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A New Generalisation of Sam-Solai's Multivariate-t-Distribution

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Abstract

This paper proposed a new generalization of family of Sarmanov type Continuous multivariate symmetric probability distributions. More specifically the authors visualize a new generalization of Sam-Solai's Multivariate t-distribution from the univariate student's t-distribution. Further, we find its Marginal, Conditional distributions and also discussed its Bi-variate case. Moreover, it is found that the conditional variances of Sam-Solai's Multivariate and Bi-variate conditional t- distribution are heteroscedastic and the population Co-variances also extracted for certain degrees of freedom to the Sam-Solai's Bi-variate t- distribution.

Keywords: Sam-Solai's Multivariate t-distribution, univariate student's t-distribution, Cumulative t- distribution

Preliminaries

The logical generalization of univariate probability distribution for a Multivariate case is an interesting task on the part of statisticians. The generalization of univariate student's t distribution to its Multivariate case based on the Sarmanov type distribution is discussed.

Introduction

Kshiragar(1961) proposed some extensions of multivariate t-distribution and visualize the distribution of regression Co-efficient.Similarly,Gupta(1964) discussed the Multivariate wishart distribution and proposed the multivariate t-distribution with reference to the Complex random variables.Miller(1968) studied the different kinds of multivariate t-distribution and Siotani (1976) discussed the conditional and stepwise multivariate t-distribution.Moreover,Arellano et al(1994) gave a Predictivistic interpretation of the multivariate t-distribution and its applications.Gupta A.K(2000) formulated and discussed the Skewed version of the multivariate t-distribution and Sahu et al.(2000) proposed a new class of multivariate skew distribution and elucidates its application to the Bayesian regression models. In the same manner, Jones (2001) discussed the multivariate t-distributions and its relationship with beta distributions and multivariate F-distribution. Fang et al (2002) gave a detailed account of the Meta elliptical distributions for given marginal and Azzalini et al (2002) studied the perturbation of symmetry distributions with special emphasis to the Multivariate skewed t distribution. Based on the past and present literatures, statisticians failed to highlight the symmetric family of Multivariate distribution of the Sarmanov type. So the authors proposed a new symmetric family of multivariate generalization of univariate Continuous distribution with special reference to the student's t- distribution and it is discussed in the next section.

Sam-Solai's Multivariate t- distribution

Definition 2.1: Let $t_1, t_2, t_3 .., t_p$ are the random variables followed Continuous univariate student's t-distribution with mean 0, variance $\frac{v_i}{v_i - 2}$ and v_i degrees of freedom for all i (i=1 to *p*), then the Multivariate Sam-Solai's t-distribution and its density is defined as

$$f(t_1, t_2, t_3 \ldots, t_p) = (1 + \sum_{i=1}^{p} \sum_{j=1}^{p} \rho_{ij} t_i t_j \sqrt{\frac{(v_i - 2)(v_j - 2)}{v_i v_j}}) \prod_{i=1}^{p} \left(\frac{(1 + \frac{t_i^2}{v_i})^{-(\frac{v_i + 1}{2})}}{\sqrt{v_i} B(\frac{1}{2}, \frac{v_i}{2})} \right) \quad (1)$$

where $i \neq j$, $-\infty \leq t_i \leq +\infty$, $v_i > 0$ $-1 \leq \rho_{ij} \leq +1$

Result 2.1: Sam-Solai's Cumulative Multivariate t- distribution

The cumulative distribution function of the Sam's Multivariate t- distribution can also be represented in terms of its Multivariate incomplete Beta Integral form and it is given as

$$F(t_1,t_2,t_3\ldots,t_p)=\int_{-\infty}^{t_1}\int_{-\infty}^{t_2}\int_{-\infty}^{t_3}\ldots\int_{-\infty}^{t_p}(1+\sum_{i=1}^{p}\sum_{j=1}^{p}\rho_{ij}u_iu_j\sqrt{\frac{(v_i-2)(v_j-2)}{v_iv_j}})\prod_{i=1}^{p}(\frac{(1+\frac{u_i^2}{v_i})^{-(\frac{v_i+1}{2})}}{\sqrt{v_i}B(\frac{1}{2},\frac{v_i}{2})})du_i \quad (3)$$

Where $i \neq j$,$-\infty \leq u_i \leq +\infty$, $v_i > 0$
$-1 \leq \rho_{ij} \leq +1$

Theorem 2.2: Sam-Solai's Multivariate Conditional t- distribution
The Probability density function of Sam-Solai's Multivariate Conditional t-distribution of t_1 on $t_2, t_3, \ldots t_P$ is

$$f(t_1/t_2,t_3\ldots,t_p)=\frac{(1+\frac{t_1^2}{v_1})^{-(\frac{v_1+1}{2})}(1+\sum_{i=1}^{p}\sum_{j=1}^{p}\rho_{ij}t_it_j\sqrt{\frac{(v_i-2)(v_j-2)}{v_iv_j}})}{\sqrt{v_1}B(\frac{1}{2},\frac{v_1}{2})(1+\sum_{i=2}^{p}\sum_{j=2}^{p}\rho_{ij}t_it_j\sqrt{\frac{(v_i-2)(v_j-2)}{v_iv_j}})} \quad (4)$$

