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Estimate the Velocity Distribution in Open Channel for Different Surface Roughness and Flow Rate

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Abstract

The present paper describe the hydraulic model, testing of various important structure of open channel as well as river flow with analytical solution of the flow distribution along the open channel. We have to consider different height factor and different flow rate to examine the flow pattern in channel. The modeling approach presented in the paper is motivated from the concept of open channel. This work also help us to predict the velocity profile of open channel flow, this would be applicable for a wide range of Reynolds number by changing the discharge flow rate. We also find the critical points which arise due to the height and the discharge rate.

Keywords: open channel, flow rate, velocity.

Nomenclature
Q = flow rate
b = breadth
g = gravitational acceleration
h = height
u = velocity
θ = incline angle
f = frictional constant
A = area of a channel

Introduction
The testing of hydraulic model of various important structures such as river training work, open channel fall structure, escape structure of canal, channel, drainage tap, is

being carried out on physical model of required scale either on flunk model or on comprehensive model [1, 2]. The observation of water level, velocity, pressure source level and flow characteristics are taken for known or required discharge. To solve the single specific hydraulic problem sufficient infrastructure facilities with experienced engineers are required and minimum duration of the study may be a year. Now a day's mathematics plays a central role in solving real world problems pertaining to any discipline but also it is difficult areas especially in the area of hydraulic structure as it requires through understanding of hydraulic problem by mathematics [3].

The present techniques are useful to solve the problem of water flow phenomena [1, 2]. But it is not helpful to solve the complicated problems those are encountered on actual practice. Hence it is highly essential to carry out the hydraulic model studies for major structure to solve the problem that may encounter during the actual operation structure. Describe technique is use to solve complex problem of irrigation structure of Dams, weir and canal.

Through a hydraulic model study at different stages of construction, the performance of the structure is viewed for different design parameters such as upstream water level, the quantity of inflow, discharge passing through the partially constructed dam portion, velocity profile, pressure profile, & profile of water level. The study of river, canal, and reservoir levels are necessarily studied on hydraulic model [4, 5]. The hydraulic model studies are also carried out to observe the effect of roughness, flow pattern and the velocity observation. The development of hydraulic model, preset the different topography and alternatives with respect to different discharge.

Mathematical Formulation

In the present model for bed transport in steady condition is presented. The model, derived in the Eulerian frame work [4]. The class of model based on two equations continuity equation and momentum equation [4, 5].

