Caterpillar is a tree in which the removal of pendant vertices results in a path.

Definition 1.8: Lobster is a tree in which the removal of pendant vertices results in a caterpillar.

Definition 1.9: The underlying path P_l of a Lobster L is a path obtained by the removal of pendant vertices two times successively.

ODD Path Decomposition of Extended Lobster

Definition 2.1: Let L be a Lobster with n^2-1 edges. Then the graph denoted by L_E is obtained by adding an edge e to L that forms a unicyclic graph is called an Extended Lobster.

Remark 2.2:
Extended Lobster is a graph which is not a Lobster. Clearly L_E has n^2 edges. Hence L_E admits AOD.

Let L_E be the extended Lobster with q = n^2. Then L_E = L + e where L is the Lobster with underlying path P_l.

The unicycle in L_E is $C_k = P_{k-1} \cup P_1$, $3 \leq k \leq n^2-1$.

Definition 2.3: If L_E admits decomposition (P_1, P_3, P_5, …, $P_{(2n-1)}$), then the decomposition is called an Arithmetic Odd Path Decomposition (OPD) of L_E.

Remark 2.4: For OPD in L_E, always we treat P_1 as e.

Remark 2.5: In this paper, we study the Extended Lobster L_E with q= n^2 and so the term Extended Lobster L_E always means L_E with q= n^2.

Our main theorem can now be stated as follows:

Theorem 2.6: If the extended Lobster L_E admits OPD (P_1, P_3, P_5, …, $P_{(2n-1)}$), then $\sqrt{l+5} \leq n \leq 2+\sqrt{l}$.

Proof: Assume that L_E admits OPD. Clearly diam (L_E) $\geq l+4$.

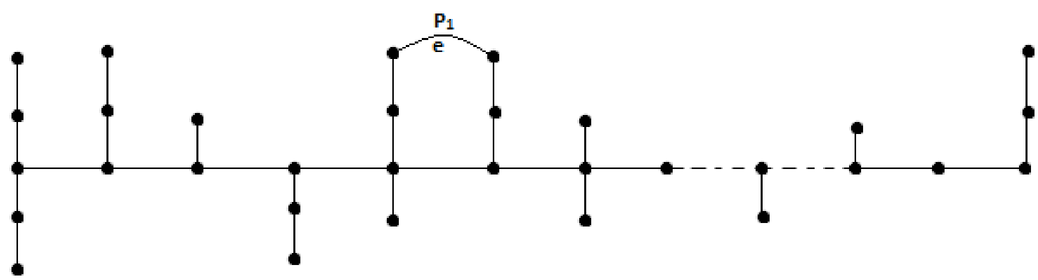

Figure (4)

Case (i): P_1 and P_3 can be obtained from L_E without taking any edge from P_l.
Then, for P_5, we must have only one edge from P_l, for P_7, we must have 3 edges from P_l, for P_9, we must have 5 edges from P_l, …, for $P_{(2n-1)}$, we must have $[(2n-1)-4]$ edges from P_l.

Hence $l = 1+3+5+\ldots+[(2i-1)-4]+\ldots+[(2n-1)-4] = (n-2)^2 \Rightarrow n = 2 \pm \sqrt{l}$

Case (ii) Each path P_{2i-1}, i = 2, 3, 4, …, n has edges from P_l.
Then, for P_3, we must have one edge from P_l, for P_5, we must have one edge from P_l, for P_7, we must have three edges from P_l, for P_9, we must have five edges from P_l …, for $P_{(2n-1)}$, we must have $[(2n-1)-4]$ edges from P_l.
Thus $l = 1+1+3+5+\ldots+[(2n-1)-4] = n^2 - 4n + 5 \Rightarrow n = 2 \pm \sqrt{l-1}$

Case (iii): atleast one edge of each $P_{(2i-1)}$, i = 2, 3, 4, …, n must be in P_l.
Then $l = 1+1+3+5+\ldots+[(2n-1)-4]$, which is same as case (ii).

Case (iv): Let $P_{(2r-1)}$ and $P_{(2s-1)}$ be two paths in the decomposition with origin v_r and v_s respectively.