Where $i \neq j$ $-\infty \leq t_i \leq +\infty$,$v_i > 0$ $-1 \leq \rho_{ij} \leq +1$

Proof: Let the Multivariate Conditional distribution of t_1 on $t_2, t_3, \ldots t_P$ is given as

$$f(t_1/t_2,t_3\ldots,t_p)=\frac{f(t_1,t_2,t_3\ldots,t_p)}{f(t_2,t_3\ldots,t_p)}$$

$$f(t_1/t_2,t_3\ldots,t_p)=\frac{(1+\sum_{i=1}^{p}\sum_{j=1}^{p}\rho_{ij}t_it_j\sqrt{\frac{(v_i-2)(v_j-2)}{v_iv_j}})\prod_{i=1}^{p}(\frac{(1+\frac{t_i^2}{v_i})^{-(\frac{v_i+1}{2})}}{\sqrt{v_i}B(\frac{1}{2},\frac{v_i}{2})})}{\int_{-\infty}^{+\infty}(1+\sum_{i=1}^{p}\sum_{j=1}^{p}\rho_{ij}t_it_j\sqrt{\frac{(v_i-2)(v_j-2)}{v_iv_j}})\prod_{i=1}^{p}(\frac{(1+\frac{t_i^2}{v_i})^{-(\frac{v_i+1}{2})}}{\sqrt{v_i}B(\frac{1}{2},\frac{v_i}{2})})dt_1}$$

$$f(t_1/t_2,t_3\dots,t_p)=\frac{(1+\frac{t_1^2}{v_1})^{-(\frac{v_1+1}{2})}(1+\sum_{i=1}^{p}\sum_{j=1}^{p}\rho_{ij}t_it_j\sqrt{\frac{(v_i-2)(v_j-2)}{v_iv_j}})}{\sqrt{v_1}B(\frac{1}{2},\frac{v_1}{2})(1+\sum_{i=2}^{p}\sum_{j=2}^{p}\rho_{ij}t_it_j\sqrt{\frac{(v_i-2)(v_j-2)}{v_iv_j}})}$$

where $i\neq j$ $,-\infty\leq t_i\leq+\infty$, $v_i>0$
$-1\leq\rho_{ij}\leq+1$

Theorem 2.3: Mean and Variance of Sam-Solai's Multivariate Conditional t-distribution are

$$E(t_1/t_2,t_3\dots,t_p)=\sum_{j=2}^{p}\rho_{1j}t_j\sqrt{\frac{v_1(v_j-2)}{v_j(v_1-2)}} \quad (5)$$

$$V(t_1/t_2,t_3\dots,t_p)=\frac{v_1}{v_1-2}-(\sum_{j=2}^{p}\rho_{1j}t_j\sqrt{\frac{v_1(v_j-2)}{v_j(v_1-2)}})^2 \quad (6)$$

Proof: The Mean and Variance are also conditional and the Conditional expectation and Conditional variance are given as

$$E(t_1/t_2,t_3\dots,t_p)=\int_{-\infty}^{+\infty}t_1f(t_1/t_2,t_3\dots,t_p)dt_1$$

$$E(t_1/t_2,t_3\dots,t_p)=\int_{-\infty}^{+\infty}t_1\frac{(1+\frac{t_1^2}{v_1})^{-(\frac{v_1+1}{2})}(1+\sum_{i=1}^{p}\sum_{j=1}^{p}\rho_{ij}t_it_j\sqrt{\frac{(v_i-2)(v_j-2)}{v_iv_j}})}{\sqrt{v_1}B(\frac{1}{2},\frac{v_1}{2})(1+\sum_{i=2}^{p}\sum_{j=2}^{p}\rho_{ij}t_it_j\sqrt{\frac{(v_i-2)(v_j-2)}{v_iv_j}})}dt_1$$

$$E(t_1/t_2,t_3\dots,t_p)=\sum_{j=2}^{p}\rho_{1j}t_j\sqrt{\frac{v_1(v_j-2)}{v_j(v_1-2)}}$$

$$V(t_1/t_2,t_3\dots,t_p)=\int_{-\infty}^{+\infty}(t_1-E(t_1/t_2,t_3\dots,t_p))^2f(t_1/t_2,t_3\dots,t_p)dt_1$$

