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Mildly (1, 2)*-πgα-Normal Spaces

K. Mohana[1] and I. Arockiarani[2]

[1]Sr. Lecturer, Department of Quantitative Techniques,
T. John College, Bangalore-560 083. India.
E-mail: mohanamaths@yahoo.co.in
[2]Associate Professor, Department of Mathematics,
Nirmala college for women, Coimbatore-641 018, India.

Abstract

The aim of this paper is to introduce and study some new classes of spaces, called Mildly $(1, 2)^{*-}\pi g\alpha$-normal spaces, almost $(1, 2)^{*-}\alpha$-normal spaces and Mildly $(1, 2)^{*-}\alpha$-normal spaces. Some properties of these separation axioms are studied by utilizing $(1, 2)^*$-$\pi g\alpha$-open sets.

Keywords: $(1, 2)^*$-$\pi g\alpha$-open set, Mildly $(1, 2)^*$-$\pi g\alpha$-normal spaces, almost $(1, 2)^{*-}\alpha$-normal spaces, mildly $(1, 2)^{*-}\alpha$-normal spaces.

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Introduction

The concept of s-regular[6] and s-normal spaces[7] in topological spaces were introduced and studied by Maheshwari and Prasad. Arya and Nour [3] obtained some characterization of s-normal spaces. Munshi [10] introduced and studied the notions of g-regular and g-normal spaces using g-closed sets in topological spaces. Further Noiri and Popa [11] investigated the concepts introduced by Munshi [10]. Recently authors [1, 2, 4, 5] studied various functions in bitopological spaces. The aim of this paper is to introduce and study some new classes of spaces, called Mildly $(1, 2)^*$-$\pi g\alpha$-normal spaces, almost $(1, 2)^*$-α-normal spaces and Mildly $(1, 2)^*$-α-normal spaces. Some properties of these separation axioms are discussed.

Preliminaries

Throughout the present paper (X, τ_1, τ_2), (Y, σ_1, σ_2), (Z, η_1, η_2) (briefly X, Y, Z)be bitopological spaces.

Definition 2. 1: [4] A subset S of a bitopological space(X, τ_1, τ_2) is said to be $\tau_{1,2}$-open if S = A U B where $A \in \tau_1$and $B \in \tau_2$. A subset S of X is said to be [1] $\tau_{1,2}$-closed if the complement of S is $\tau_{1,2}$-open. [2] $\tau_{1,2}$-clopen if S is both $\tau_{1,2}$-open and $\tau_{1,2}$-closed.

Definition 2. 2: [4] Let S be a subset of the bitopological space (X, τ_1, τ_2). Then

[1] The $\tau_{1,2}$-interior of S, denoted by $\tau_{1,2}$-int(S) is defined by $\cup\{G{:}G \subseteq S$ and G is $\tau_{1,2}$ –open$\}$.

[2] The $\tau_{1,2}$-closure of S, denoted by $\tau_{1,2}$-cl(S) is defined by $\cap\{F{:}S \subseteq F$ and F is $\tau_{1,2}$ –closed$\}$.

Remark 2. 3: [4] $\tau_{1,2}$-open sets need not form a topology.

Definition 2. 4: A subset A of a bitoplogical space (X, τ_1, τ_2) is called

[1] $(1, 2)^*$-regular open[4] if $A = \tau_{1,2}\text{-int}(\tau_{1,2} - \text{cl}(A))$.

[2] $(1, 2)^* - \alpha-$open[4] if $A = \tau_{1,2} - \text{int}(\tau_{1,2} - \text{cl}(\tau_{1,2} - \text{int}(A)))$.

[3] $(1, 2)^* - \pi g\alpha$-closed [1] if $(1, 2)^* - \alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $\tau_{1,2} - \pi$-open in X.

[4] $(1, 2)^{*-}g\alpha$-closed if $(1, 2)^* - \alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $(1, 2)^* - \alpha$-open in X.

The complement of the mentioned in [1], [2] are called their respective closed sets and the complement of the mentioned in [3] and [4] are called their respective open sets.

Definition 2. 5: A map f: $(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is called

[1] $(1, 2)^{*-}\pi g\alpha$-irresolute[2] if the inverse image of every $(1, 2)^*\pi g\alpha$-closed in Y is $(1, 2)^*\pi g\alpha$-closed in X.

[2] completely $(1, 2)^*$-continuous[5] if the inverse image of every $\tau_{1,2}$-open in Y is $(1, 2)^*$-regular open in X.