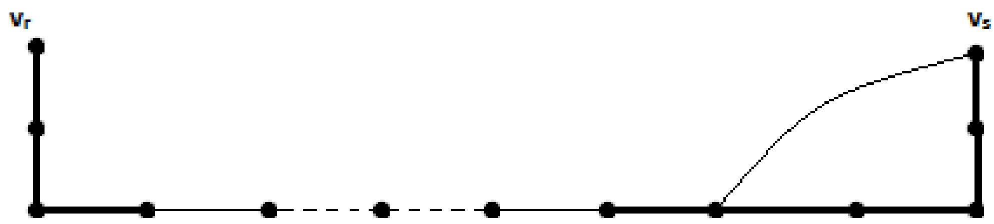

Figure (5)

Then we have $l = 1+3+7+9+\ldots+(2n-1) = n^2 - 5 \Rightarrow n = \pm\sqrt{l+5}$.

Hence $\sqrt{l+5} \le n \le 2 + \sqrt{l}$. ∎

Remark 2.7: Let L_E be an extended Lobster and P_l be the underlying path obtained from $L_E - e$. Let N_1 denotes the set of vertices in $L_E - e$ which are at a distance one from P_l. Let $n_1 = |N_1|$. Let N_2 denotes the set of pendant vertices of $L_E - e$ which are at a distance two from P_l. Let $n_2 = |N_2|$.

Theorem 2.8: Let L_E be the Extended Lobster with underlying path P_l of length l and $n = 2 + \sqrt{l}$. If L_E admits OPD $(P_1, P_3, P_5, \ldots, P_{(2n-1)})$, then $n_2 = 2n-3$.

Proof: Suppose L admits OPD. Since $n = 2 + \sqrt{l}$, no edge of P_1 and P_3 must be in P_l. Thus P_1 contributes 0 for n_2, P_3 contributes 1 for n_2, P_5 contributes atmost 2 for n_2, P_7

contributes atmost 2 for n_2, …, $P_{(2n-1)}$ contributes atmost 2 for n_2.

Thus $n_2 = 1+2(n-2) = 2n-3$. ∎

Example 2.9: We take L_E with $q=5^2$. Consider $L_E - e$ Then $n = 2 + \sqrt{l} \Rightarrow l=9$.

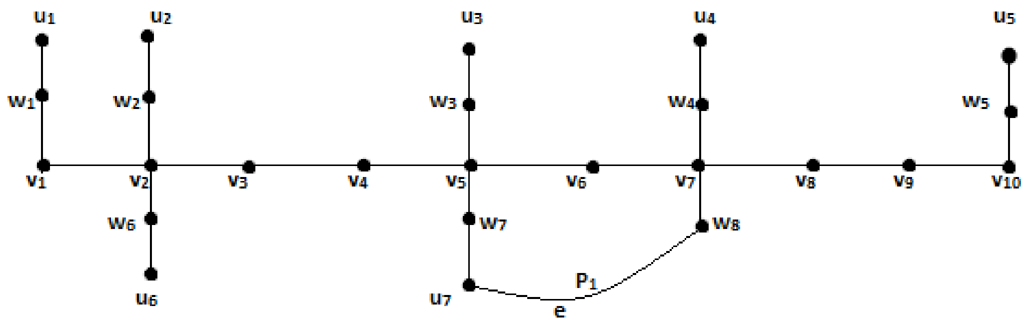

Figure (6)

Here the underlying path P_l of $L_E - e$ is $v_1v_2v_3v_4v_5v_6v_7v_8v_9v_{10}$. Clearly $P_1 = e = u_7w_8$.From the Lobster $L_E - e$, we can easily construct P_3 as $u_4w_4v_7w_8$, since no edge of P_3 must be in P_l. Also P_5 is $u_1w_1v_1v_2w_2u_2$, P_7 is $u_3w_3v_5v_4v_3v_2w_6u_6$ and P_9 is $u_5w_5v_{10}v_9v_8v_7v_6v_5w_7u_7$.