$$V(t_1/t_2,t_3\ldots,t_p)=\int_{-\infty}^{+\infty}(t_1-E(t_1/t_2,t_3\ldots,t_p))^2\frac{(1+\frac{t_1^2}{v_1})^{-(\frac{v_1+1}{2})}(1+\sum_{i=1}^{p}\sum_{j=1}^{p}\rho_{ij}t_it_j\sqrt{\frac{(v_i-2)(v_j-2)}{v_iv_j}})}{\sqrt{v_1}B(\frac{1}{2},\frac{v_1}{2})(1+\sum_{i=2}^{p}\sum_{j=2}^{p}\rho_{ij}t_it_j\sqrt{\frac{(v_i-2)(v_j-2)}{v_iv_j}})}dt_1$$

$$V(t_1/t_2,t_3\ldots,t_p)=\frac{v_1}{v_1-2}-(\sum_{j=2}^{p}\rho_{1j}t_j\sqrt{\frac{v_1(v_j-2)}{v_j(v_1-2)}})^2$$

Theorem 2.4: If there are ***p=q+k*** random variables,such that ***q*** random variables $t_1, t_2, t_3, \ldots t_q$ conditionally depends on the ***k*** variables $t_{q+1}, t_{q+2}, t_{q+3}, \ldots t_{q+k}$,then the density function of Sam-Solai's multivariate conditional t-distribution is

$$f(t_1,t_2,t_3,\ldots t_q/t_{q+1},t_{q+2},t_{q+3},\ldots t_{q+k})=\frac{(1+\sum_{i=1}^{q}\sum_{j=1}^{q}\rho_{ij}t_it_j\sqrt{\frac{(v_i-2)(v_j-2)}{v_iv_j}})\prod_{i=1}^{q}(\frac{(1+\frac{t_i^2}{v_i})^{-(\frac{v_i+1}{2})}}{\sqrt{v_i}B(\frac{1}{2},\frac{v_i}{2})})}{(1+\sum_{i=q+1}^{q+k}\sum_{j=q+1}^{q+k}\rho_{ij}t_it_j\sqrt{\frac{(v_i-2)(v_j-2)}{v_iv_j}})} \tag{7}$$

Proof: Let the multivariate conditional law for ***q*** random variables $t_1, t_2, t_3, \ldots t_q$ conditionally depends on the ***k*** variables $t_{q+1}, t_{q+2}, t_{q+3}, \ldots t_{q+k}$ is given as

$$f(t_1,t_2,t_3,\ldots t_q/t_{q+1},t_{q+2},t_{q+3},\ldots t_{q+k})=\frac{f(t_1,t_2,t_3,\ldots t_q,t_{q+1},t_{q+2},t_{q+3},\ldots t_{q+k})}{f(t_{q+1},t_{q+2},t_{q+3},\ldots t_{q+k})}$$

$$f(t_1,t_2,t_3,\ldots t_q/t_{q+1},t_{q+2},t_{q+3},\ldots t_{q+k})=\frac{(1+\sum_{i=1}^{q+k}\sum_{j=1}^{q+k}\rho_{ij}t_it_j\sqrt{\frac{(v_i-2)(v_j-2)}{v_iv_j}})\prod_{i=1}^{q+k}(\frac{(1+\frac{t_i^2}{v_i})^{-(\frac{v_i+1}{2})}}{\sqrt{v_i}B(\frac{1}{2},\frac{v_i}{2})})}{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\cdots\int_{-\infty}^{+\infty}(1+\sum_{i=1}^{q+k}\sum_{j=1}^{q+k}\rho_{ij}t_it_j\sqrt{\frac{(v_i-2)(v_j-2)}{v_iv_j}})\prod_{i=1}^{q+k}(\frac{(1+\frac{t_i^2}{v_i})^{-(\frac{v_i+1}{2})}}{\sqrt{v_i}B(\frac{1}{2},\frac{v_i}{2})}))\prod_{i=1}^{q}dx_i}$$

$$f(t_1,t_2,t_3,..t_q/t_{q+1},t_{q+2},t_{q+3},..t_{q+k})=\frac{(1+\sum_{i=1}^{q}\sum_{j=1}^{q}\rho_{ij}t_it_j\sqrt{\frac{(v_i-2)(v_j-2)}{v_iv_j}})\prod_{i=1}^{q}(\frac{(1+\frac{t_i^2}{v_i})^{-(\frac{v_i+1}{2})}}{\sqrt{v_i}B(\frac{1}{2},\frac{v_i}{2})})}{(1+\sum_{i=q+1}^{q+k}\sum_{j=q+1}^{q+k}\rho_{ij}t_it_j\sqrt{\frac{(v_i-2)(v_j-2)}{v_iv_j}})}$$

where $i\neq j$,$-\infty\leq t_i\leq+\infty$, $v_i>0$ $-1\leq\rho_{ij}\leq+1$, ***p=q+k***