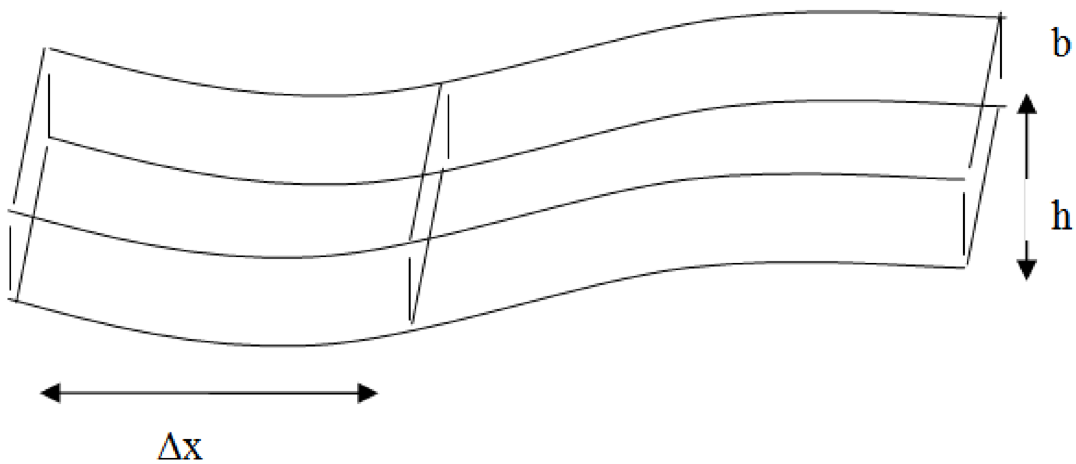

Figure 1: Two dimensional flow of on bed

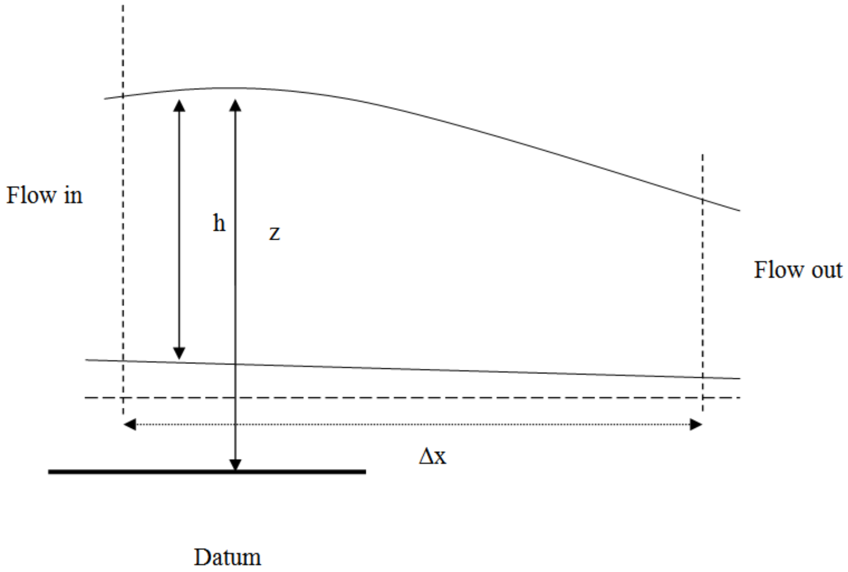

Figure 2: and short length of river bed

Assuming that there is no lateral inflow, then

$$Q_2 - Q_1 = \frac{\partial Q}{\partial x}\Delta x \quad (1)$$

Now volume of the water between section 1 and section 2 is increasing as a rate of

$$-(Q_2 - Q_1) = b\frac{\partial h}{\partial t}\Delta x \quad (2)$$

Form (1) and (2) we get

$$\frac{\partial Q}{\partial x}\Delta x + b\frac{\partial h}{\partial t}\Delta x = 0$$

But

$$\frac{\partial Q}{\partial x} = \frac{\partial (bhu)}{\partial x} = bh\frac{\partial u}{\partial x} + u\frac{\partial (bh)}{\partial x}$$

Therefore, we have

$$bh\frac{\partial u}{\partial x} + u\frac{\partial (bh)}{\partial x} + b\frac{\partial h}{\partial t} = 0 \quad (3)$$

$$A\frac{\partial u}{\partial x} + u\frac{\partial A}{\partial x} + b\frac{\partial h}{\partial t} = 0$$

Now we use Newton's second law of motion in an arbitrary coordinate system to our elemental length of channel,

Force = mass * acceleration

$$= bh\Delta x \ \frac{Du(x,t)}{Dt}$$

$$= bh\Delta x \ \left[u\frac{\partial u}{\partial x} + \frac{\partial u}{\partial t}\right] \tag{4}$$

Following, we assume the external forces, which cause this acceleration

change in static pressure: $-gbh\frac{\partial h}{\partial x}$

frictional resistance of channel walls and bed: f

acceleration due to gravitational force: g

If θ is the bed slope then the sum of these forces is

$$-gbh\frac{\partial h}{\partial x}\cos\theta\,\Delta x - gfu^2\Delta x + gbh\sin\theta\ \Delta x \tag{5}$$

The equation of these external forces to change in motion yields (4) = (5)

$$bh\Delta x\ \left[u\frac{\partial u}{\partial x} + \frac{\partial u}{\partial t}\right] = -gbh\frac{\partial h}{\partial x}\cos\theta\,\Delta x - gfu^2\Delta x + gbh\sin\theta\ \Delta x$$

$$\Longrightarrow \left[bhu\frac{\partial u}{\partial x} + bh\frac{\partial u}{\partial t}\right]\Delta x\ = \left[-gbh\frac{\partial h}{\partial x}\cos\theta - gfu^2 + gbh\sin\theta\right]\ \Delta x$$

$$\Longrightarrow bhu\frac{\partial u}{\partial x} + bh\frac{\partial u}{\partial t}\ = -gbh\frac{\partial h}{\partial x}\cos\theta - gfu^2 + gbh\sin\theta \tag{6}$$

Case 1: We consider, b and h are constant (A is constant) throughout the domain and u is the function of *x* only.

