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Study of Exact Solution of Deflection of Thermoelastic Circular Plate by using Marchi-Fasulo Integral Transform

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Abstract

This work consists of determination of unknown temperature and thermal deflection of thin circular plate with the stated conditions. The inverse heat conduction equation is solved by using Marchi-Fasulo transform and the results for unknown temperature and thermal deflection are obtained in terms of infinite series of Bessel's function and it is solved for special case by using Math-cad software and illustrated graphically by using origin software.

Keywords: Inverse Heat conduction, Thermal deflection problem, Marchi-Fasulotransform, Circular plate.

2000 Mathematics Subject Classification: Primary 35A25, Secondary74M99, 74K20.

Introduction

The inverse heat conduction problem is one of the most frequently encountered problems by scientists. The wide varieties of problems that are covered under conduction also make it one of the most researched and thought about problems in the field of engineering and technology. This kind of problems can be solved by various methods. These inverse problems consist of determination of unknown temperature and thermal deflection of solids when the conditions of temperature and deflection are known at the some points of the solid under consideration. Grysa and Cialkowski [1], Grysa and Koalowski [2] studied one-dimensional transient thermo elastic problems and derived the heating temperature and heat flux on the surface of an isotropic infinite slab. Khobragade [3] and [4] discuss an inverse steady state and transient thermo elastic problem of thin circular plate and annular disc in Marchi-Fasulo transform domain. Deshmukh et.al [5] investigated inverse heat conduction problem of semi-infinite, clamped thin circular plate and their thermal deflection by quasi-static approach. Tikhe and Deshmukh[6] introduced inverse problem of a thin circular plate and its thermal deflection.

In this work we modify the problem of Tikhe and Deshmukh [6] which consist of given temperature distribution on the interior surface of thin circular plate. In this work, the temperature, unknown temperature on outer surface and quasi-static thermal deflection due to unknown temperature g(z) are discuss. The inverse heat conduction equation is solved by using Marchi-Fasulointegral transform and the results for unknown temperature and thermal deflection are obtained in terms of infinite series of Bessel's function and it is solved for special case by using Math-cad software and illustrated graphically by using Origin software.

Formulation of the problem

Consider a thin circular plate of thickness 2h occupying the space $D:\left\{(x,y,z)\,\middle|\,0\le r\le\sqrt{x^2+y^2}\le a,-h\le z\le h\right\}$. Suppose the plate is subjected to arbitrary known interior temperature $f(z)$ within the region $0\le r\le a$ with third kind condition which assumes to be zero at upper surface $z=h$ and lower surface $z=-h$. Under these more realistic prescribed conditions, the unknown temperature on lower surface and quasi-static thermal deflection due to unknown temperature $g(z)$ are required to determine. The differential equations satisfying the deflection function as in Noda et. al [7] is given as

$$\nabla^4 w=\frac{-1}{(1-\nu)D}\nabla^2 M_T \tag{2.1}$$

where, the operator ∇^2 is defined by

$$\nabla^2=\frac{\partial}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r} \tag{2.2}$$

M_T is the thermal moment of the plate defined as

$$M_T=\alpha E\int_{-h}^{h} zT(r,z)dz \tag{2.3}$$

and D is the flexural rigidity of the plate denoted as

$$D=\frac{Eh^3}{12(1-v^2)} \tag{2.4}$$

where α , E and v are the coefficients of the linear thermal expansion, the Young's modulus and Poisson's ratio of the plate material respectively.

Since the edge of the circular plate is fixed and clamped;

$$w = \frac{\partial w}{\partial r} = 0 \text{ at } r = a \tag{2.5}$$

The temperature of the circular plate satisfying the heat conduction equation as in Ozisik [8] is as

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = 0 \text{ in } 0 \le r \le a, -h \le z \le h \tag{2.6}$$

Subject to the conditions

$$\frac{\partial T}{\partial r} = g(z) \text{ (unknown) at } r = a, -h \le z \le h \tag{2.7}$$

$$\left[T(r,z) + k_1 \frac{\partial T(r,z)}{\partial z} \right]_{z=h} = 0 \tag{2.8}$$

$$\left[T(r,z) + k_2 \frac{\partial T(r,z)}{\partial z} \right]_{z=-h} = 0 \tag{2.9}$$

$$\left[T(r,z) + \frac{\partial T(r,z)}{\partial r} \right]_{z=\xi} = f(z) \text{ (known)} \tag{2.10}$$

where k is the thermal conductivity of the circular plate. The equations (2.1)to (2.10) constitute the mathematical formulation of the inverse thermo elastic deflection problem of circular plate.