Theorem 2.5.1 (Constants of Sam-Solai's multivariate t- distribution): The Marginal Co-variance and Population Correlation Co-efficient between the random variables t_1 and t_2 is given as

$$COV(t_1,t_2)=\rho_{12}\sqrt{\frac{v_1v_2}{(v_1-2)(v_2-2)}} \tag{8}$$

Proof: Let the product moment of the Sam-Solai's multivariate t- distribution is given as in terms of Co-variance

$$COV(t_1,t_2)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\dots\int_{-\infty}^{+\infty}t_1t_2\,f(t_1,t_2,t_3,\dots t_q)\prod_{i=1}^{p}dt_i$$

$$COV(t_1,t_2)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\dots\int_{-\infty}^{+\infty}t_1t_2(1+\sum_{i=1}^{p}\sum_{j=1}^{p}\rho_{ij}t_it_j\sqrt{\frac{(v_i-2)(v_j-2)}{v_iv_j}})\prod_{i=1}^{p}(\frac{(1+\frac{t_i^2}{v_i})^{-(\frac{v_i+1}{2})}}{\sqrt{v_i}B(\frac{1}{2},\frac{v_i}{2})})\prod_{i=1}^{p}dt_i$$

$$COV(t_1,t_2)=\rho_{12}\sqrt{\frac{v_1v_2}{(v_1-2)(v_2-2)}}$$

Note: (1). The result can be generalized to the Co-variance between the i^{th} and j^{th} random variable are given as

$$COV(t_i,t_j)=\rho_{ij}\sqrt{\frac{v_iv_j}{(v_i-2)(v_j-2)}} \tag{9}$$

$$\rho_{ij} = \frac{COV(t_i, t_j)}{\sqrt{\frac{v_i v_j}{(v_i - 2)(v_j - 2)}}}$$

Where $i \neq j$, $-1 \leq \rho_{ij} \leq +1$

(2). The Moment generating function, Cumulant of Moment Generating function and Characteristic function of Sam-Solai's Multivariate t- distribution are also undefined, due to the non-convergence of the incomplete beta integral.

Some special cases

Result 3.1 If ρ_{ij} = 0, then there is no correlation between the variables and the Sam-Solai's multivariate t-distribution is reduced to product of students uni-variate t-distribution.

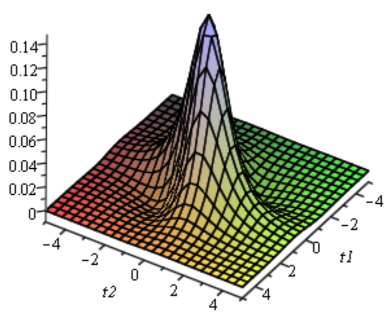
Result 3.2 If ***P*=2**, then the density of Sam-Solai's Multivariate t- distribution was reduced into

$$f(t_1,t_2) = \frac{(1+\frac{t_1^2}{v_1})^{-(\frac{v_1+1}{2})}(1+\frac{t_2^2}{v_2})^{-(\frac{v_2+1}{2})}}{\sqrt{v_1}B(\frac{1}{2},\frac{v_1}{2})\sqrt{v_2}B(\frac{1}{2},\frac{v_2}{2})}(1+\rho_{12}t_1t_2\sqrt{\frac{(v_1-2)(v_2-2)}{v_1v_2}}) \quad (13)$$

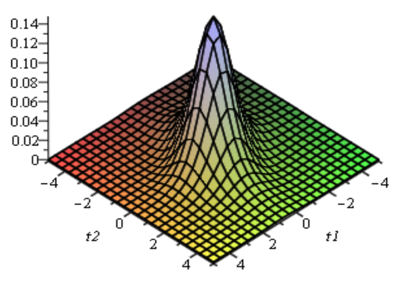
where $-\infty \leq t_1, t_2 \leq +\infty, v_i > 0, \quad -1 \leq \rho_{12} \leq +1$

This is called Sam-Solai's Bi-variate t- distribution.

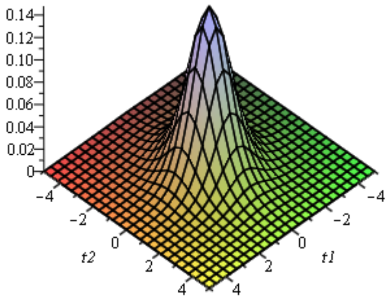
Result 3.4: Below the diagram shows the Bi-variate probability surface of the Sam's Bi-variate t- distribution for (***v₁*=5, *v₂*=10**) degrees of freedom and various values of population correlation coefficient (ρ =-1,-0.5, 0, +0.5, +1) are given.