Then, equation (6) becomes

$$bhu\frac{\partial u}{\partial x} = -gfu^2 + gbh\sin\theta$$

$$\frac{\partial u}{\partial x} = \frac{-gfu^2 + gbh\sin\theta}{bhu} \tag{7}$$

Case 2: We consider, b is constant throughout the domain and both u and h are the function of *x* only

Form equation (3) we assume that the steady flow

$$\therefore \frac{\partial h}{\partial t} = 0$$

$$\therefore bh\frac{du}{dx} + u\frac{d(bh)}{dx} = 0$$

$$\therefore bu\frac{dh}{dx} = -bh\frac{du}{dx}$$

$$\therefore \frac{dh}{dx} = -\frac{h}{u}\frac{du}{dx} \tag{8}$$

Substitute (7) in (6) we get,

$$\Longrightarrow bhu\frac{du}{dx} = -gbh\left(-\frac{h}{u}\frac{du}{dx}\right)\cos\theta - gfu^2 + gbh\sin\theta$$

$$\Longrightarrow bhu^2\frac{du}{dx} - gbh^2\frac{du}{dx}\cos\theta = -\ gfu^3 + gbhu\sin\theta$$

$$\Longrightarrow \frac{du}{dx} = \frac{-gfu^3 + gbhu\sin\theta}{bhu^2 - bgh^2\cos\theta} \tag{9}$$

Result and Conclusion

Case 1: h is constant

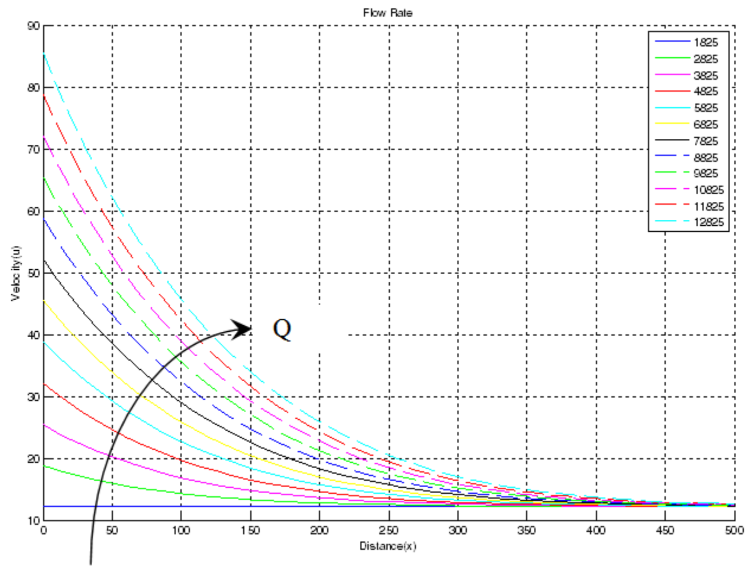

Figure 3: Flow Rate that generate velocity with respect to distance where A is constant