Solution of the Problem

Results Required

Finite Marchi-Fasulo Integral Transform: The finite Marchi-Fasulo integral transform of $f(x)$, in $-h \le z \le h$ as in [9] is defined to be

$$\overline{F}(n) = \int_{-h}^{h} f(z) P_n(z) dz \tag{3.1}$$

$$f(z) = \sum_{n=1}^{\infty} \left(\frac{\overline{F}(n)}{\lambda_n} P_n(z) \right) \tag{3.2}$$

where

$$P_n(z) = Q_n \cos(a_n z) - W_n \sin(a_n z)$$

$$Q_n = a_n(\alpha_1 + \alpha_2)\cos(a_n h) + (\beta_1 - \beta_2)\sin(a_n h)$$

$$W_n = (\beta_1 + \beta_2)\cos(a_n h) + a_n(\alpha_2 - \alpha_1)\sin(a_n h)$$

$$\lambda_n = \int_{-h}^{h} P_n^2(z)dz = h\left[Q_n^2 + W_n^2\right] + \frac{\sin(2a_n h)}{2a_n}\left[Q_n^2 - W_n^2\right]$$

Determination of Temperature Function

By applying finite Marchi-Fasulo transform as defined in (3.1) to the equations(2.6),(2.7),(2.10) and using (2.8),(2.9) and then using (3.2), once we get the temperature function as

$$T(r,z) = \sum_{n=1}^{\infty}\left(\frac{\overline{f}(n)}{\lambda_n} P_n(z)\right)\frac{I_0(a_n a)}{I_0(a_n\xi) + c a_n I_0^{'}(a_n\xi)} \tag{3.3}$$

Determination of Unknown Temperature Function

Using (3.3) in (2.7), once obtain the unknown temperature function $g(z)$ as

$$g(z) = \sum_{n=1}^{\infty}\left(\frac{a_n\overline{f}(n)}{\lambda_n} P_n(z)\right)\frac{I_0^{'}(a_n a)}{I_0(a_n\xi) + c a_n I_0^{'}(a_n\xi)} \tag{3.4}$$

Where

$$\overline{f}(n) = \int_{-h}^{h} f(z)P_n(z)dz, \lambda_n = \int_{-h}^{h} P_n^2(z)dz$$

$$P_n(z) = Q_n\cos(a_n z) - W_n\sin(a_n z)$$

$$Q_n = a_n(\alpha_1 + \alpha_2)\cos(a_n h) + (\beta_1 - \beta_2)\sin(a_n h)$$

$$W_n = (\beta_1 + \beta_2)\cos(a_n h) + a_n(\alpha_2 - \alpha_1)\sin(a_n h)$$

equations (3.3) and (3.4) are the desired solution of equations (2.6) and (2.7)with $\alpha_1 = k_1$ and $\alpha_2 = k_2$.

Determination of Quasi-Static Thermal Deflection Function

Using (3.3) in equation (2.3), we obtain

$$M_T = 2\alpha E\sum_{n=1}^{\infty}\frac{\overline{f}(n)W_n\left[a_n h\cos(a_n h) - \sin(a_n h)\right]I_0(a_n r)}{\lambda_n a_n^2\left[I_0(a_n\xi) + c a_n I_0^{'}(a_n\xi)\right]} \tag{3.5}$$

Assume the solution of (2.1) satisfying the (2.5) as

$$w(r) = \sum_{n=1}^{\infty} C_n\left[I_0(a_n r) - I_0(a_n a)\right] \tag{3.6}$$

Using the equations (3.5),(3.6) and the result $\left[\frac{d}{dr^2} + \frac{1}{r}\frac{d}{dr}\right]I_0(a_n r) = -a_n^2 I_0(a_n r)$ in (2.1), We obtain the expression for C_n as

$$C_n = \frac{2\alpha E}{D(1-v)} \frac{\overline{f}(n) W_n \left[a_n h \cos(a_n h) - \sin(a_n h)\right]}{\lambda_n a_n^4 \left[I_0(a_n \xi) + c a_n I_0^{'}(a_n \xi)\right]} \tag{3.7}$$