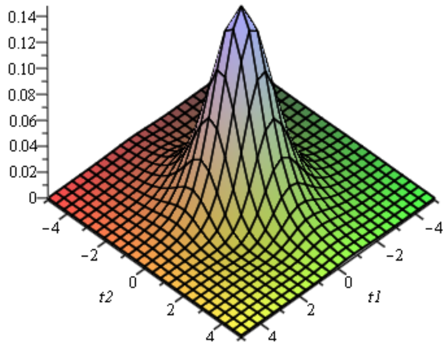
ρ = 1, v_1 = 5, v_2 = 10



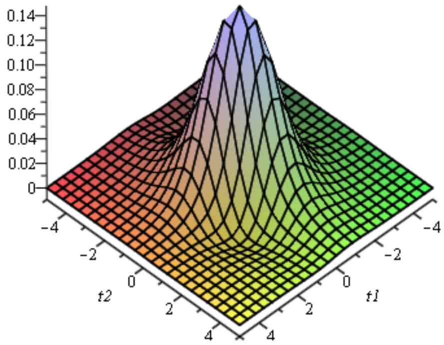
ρ = 0.5, v_1 = 5, v_2 = 10



$\rho = 0, \nu_1 = 5, \nu_2 = 10$



$\rho = -0.5, \nu_1 = 5, \nu_2 = 10$



$\rho = -1, \nu_1 = 5, \nu_2 = 10$

Result 3.5: The Sam-Solai's Bi-variate Cumulative t- distribution is given as

$$F(t_1,t_2)=\int_{-\infty}^{t_1}\int_{-\infty}^{t_2}\frac{(1+\frac{u_1^2}{v_1})^{-(\frac{v_1+1}{2})}(1+\frac{u_2^2}{v_2})^{-(\frac{v_2+1}{2})}}{\sqrt{v_1}B(\frac{1}{2},\frac{v_1}{2})\sqrt{v_2}B(\frac{1}{2},\frac{v_2}{2})}(1+\rho_{12}u_1u_2\sqrt{\frac{(v_1-2)(v_2-2)}{v_1v_2}})du_1du_2 \qquad (15)$$

Result 3.6: Marginal distributions of t_1 and t_2 for Sam-Solai's Bi-variate t-distribution are

$$f(t_1)=\frac{(1+\frac{t_1^2}{v_1})^{-(\frac{v_1+1}{2})}}{\sqrt{v_1}\,B(\frac{1}{2},\frac{v_1}{2})} \qquad (16)$$

where $-\infty \le t_1 \le +\infty \;,\; v_1 > 0$

$$f(t_2)=\frac{(1+\frac{t_2^2}{v_2})^{-(\frac{v_2+1}{2})}}{\sqrt{v_2}\,B(\frac{1}{2},\frac{v_2}{2})}$$

Where $-\infty\leq t_2\leq+\infty,\quad v_2>0$

Proof: The Marginal distribution of t_1 for the Sam-Solai's Bi-variate t- distribution is given as

$$f(t_1)=\int_{-\infty}^{+\infty}f(t_1,t_2)dt_2$$

$$f(t_1)=\int_{-\infty}^{+\infty}\frac{(1+\frac{t_1^2}{v_1})^{-(\frac{v_1+1}{2})}(1+\frac{t_2^2}{v_2})^{-(\frac{v_2+1}{2})}}{\sqrt{v_1}\,B(\frac{1}{2},\frac{v_1}{2})\sqrt{v_2}\,B(\frac{1}{2},\frac{v_2}{2})}(1+\rho_{12}t_1t_2\sqrt{\frac{(v_1-2)(v_2-2)}{v_1v_2}})dt_2$$

$$f(t_1)=\frac{(1+\frac{t_1^2}{v_1})^{-(\frac{v_1+1}{2})}}{\sqrt{v_1}\,B(\frac{1}{2},\frac{v_1}{2})}$$

Where $-\infty\leq t_1\leq+\infty\;,\quad v_1>0$

Similarly, the Marginal distribution of t_2 for the Sam's Bi-variate t- distribution is given as

$$f(t_2)=\int_{-\infty}^{+\infty}f(t_1,t_2)dt_1$$

$$f(t_2)=\int_{-\infty}^{+\infty}\frac{(1+\frac{t_1^2}{v_1})^{-(\frac{v_1+1}{2})}(1+\frac{t_2^2}{v_2})^{-(\frac{v_2+1}{2})}}{\sqrt{v_1}\,B(\frac{1}{2},\frac{v_1}{2})\sqrt{v_2}\,B(\frac{1}{2},\frac{v_2}{2})}(1+\rho_{12}t_1t_2\sqrt{\frac{(v_1-2)(v_2-2)}{v_1v_2}})dt_1$$

$$f(t_2) = \frac{(1+\frac{t_2^2}{v_2})^{-(\frac{v_2+1}{2})}}{\sqrt{v_2}\,B(\frac{1}{2},\frac{v_2}{2})} \qquad (17)$$

where $-\infty \le t_2 \le +\infty, \quad v_2 > 0$

Theorem 3.6: Conditional distribution of t_1 on t_2 of Sam-Solai's Bi-variate normal distribution is

$$f(t_1/t_2) = \frac{(1+\frac{t_1^2}{v_1})^{-(\frac{v_1+1}{2})}}{\sqrt{v_1}\,B(\frac{1}{2},\frac{v_1}{2})}(1+\rho_{12}t_1t_2\sqrt{\frac{(v_1-2)(v_2-2)}{v_1v_2}}) \qquad (18)$$