[3] strongly-$(1, 2)^*$-$\pi g\alpha$-closed [8] if the image of every $(1, 2)^* - \pi g\alpha$-closed set in X

is $(1, 2)^* - \pi g\alpha$-closed in Y.

Definition 2. 6: A space (X, τ_1, τ_2) is said to be $(1, 2)^*$-α-normal if for every pair of disjoint $\tau_{1,2}$-closed sets A and B of X, there exist disjoint $(1, 2)^*$-α-open sets U and V such that $A \subset U$ and $B \subset V$.

Definition 2. 7: [9] A space (X, τ_1, τ_2) is said to be $(1, 2)^*$-πgα-normal if for every pair of disjoint $\tau_{1,2}$-closed sets A and B of X, there exist disjoint $(1, 2)^* - \pi g\alpha$-open sets U and V such that $A \subset U$ and $B \subset V$.

Mildly (1, 2)*-πgα-Normal Spaces

Definition 3. 1: A space (X, τ_1, τ_2) is said to be Mildly $(1, 2)^*$-πgα-normal if for every pair of disjoint $(1, 2)^*$−regular closed sets A and B of X, there exist disjoint $(1, 2)^*$-πgα-open sets U and V of X such that $A \subset U$ and $B \subset V$.

Example 3. 2 : Let X={a, b, c} and τ_1={φ, X, {b}, {b, c}}, τ_2={φ, X, {a}, {a, b}} then X is mildly $(1, 2)^*$-πgα-normal.

Theorem 3. 3: Every mildly $(1, 2)^*$-normal space is mildly $(1, 2)^*$-πgα-normal. Proof: Straight forward.

Theorem 3. 4: For a space (X, τ_1, τ_2) the following are equivalent. [1] X is mildly $(1, 2)^*$-πgα-normal.

[2] For every pair of $(1, 2)^*$−regular open sets U and V whose union is X, there exist $(1, 2)^*$-πgα-closed sets G and H such that $G \subset U$, $H \subset V$ and $G \cup H = X$.

[3] For any $(1, 2)^*$−regular closed set A and every $(1, 2)^*$−regular open set B containing A, there exist a $(1, 2)^* - \pi g\alpha$-open set U such that $A \subset U \subset (1, 2)^*\text{-}\pi g\alpha - cl(U) \subset B$.

Proof: [1] ⇒ [2]. Let U and V be a pair of $(1, 2)^*$−regular open sets such that $X = U \cup V$. Then $(X - U) \cap (X - V) = \varphi$. Since X is mildly $(1, 2)^{*-}\pi g\alpha$-normal there exist disjoint $(1, 2)^{*-}\pi g\alpha$-open sets U_1 and V_1 such that $X - U \subset U_1$ and $X - V \subset V_1$. Let $G = X - U_1$ and $H = X - V_1$. Then G and H are $(1, 2)^{*-}\pi g\alpha$-closed sets such that $G \subset U$, $H \subset V$ and $G \cup H = X$.

[2] ⇒ [3]. Let A be a $(1, 2)^*$-regular closed set and B be an $(1, 2)^*$-regular open set containing A. Then X-A and B are $(1, 2)^*$-regular open sets whose union is X. Then by[2] there exist $(1, 2)^*$-$\pi g\alpha$-closed sets W_1 and W_2 such that $W_1 \subset X - A$ and $W_2 \subset B$ and $W_1 \cup W_2 = X$. Then $A \subset X - W_1$, $X - B \subset X - W_2$ and $(X - W_1) \cap (X - W_2) = \varphi$. Let $U = X - W_1$ and $V = X - W_2$. Then U and V are disjoint $(1, 2)^{*-}\pi g\alpha$-open set such that $A \subset U \subset X - V \subset B \Rightarrow A \subset U \subset (1, 2)^{*-}\pi g\alpha\text{-cl}(U) \subset X - V \subset B$.

[3] ⇒ [1]. Let A and B be any two disjoint $(1, 2)^*$-regular closed subsets of X. Then $A \subset X - B$. Put $G = X - B$. Then G is an $(1, 2)^*$-regular open set containing A. By[3] there exist a $(1, 2)^{*-}\pi g\alpha$-open set U of X such that $A \subset U \subset (1, 2)^{*-}\pi g\alpha\text{-cl}(U) \subset G$. It follows that $B \subset X - (1, 2)^{*-}\pi g\alpha\text{-cl}(U)$. Let $V = X - (1, 2)^{*-}\pi g\alpha\text{-cl}(U)$. Then V is a $(1, 2)^{*-}\pi g\alpha$-open set and $U \cap V = \varphi$. Therefore X is mildly $(1, 2)^{*-}\pi g\alpha$-normal.