Substituting the equation (3.7) in the equation (3.6), once obtain the expression for thermal deflection function as

$$w(r) = \frac{2\alpha E}{D(1-v)} \sum_{n=1}^{\infty} \frac{\overline{f}(n) W_n \left[a_n h \cos(a_n h) - \sin(a_n h)\right]\left[I_0(a_n r) - I_0(a_n a)\right]}{\lambda_n a_n^4 \left[I_0(a_n \xi) + c a_n I_0^{'}(a_n \xi)\right]} \tag{3.8}$$

Special Case

For formulation of special case of an analytical behavior of a circular plate

Set $f(z) = (z-h)^2 (z+h)^2 \xi$ (4.1)

Applying finite Marchi-Fasulo transform as define in equation (3.1) to the equation (4.1),one obtains,

$$\overline{f}(n) = 4(k_1 + k_2)\xi \left[\frac{a_n h \cos^2(a_n h) - \cos(a_n h)\sin(a_n h)}{a_n^2}\right] \tag{4.2}$$

Substituting the value of $\overline{f}(n)$ from equation (4.2) in equations (3.3), (3.4) and(3.8), ones obtain

$$T(r,z) = 4(k_1 + k_2)\xi \sum_{n=1}^{\infty} \frac{P_n(z) I_0(a_n r)\left[a_n h \cos^2(a_n h) - \cos(a_n h)\sin(a_n h)\right]}{\lambda_n a_n^2 \left[I_0(a_n \xi) + c a_n I_0^{'}(a_n \xi)\right]} \tag{4.3}$$

$$g(z) = 4(k_1 + k_2)\xi \sum_{n=1}^{\infty} \frac{P_n(z) I_0^{'}(a_n a)\left[a_n h \cos^2(a_n h) - \cos(a_n h)\sin(a_n h)\right]}{\lambda_n a_n \left[I_0(a_n \xi) + c a_n I_0^{'}(a_n \xi)\right]} \tag{4.4}$$

$$w(r) = \frac{8\alpha E(k_1 + k_2)\xi}{D(1-v)} \sum_{n=1}^{\infty} \frac{W_n \cos(a_n h)\left[a_n h \cos(a_n h) - \sin(a_n h)\right]^2 \left[I_0(a_n r) - I_0(a_n a)\right]}{\lambda_n a_n^6 \left[I_0(a_n \xi) + c a_n I_0^{'}(a_n \xi)\right]} \tag{4.5}$$

Numerical Results

Set $A = 4(k_1 + k_2)\xi, B = \dfrac{8\alpha E(k_1 + k_2)\xi}{D(1-v)}, a = 1m, \xi = 0.5m$ and $h = 0.1m$ in equations (4.3), (4.2) and (4.4), we get

$$\frac{T(r,z)}{A} = \sum_{n=1}^{\infty} \frac{P_n(z) I_0(a_n r)\left[a_n (0.1)\cos^2(0.a_n) - \cos(0.a_n)\sin(0.a_n)\right]}{\lambda_n a_n^2 \left[I_0(0.5 a_n) + a_n I_0^{'}(0.5 a_n)\right]} \tag{5.1}$$

$$\frac{g(z)}{A} = \sum_{n=1}^{\infty} \frac{P_n(z) I_0'(a_n) \left[a_n (0.1) \cos^2(0.a_n) - \cos(0.a_n) \sin(0.a_n) \right]}{\lambda_n a_n \left[I_0(0.5a_n) + a_n I_0'(0.5a_n) \right]} \tag{5.2}$$

$$\frac{w(r)}{B} = \sum_{n=1}^{\infty} \frac{W_n \cos(0.a_n) \left[a_n (0.1) \cos(0.a_n) - \sin(0.a_n) \right]^2 \left[I_0(a_n r) - I_0(a_n) \right]}{\lambda_n a_n^6 \left[I_0(0.5a_n) + a_n I_0'(0.5a_n) \right]} \tag{5.3}$$