Hence $N_2= \{u_1, u_2, u_3, u_4, u_5, u_6, u_7\}$ and $n_2=7$.

Theorem 2.10: If L_E be an extended Lobster with underlying path P_l of length l and $n = \sqrt{l+5}$. Then L_E admits OPD $(P_1, P_3, P_5, \ldots, P_{(2n-1)})$ if and only if L_E –e is a path.

Proof: Assume that L_E admits OPD. To prove all the internal vertices of vertices of L_E –e are of degree 2. Suppose not. Let u be an internal vertex of L_E –e of degree > 2 as shown in figure (7).

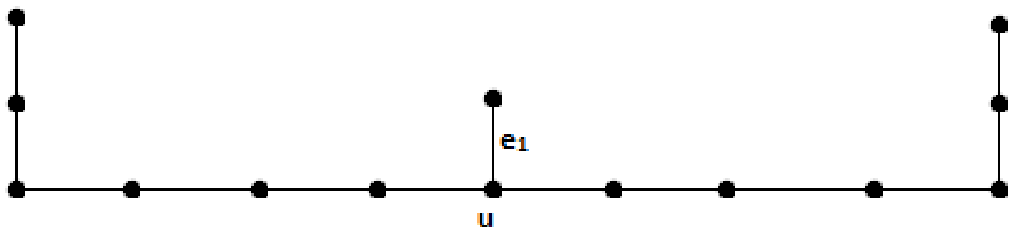

Figure (7)

Let e_1 be an edge incident with u, which is not in L_E –e. Then e_1 is the first or the last edge of some path $P_{(2k-1)}$. Thus $l + 4$ = 1+3+5+7+ …+(2k-3)+(2k-1-1)+(2k+1)+ …+(2n-1).

$\Rightarrow l + 5 = n^2 - 1 \Rightarrow n = \pm\sqrt{l+6}$ which is a contradiction.

Hence L_E –e is a path. The converse part is obvious, since L_E has $q = n^2$. ∎

ODD Star Decomposition of Extended Lobster

Definition 3.1: If L_E admits decomposition (S_1, S_3, S_5, …,$S_{(2n-1)}$), then the decomposition is known as Arithmetic odd Star Decomposition or simply odd star decomposition(OSD).

Remark 3.2: Let L_E be the extended Lobster with $q = n^2$. Then L_E –e is a Lobster with the longest path P.

Remark 3.3: For OSD in L_E, always we treat S_1 as e.

Result 3.4: If L_E admits OSD (S_1, S_3, S_5, …, $S_{(2n-1)}$), then diam (L_E –e) ≤ 2n-2.

Proof: diam (L_E –e) ≤ diam (S_3) + diam (S_5) + diam (S_7) + …+ diam ($S_{(2n-1)}$) = 2n-2.

Hence diam (L_E –e) ≤ 2n-2. ∎

Now we are ready to prove Theorem 3.5.

Theorem 3.5: Let L_E be an Extended Lobster with $q = n^2$ and diam (L_E –e) = 2n-2. If L_E admits OSD (S_1, S_3, S_5, …, $S_{(2n-1)}$) with S_1 = e if and only if

L_E –e is a caterpillar with (n-1) non-adjacent junctions and

There is no junction – neighbour in L_E.

Proof: Suppose L_E admits OSD. Since diam (L_E –e) = 2n-2, the centres of S_3, S_5, …, $S_{(2n-1)}$ lie in P. Thus L_E –e is a caterpillar. Since S_1 is e and diam (L_E –e) = 2n-2, there is exactly one non-support in between any two centres. Hence there are (n-1) non-adjacent junctions in L_E –e.

Next to prove there is no junction-neighbor in L_E - e. Suppose there is atleast one junction – neighbor in L_E – e. Let the junction – neighbor be $e_i = x_i y_j$. Then there exist junction supports v_i and v_j such that d (v_i, v_j) 3. Therefore $< E(L_E - e) - E(S_3 \cup S_5 \cup \ldots \cup S_{2n-1}) > = 2S_1$, which is a contradiction. Hence there is no junction – neighbor in L_E – e. The converse part is obvious. ∎

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