Theorem 3. 5: If $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is a M-$(1, 2)^{*-}\pi g\alpha$-open $(1, 2)^*$-rc continuous and $(1, 2)^{*-}\pi g\alpha$-irresolute surjection from a mildly $(1, 2)^{*-}\pi g\alpha$-normal space X onto Y then Y is mildly $(1, 2)^{*-}\pi g\alpha$-normal.

Proof: Let A be a $(1, 2)^*$-regular closed set and B be a $(1, 2)^*$-regular open set in Y containing A. Then by $(1, 2)^*$-rc continuity, $f^{-1}(A)$ is a $(1, 2)^*$-regular closed set contained in the $(1, 2)^*$-regular open set $f^{-1}(B)$. Since X is mildly $(1, 2)^{*-}\pi g\alpha$-normal, there exist a $(1, 2)^{*-}\pi g\alpha$-open set V such that $f^{-1}(A) \subset V \subset (1, 2)^{*-}\pi g\alpha\text{-cl}(V) \subset f^{-1}(B)$. As f is M-$(1, 2)^* - \pi g\alpha$-open and almost $(1, 2)^{*-}\pi g\alpha$-irresolute surjection, it follows that $A \subset f(V) \subset (1, 2)^{*-}\pi g\alpha\text{-cl}(f(V)) \subset B$. Hence Y is mildly $(1, 2)^{*} - \pi g\alpha$-normal.

Theorem 3. 6: If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is $(1, 2)^{*-}\pi g\alpha$-irresolute, $(1, 2)^{*}$-rc-preserving injection and Y is mildly $(1, 2)^{*} - \pi g\alpha$-normal then X is mildly $(1, 2)^{*-}\pi g\alpha$-normal.

Proof: Let A and B be any two disjoint $(1, 2)^{*}-$regular closed subsets of X. Since f is $(1, 2)^{*}$-rc-preserving injection, f(A) and f(B) are disjoint $(1, 2)^{*}$-regular closed sets of Y. By mildly $(1, 2)^{*-}\pi g\alpha$-normality of Y, there exist $(1, 2)^{*-}\pi g\alpha$-open sets U and V of Y such that $f(A) \subset U$ and $f(B) \subset V$. $f^{-1}(U)$, $f^{-1}(V)$ are disjoint $(1, 2)^{*} - \pi g\alpha-$open sets A and B respectively. By Theorem [3. 4] X is mildly $(1, 2)^{*} - \pi g\alpha-$normal.

Theorem 3. 7: If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is an $(1, 2)^{*-}\pi g\alpha$-irresolute, almost-$(1, 2)^{*}$-closed injection and Y is $(1, 2)^{*-}\pi g\alpha$-normal, then X is mildly $(1, 2)^{*-}\pi g\alpha$-normal.

Proof: Let A and B be disjoint $(1, 2)^{*}$-regular closed sets in X. Since f is almost $(1, 2)^{*}$-closed injection, f(A) and f(B) are disjoint $\sigma_{1,2}$-closed sets in Y. Since Y is $(1, 2)^{*-}\pi g\alpha$-normal, there exist a $(1, 2)^{*-}\pi g\alpha$-open sets U and V such that $f(A) \subset U$ and $f(B) \subset V$ such that $U \cap V = \varphi$. Since f is $(1, 2)^{*-}\pi g\alpha$-irresolute, $f^{-1}(V)$, $f^{-1}(U)$ are $(1, 2)^{*-}\pi g\alpha$-open sets such that $A \subset f^{-1}(U)$ and $B \subset f^{-1}(V)$. By Theorem[3. 4], X is mildly $(1, 2)^{*}$-$\pi g\alpha$-normal.

Theorem 3. 8: If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is completely $(1, 2)^{*}$-continuous, M-$(1, 2)^{*-}\pi g\alpha$-open surjection and X is mildly $(1, 2)^{*-}\pi g\alpha$-normal then Y is $(1, 2)^{*-}\pi g\alpha$-normal

Proof: Let A and B be disjoint $\sigma_{1,2}$-closed subsets of Y. Since f is completely (1, 2)*-continuous, $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint (1, 2)*-regular closed subsets of X. X is mildly (1, 2)*-πgα-normal implies that there exist (1, 2)*-πgα-open sets U and V in X such that $f^{-1}(A) \subset U$ and $f^{-1}(B) \subset V$ .Since f is M-(1, 2)*-πgα-open surjection. Hence Y is (1, 2)*-πgα-normal.