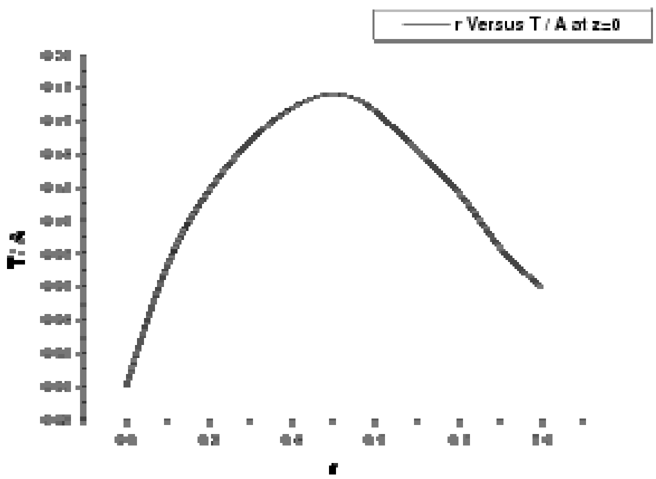

Figure 1: Temperature Distribution in Circular Plate

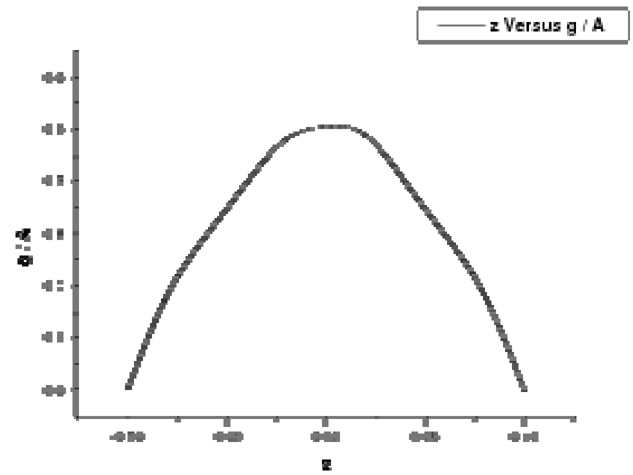

Figure 2: Unknown Temperature Distribution in Circular Plate

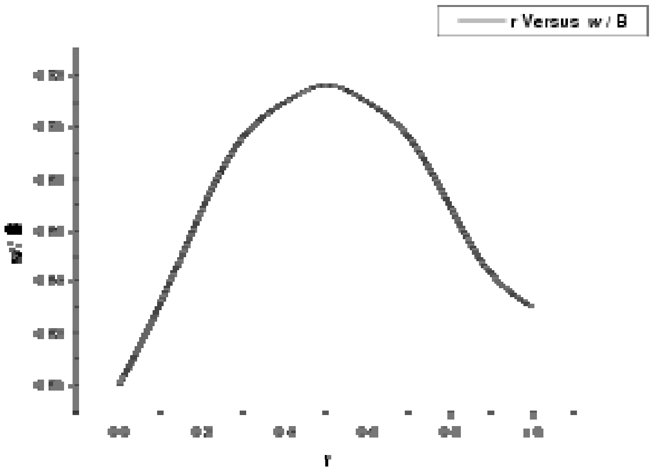

Figure 3: Quasi-static thermal deflection in Circular Plate

Figure 1: show that the temperature goes on increasing from center up to $r = 0.5$ and then slowly goes on decreasing towards outer surface of the plate.

Figure 2: show that the unknown temperature goes on increasing from lower surface upto $z = 0$ and then vanishes on upper surface of the plate.

Figure 3: show that the quasi-static thermal deflection which increases from center upto $r = 0.5$ and then analytically goes on decreases towards outer surface of the plate.

Conclusion

In this work we applied Marchi-Fasulo integral transform method to find the analytical solution of inverse heat conduction equation. The results are obtained in terms of Bessel's function in the form of Marchi-Fasulo transform series. The series solution converges provided we take sufficient number of terms in the series. Since the thickness of the plate is very small, the series solution given here will be definitely convergent. Any particular case can be derived by assigning suitable values to the parameters and function in the series expressions. The temperature and quasi-static deflection can be applied to the design of useful structures or machines in engineering applications.

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