Case 2: h is a function of space variable

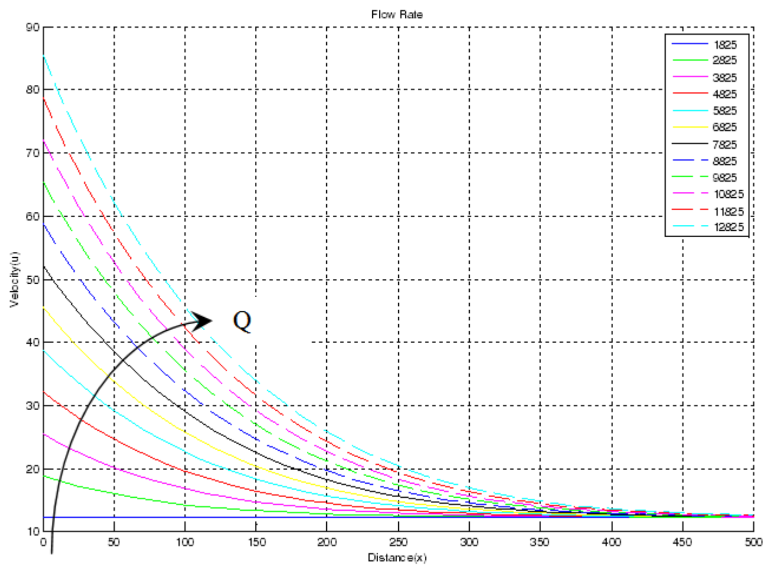

Figure 4: Flow Rate that generate velocity with respect to distance where A is function of space

An attempt has also been made to compute the values for the range of discharge (Q=12825 m/s, 11825 m/s, 10825 m/s, 9825 m/s, 8825 m/s, 7825 m/s, 6825 m/s, 5825 m/s, 4825 m/s, 3825 m/s, 2825 m/s, 1825 m/s) against various water levels in both the cases. It gives the velocity profile of fluids with unit density at steady state with unit friction incline angle 0.1 and breadth of the profile is constant 150 m. This observation is based on the flow discharge, is generate the velocity profile and how the velocity profile work with the distance. Here flow rates create the velocity profile at initial point are 85.5m/s, 79 m/s, 72.1 m/s, 65.2 m/s, 59 m/s, 52.1 m/s 45.5 m/s, 39 m/s, 32.1 m/s, 25.5 m/s, 19 m/s and 12 m/s. In the result, we observe that the velocity profile with the distance with specific initial velocity which applicable for wide rage of Reynolds number by changing the flow rate.

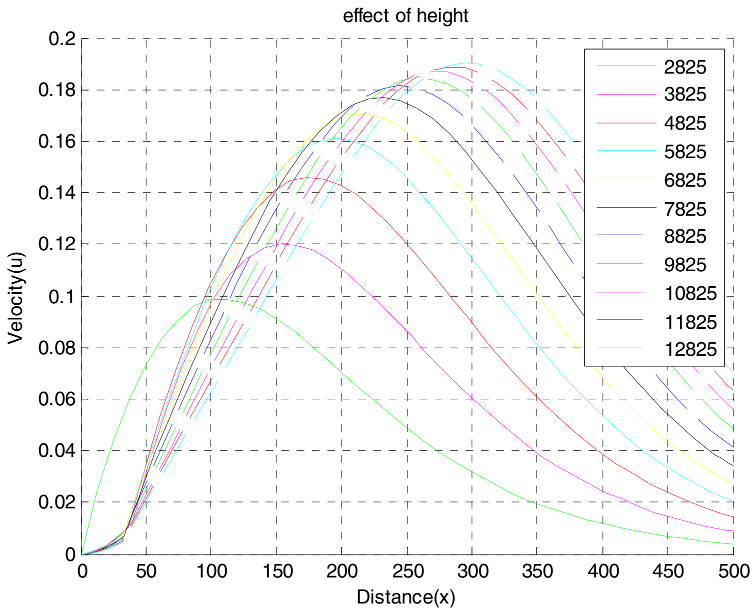

Figure 5: velocity difference in case 1 and case 2 with respect to distance

In the above figure we estimate the change in velocity due to the variation in height with respect to space which gives the critical point at which the velocity in case2, much higher then the case 1.

Future Work

This analysis could o be extended to address non-uniform channel geometry, allowing one to modify the through flow velocity profile by changing the channel height or width in order to satisfy application requirements. This analytical approach, based on momentum theory, appears to be relatively versatile and changeable for a broader range of fluid geometries.

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