Proof: Let the conditional distribution of t_1 **on** t_2 based on the conditional probability is given as

$$f(t_1/t_2) = \frac{f(t_1,t_2)}{f(t_2)}$$

$$f(t_1/t_2) = \frac{\frac{(1+\frac{t_1^2}{v_1})^{-(\frac{v_1+1}{2})}(1+\frac{t_2^2}{v_2})^{-(\frac{v_2+1}{2})}}{\sqrt{v_1}\,B(\frac{1}{2},\frac{v_1}{2})\sqrt{v_2}\,B(\frac{1}{2},\frac{v_2}{2})}(1+\rho_{12}t_1t_2\sqrt{\frac{(v_1-2)(v_2-2)}{v_1v_2}})}{\int_{-\infty}^{+\infty}\frac{(1+\frac{t_1^2}{v_1})^{-(\frac{v_1+1}{2})}(1+\frac{t_2^2}{v_2})^{-(\frac{v_2+1}{2})}}{\sqrt{v_1}\,B(\frac{1}{2},\frac{v_1}{2})\sqrt{v_2}\,B(\frac{1}{2},\frac{v_2}{2})}(1+\rho_{12}t_1t_2\sqrt{\frac{(v_1-2)(v_2-2)}{v_1v_2}})dt_1}$$

$$f(t_1/t_2) = \frac{(1+\frac{t_1^2}{v_1})^{-(\frac{v_1+1}{2})}}{\sqrt{v_1}\,B(\frac{1}{2},\frac{v_1}{2})}(1+\rho_{12}t_1t_2\sqrt{\frac{(v_1-2)(v_2-2)}{v_1v_2}})$$

For the conditional density of Bi-variate Sam-Solai's t- distribution, we can easily derive the Conditional expectation and Variance. The conditional Variance of t_1 **on** t_2 is heteroscedastic and it is given as

$$E(t_1/t_2)=\int_{-\infty}^{+\infty} t_1 f(t_1/t_2)dt_1$$

$$E(t_1/t_2)=\int_{-\infty}^{+\infty} t_1 \frac{(1+\frac{t_1^2}{v_1})^{-(\frac{v_1+1}{2})}(1+\frac{t_2^2}{v_2})^{-(\frac{v_2+1}{2})}}{\sqrt{v_1}B(\frac{1}{2},\frac{v_1}{2})\sqrt{v_2}B(\frac{1}{2},\frac{v_2}{2})}(1+\rho_{12}t_1t_2\sqrt{\frac{(v_1-2)(v_2-2)}{v_1v_2}})dt_1$$

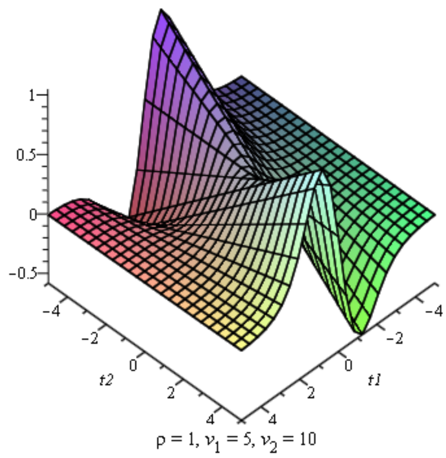
$$E(t_1/t_2)=\rho_{12}t_2\sqrt{\frac{v_1(v_2-2)}{v_2(v_1-2)}} \tag{19}$$

$$V(t_1/t_2)=\int_{-\infty}^{+\infty}(t_1-E(t_1/t_2))^2 f(t_1/t_2)dt_1$$

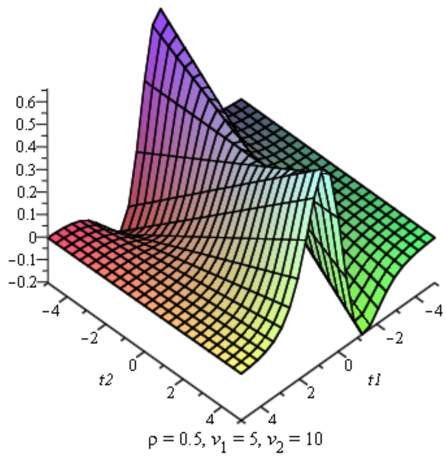
$$V(t_1/t_2)=\int_{-\infty}^{+\infty}(t_1-E(t_1/t_2))^2 \frac{(1+\frac{t_1^2}{v_1})^{-(\frac{v_1+1}{2})}(1+\frac{t_2^2}{v_2})^{-(\frac{v_2+1}{2})}}{\sqrt{v_1}B(\frac{1}{2},\frac{v_1}{2})\sqrt{v_2}B(\frac{1}{2},\frac{v_2}{2})}(1+\rho_{12}t_1t_2\sqrt{\frac{(v_1-2)(v_2-2)}{v_1v_2}})dt_1$$