Almost (1, 2)*-α-normal and Mildly (1, 2)*-α-normal spaces

Definition 4. 1: A space (X, τ_1, τ_2) is said to be

[1] almost (1, 2)*-α-normal if for each $\tau_{1,2}$-closed set A and (1, 2)*−regular closed set B of X, such that $A \cap B = \varphi$ there exist disjoint (1, 2)*-α-open sets U and V such that $A \subset U$ and $B \subset V$.

[2] Mildly (1, 2)*-α-normal if for every pair of disjoint (1, 2)*−regular closed sets A and B of X, there exist disjoint (1, 2)*-α-open sets U and V such that $A \subset U$ and $B \subset V$.

Lemma 4. 2: A subset A of a space X is (1, 2)*-πgα-open (resp.(1, 2)*-gα-open) iff $F \subset$ (1, 2)*-αintA whenever F is (1, 2)*-regular closed (resp. $\tau_{1,2}$-closed) and $F \subset A$.

Theorem 4. 3: For a space (X, τ_1, τ_2) the following are equivalent.

[1] X is almost (1, 2)*-α-normal.

[2] For each $\tau_{1,2}$-closed set A and (1, 2)*−regular closed set B such that $A \cap B = \varphi$, there exist disjoint (1, 2)*-gα-open sets U and V such that $A \subset U$, $B \subset V$.

[3] For each $\tau_{1,2}$-closed set A and (1, 2)*−regular closed set B such that $A \cap B = \varphi$, there exist disjoint (1, 2)*-πgα-open sets U and V such that $A \subset U$, $B \subset V$.

[4] For each $\tau_{1,2}$-closed set A and each (1, 2)*−regular open set B containing A, there exist a (1, 2)*-πgα-open set V of X such that $A \subset V \subset (1, 2)^* - \alpha cl(V) \subset B$.

Proof: [1] ⇒ [2], [2] ⇒ [3] is obvious.

[3] ⇒ [4]. Let A be a $\tau_{1,2}$-closed set and B be a (1, 2)*-regular open subset of X containing A. X-B is (1, 2)*-regular closed and by[3], there exist (1, 2)*-πgα-open sets

V and W such that $A \subset V$ and $X - B \subset W$. By Lemma [4. 2] $X - B \subset (1, 2)^* - \alpha int(W)$ and $V \cap (1, 2)^* - \alpha int(W) = \varphi$. Therefore $(1, 2)^* - \alpha cl(V) \cap (1, 2)^* - \alpha int(W) = \varphi$ and hence $A \subset V \subset (1, 2)^* - \alpha cl(V) \subset X - (1, 2)^* - \alpha int(W) \subset B$.

[4] ⇒ [1]. Let A and B be $\tau_{1, 2}$-closed set and $(1, 2)^*$-regular closed sets respectively. Then X-B is $(1, 2)^*$-regular open set containing A. By[4] there exist a $(1, 2)^*$-πgα-open set G of X such that $A \subset G \subset (1, 2)^* - \alpha cl(G) \subset X - B$, put $U = (1, 2)^* - \alpha int(G)$ and $V = X - (1, 2)^* - \alpha cl(G)$. Then U and V are disjoint $(1, 2)^*$-α-open sets of X such that $A \subset U$ and $B \subset V$. Hence X is almost $(1, 2)^*$-α-normal.

Theorem 4. 4: For a space (X, τ_1, τ_2) the following are equivalent.

[1] X is mildly $(1, 2)^* - \alpha$-normal.

[2] For any disjoint $(1, 2)^*$-regular closed sets A and B of X, there exist disjoint $(1, 2)^*$-gα-open sets U and V such that $A \subset U$, $B \subset V$.

[3] For any disjoint $(1, 2)^*$-regular closed sets A and B of X, there exist disjoint $(1, 2)^*$-πgα-open sets U and V such that $A \subset U$, $B \subset V$.

[4] For each $(1, 2)^*$-regular closed set A and each $(1, 2)^*$-regular open set B containing A, there exist a $(1, 2)^*$-gα-open set V of X such that $A \subset V \subset (1, 2)^* - \alpha cl(V) \subset B$.

[5] For each $(1, 2)^*$−regular closed set A and each $(1, 2)^*$−regular open set B containing A, there exist a $(1, 2)^* - \pi g\alpha$-open set V of X such that $A \subset V \subset (1, 2)^* - \alpha cl(V) \subset B$.

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