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Fuzzy Dot Algebras over Fuzzy Dot Fields

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Abstract

S. Nanda, N.R. Das (2010), introduced the notation of fuzzy algebras over fuzzy field. In this paper some of their basic properties are discussed in fuzzy dot Algebras over fuzzy dot field.

Introduction
Xi (1991) applied the concept of fuzzy sets to BCK-algebras which are introduced by Imai and Iseki (1966). J. Negges and H. S. Kim (1996) analyzed the class of d-algebras which is generalization of BCK-algebras, and investigated relations between d-algebras and BCK-algebras. J. Neggers, Y. B. Jum and H. S. Kim analyzed the notion of d-ideals in d-algebras. M. Akram and K. H. Dar (2005) provided the concepts fuzzy d-algebras, they introduced fuzzy subalgebra and fuzzy d-ideal of d-algebras.

Preliminaries
Definition 1.1:
A non empty set x with a constant o and a binary operation * is called a d-algebra, if it satisfies the following axioms:
(d_1) $x * x = 0$
(d_2) $0 * x = 0$
(d_3) $x * y = 0$ and $y * x = 0$, $x = y$
For all x, y € X

Definition 1.2:
Let x be a d-algebra and I be a subset of x. Then I is called d-ideal of x if it satisfies following conditions.

(Id_1) 0 € I
(Id_2) x * y € I and y € I =>x € I
(Id_3) x € I and y € x => x * y € I

Definition 1.3:
A fuzzy set μ in x is called fuzzy ideal of x if it satisfies the following in equalities:
(Fd_1) $\mu(0) \geq \mu(x)$
(Fd_2) $\mu(x) \geq \min\{\mu(x*y), \mu(y)\}$
(Fd_3) $\mu(x*y) \geq \min\{\mu(x), \mu(y)\}$ for all x, y € X

Definition 1.4:
A fuzzy subset μ of x is called a fuzzy dot d-ideal of x if it satisfies (Fd_1) and the following conditions.
(Fd_4) $\mu(x) \geq \mu(x*y). \mu(y)$
(Fd_5) $\mu(x*y) \geq \mu(x).\mu(y)$ for all x, y € X

Let X be a field and F a fuzzy field in X

Definition 1.5:
Let Y be an algebra over X and let A be a fuzzy subset of Y. then A is called a fuzzy dot algebra in Y if for all x,y €Y and λ€X.
$\mu_A(x+y) \geq \{ \mu_A(x). \mu_A(y) \}$
$\mu_A(\lambda x) \geq \{ \mu_F(x). \mu_A(x)\}$
$\mu_A(xy) \geq \{ \mu_A(x). \mu_A(y)\}$

if F is an ordinary fields (in particular F=X),then
ii. is replaced by
ii. $\mu_A(\lambda x) \geq \mu_A(x)$

Definition 1.6:
Let A be a fuzzy dot algebra in Y. Then A is called a fuzzy dot left ideal if for all x,y,€y.

$$\mu_A(xy) \geq \mu_A(y)$$

A is a fuzzy dot right ideal if

$$\mu_A(xy) \geq \mu_A(y)$$

A is a fuzzy dot ideal if it is both a fuzzy dot left ideal and a fuzzy right ideal.
Let X be a field and F a fuzzy field in X

Definition 1.7:
Let Y be an algebra over X and let A be a fuzzy subset of Y. then A is called a fuzzy dot algebra in Y if for all x,y €Y and λ€X.
$\mu_A(x+y) \geq \{ \mu_A(x). \mu_A(y) \}$
$\mu_A(\lambda x) \geq \{ \mu_F(x). \mu_A(x)\}$

$\mu_A(xy) \geq \{ \mu_A(x). \mu_A(y)\}$

if F is an ordinary fields (in particular F=X),then

ii. is replaced by

ii. $\mu_A(\lambda x) \geq \mu_A(x)$

Definition 1.8:

Let A be a fuzzy dot algebra in Y. Then A is called a fuzzy dot left ideal if for all x,y,€y.

$\mu_A(xy) \geq \mu_A(y)$

A is a fuzzy dot right ideal if

$\mu_A(xy) \geq \mu_A(y)$

A is a fuzzy dot ideal if it is both a fuzzy dot left ideal and a fuzzy right ideal.