$$V(t_1/t_2)=\frac{v_1}{v_1-2}-(\rho_{12}t_2\sqrt{\frac{v_1(v_2-2)}{v_2(v_1-2)}})^2 \tag{20}$$

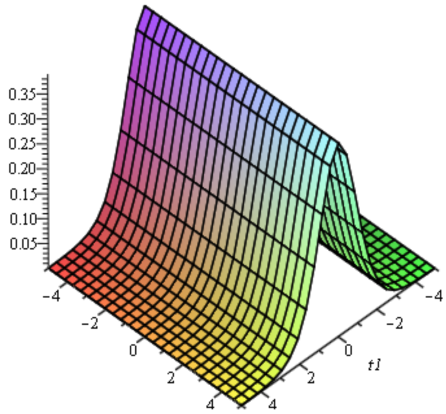
Result 3.7: Below the diagram shows the Bi-variate Conditional probability surface of the Sam's Bi-variate Conditional t- distribution of t_1 on t_2 for (**v_1=5, v_2=10**) degrees of freedom and various values of population correlation coefficient (ρ =-1,-0.5, 0, +0.5, +1) are given.



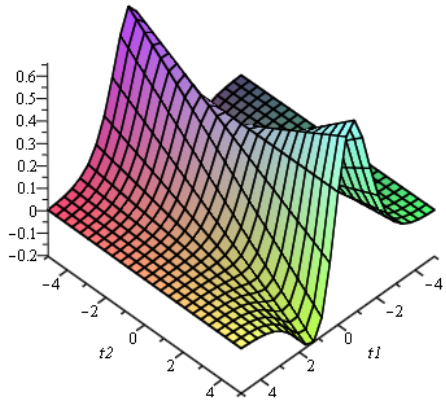
$\rho = 1, v_1 = 5, v_2 = 10$



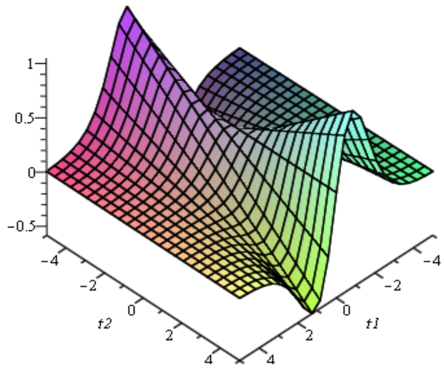
$\rho = 0.5, v_1 = 5, v_2 = 10$



$\rho = 0, \nu_1 = 5, \nu_2 = 10$



$\rho = -0.5, \nu_1 = 5, \nu_2 = 10$



$\rho = -1, \nu_1 = 5, \nu_2 = 10$

Theorem 3.7: The Conditional distribution of t_2 on t_1 of Sam-Solai's Bi-variate t-distribution is

$$f(t_2/t_1) = \frac{\left(1+\frac{t_2^2}{v_2}\right)^{-\left(\frac{v_2+1}{2}\right)}}{\sqrt{v_2}\,B\left(\frac{1}{2},\frac{v_2}{2}\right)}\left(1+\rho_{12}t_1t_2\sqrt{\frac{(v_1-2)(v_2-2)}{v_1v_2}}\right) \tag{21}$$

Proof: Let the conditional distribution of t_2 **on** t_1 based on the conditional probability is given as

$$f(t_2/t_1) = \frac{f(t_1,t_2)}{f(t_1)}$$

$$f(t_2/t_1)=\frac{\dfrac{(1+\frac{t_1^2}{v_1})^{-(\frac{v_1+1}{2})}(1+\frac{t_2^2}{v_2})^{-(\frac{v_2+1}{2})}}{\sqrt{v_1}B(\frac{1}{2},\frac{v_1}{2})\sqrt{v_2}B(\frac{1}{2},\frac{v_2}{2})}(1+\rho_{12}t_1t_2\sqrt{\frac{(v_1-2)(v_2-2)}{v_1v_2}})}{\displaystyle\int_{-\infty}^{+\infty}\frac{(1+\frac{t_1^2}{v_1})^{-(\frac{v_1+1}{2})}(1+\frac{t_2^2}{v_2})^{-(\frac{v_2+1}{2})}}{\sqrt{v_1}B(\frac{1}{2},\frac{v_1}{2})\sqrt{v_2}B(\frac{1}{2},\frac{v_2}{2})}(1+\rho_{12}t_1t_2\sqrt{\frac{(v_1-2)(v_2-2)}{v_1v_2}})dt_2}$$

$$f(t_2/t_1)=\frac{(1+\frac{t_2^2}{v_2})^{-(\frac{v_2+1}{2})}}{\sqrt{v_2}B(\frac{1}{2},\frac{v_2}{2})}(1+\rho_{12}t_1t_2\sqrt{\frac{(v_1-2)(v_2-2)}{v_1v_2}})$$

From the Conditional density, we can easily derive the Conditional expectation and Variance. The conditional Variance of t_2 **on** t_1 is heteroscedastic and it is given as

$$E(t_2/t_1)=\int_{-\infty}^{+\infty}t_2 f(t_2/t_1)dt_2$$

$$E(t_2/t_1)=\int_{-\infty}^{+\infty}t_2\frac{(1+\frac{t_1^2}{v_1})^{-(\frac{v_1+1}{2})}(1+\frac{t_2^2}{v_2})^{-(\frac{v_2+1}{2})}}{\sqrt{v_1}B(\frac{1}{2},\frac{v_1}{2})\sqrt{v_2}B(\frac{1}{2},\frac{v_2}{2})}(1+\rho_{12}t_1t_2\sqrt{\frac{(v_1-2)(v_2-2)}{v_1v_2}})dt_2$$

$$E(t_2/t_1)=\rho_{12}t_1\sqrt{\frac{v_2(v_1-2)}{v_1(v_2-2)}}\qquad(22)$$

$$V(t_2/t_1)=\int_{-\infty}^{+\infty}(t_2-E(t_2/t_1))^2 f(t_2/t_1)dt_2$$

$$V(t_2/t_1)=\int_{-\infty}^{+\infty}(t_2-E(t_2/t_1))^2\frac{(1+\frac{t_1^2}{v_1})^{-(\frac{v_1+1}{2})}(1+\frac{t_2^2}{v_2})^{-(\frac{v_2+1}{2})}}{\sqrt{v_1}B(\frac{1}{2},\frac{v_1}{2})\sqrt{v_2}B(\frac{1}{2},\frac{v_2}{2})}(1+\rho_{12}t_1t_2\sqrt{\frac{(v_1-2)(v_2-2)}{v_1v_2}})dt_2$$

$$V(t_2/t_1) = \frac{v_2}{v_2-2} - (\rho_{12} t_1 \sqrt{\frac{v_2(v_1-2)}{v_1(v_2-2)}})^2 \tag{23}$$

Theorem 3.8: Some Standard Constants of Sam-Solai's Bi-variate t- distribution are obtained.

(1) The Co-variance and Population Correlation Co-efficient between the random variables t_1 and t_2 is given as

$$COV(t_1, t_2) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} t_1 t_2 \, f(t_1, t_2) \, dt_1 dt_2$$

$$COV(t_1,t_2) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} t_1 t_2 \frac{(1+\frac{t_1^2}{v_1})^{-(\frac{v_1+1}{2})}(1+\frac{t_2^2}{v_2})^{-(\frac{v_2+1}{2})}}{\sqrt{v_1}B(\frac{1}{2},\frac{v_1}{2})\sqrt{v_2}B(\frac{1}{2},\frac{v_2}{2})}(1+\rho_{12}t_1t_2\sqrt{\frac{(v_1-2)(v_2-2)}{v_1v_2}})dt_1dt_2$$

$$COV(t_1,t_2) = \rho_{12}\sqrt{\frac{v_1v_2}{(v_1-2)(v_2-2)}} \tag{24}$$

$$\rho_{12} = \frac{COV(t_1,t_2)}{\sqrt{\frac{v_1v_2}{(v_1-2)(v_2-2)}}}$$

(2) The Moment generating function, Cumulant of Moment Generating function and Characteristic function of Sam-Solai's Bi-variate t- distribution are also undefined, due to the non-convergence of the incomplete beta integral.

Conclusion

The multivariate generalization of student's t-distribution is a natural density and the Sam-Solai's generalization of multivariate t- distribution is similar in certain aspect with matrix variant of multivariate student's t-distribution. At first, the marginal uni-variate distributions of the Sam-Solai's Multivariate t-distribution are uni-variate student's t-distribution. Secondly, the Population Co-variance and Correlation co-efficient of the proposed distribution are similar with Karl Pearson's Co-variance and correlation Co-efficient. Finally, the Conditional variance of Sam-Solai's Multivariate Conditional t- distribution is heteroscedastic in nature and this feature is a unique for the proposed distribution. Thus the new generalization of sarmonav type family of symmetric multivariate distribution open the way for logical extension of the generalization of symmetric family of all uni-variate continuous probability distributions.

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