Theorem: 2.1

A is fuzzy dot algebra in Y over F if and only if for all x,y€ Y and λ_1, λ_2€X

i. $\mu_A(\lambda_1 x+ \lambda_2 y) \geq \{ \mu_F(\lambda_1). \mu_A(x)\}.\{ \mu_F(\lambda_2). \mu_A(Y)\}$
ii. $\mu_A(xy) \geq \{ \mu_A(x), \mu_A(y)\}$

A is fuzzy dot ideal F if and only if (i) holds and

ii. $\mu_A(xy) \geq \mu_A(y)[\mu_A(xy) \geq \mu_A(x)]$

If F is an ordinary fields (in particular if F=X), then (i) is replaced by

$\mu_A(\lambda_1 x+ \lambda_2 y) \geq \{ \mu_A(x). \mu_A(y)\}$

Proof:

$\mu_A(\lambda_x+ \lambda_y) \geq \{ \mu_A(\lambda_x). \mu_A(\mu_y)\}$

$\geq \{ \mu_A(\lambda). \mu_A(x)\}. \{ \mu_F(\mu). \mu_A(y)\}$

Conversely

$\mu_A(1.x+1.y) \leq \{ \mu_F(1). \mu_A(x)\}.\{ \mu_F(1). \mu_A(y)\}$

$\geq \mu_A(x). \mu_A(y)$

$\mu_A(\lambda x) = \mu_A(\lambda x+0.x)$

$\{\mu_F(\lambda). \mu_A(x)\}.\{ \mu_F(0). \mu_A(y)\}$

$= \{\mu_F(\lambda).\mu_A(x)\} = \mu_A(xy)$

$\geq \{ \mu_A(x). \{\mu_F(y)\}$ follows automatically

This completes the first part of the proof.

For the second part

$\mu_A(xy) \geq \mu_A(x)$

and
μ_A (xy)≥ μ_A (y)

It follows that
μ_A (xy)≥{ μ_A (x). μ_A (y)}

Conversely,
μ_A (xy)≥{ μ_A (x). μ_A (y)} . { μ_A (x). μ_A (y)}

So A is fuzzy dot algebra
μ_A (xy)≥{ μ_A (x). μ_A (y)}

this implies μ_A (xy)≥ μ_A (x)
andμ_A (xy)≥ μ_A (y)

For the ideal, necessary part is obvious.
For the sufficient part,
μ_A (xy)≥ μ_A (y)

implies
μ_A (xy)= μ_A (yx) ≥ μ_A (x)

It given that
μ_A (xy)≥{ μ_A (x). μ_A (y)}

Similarly if F is an ordinary field, then (i)is replaced by
μ_A (λ_1x+λ_2y)≥{ μ_A (x). μ_A (y)}

This completes the proof.

Theorem: 2.2
Let Y and Z be algebra over a fuzzy dot fields F in a filed X and F an algebra homomorphism of Y in to Z. Let W be the fuzzy dot algebra (ideal) in Z. Then the inverse image f^{-1} [W] of W is fuzzy dot algebra (ideal) in Y.

Proof
for all x,y €Y and λ_1, λ_2€X
$\mu\ f^{-1[w]}$ (λ_1x+ λ_2y) = μ_A (f(λ_1A(f(λ_1x+ λ_1y))
= μ_w(λ_1f (x)+ λ_1f(y))
{μ_F(λ_1).μ_W(f(y))}
{ μ_F(λ_1).$\mu_{f^{-1}[w]}$(x)}
{ μ_F(λ_2). $\mu_{f^{-1}[w]}$ (y)}

$\mu\ f^{-1[w]}$ (xy)= μ_w(f(xy))
μ(f(x)f(v)) ≥ {μ_w(f(x)). μ_w(f(y))}

$= \{ \mu_{f^{-1}[w]}(x). \mu_{f^{-1}[w]}(y)\}$

For ideal it is similarly proved that

$\mu_{f^{-1}[w]}(xy) \geq \mu_w(f(x).f(y))$

$\geq \{\mu_w(f(x)). \mu_w(f(y))\}$

$= \{ \mu_{f^{-1}[w]}(x). \mu_{f^{-1}[w]}(y)\}$

This completes the proof

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