Global Journal of Mathematical Sciences: Theory and Practical.
ISSN 0974-3200 Volume 4, Number 3 (2012), pp. 231-239

http://www.irphouse.com

MHD Oscillatory Flow in a Porous Plate

Monika Kalra and H.R. Gupta

Department of Mathematics
Haryana College of Technology & Management, Kaithal, Haryana, India
E-mail: monika.kalra01@gmail.com

Abstract

The study reported herein deals with the MHD heat and mass transfer flow of a mixed convection, incompressible, electrically conducting, and viscous fluid past an infinite vertical porous plate with time dependent suction velocity. A uniform magnetic field is applied in the direction normal to the plate. Solutions for the velocity field and temperature distributions are obtained using multiparameter perturbation technique. Approximate solutions for velocity, temperature, skin friction and rate of heat transfer have been obtained.

Keywords: Unsteady flow, MHD, Heat Transfer, Skin-friction, Free stream.

Introduction

Unsteady mixed Convection MHD flows are of great importance in aeronautics, missile aerodynamics etc. Due to these prospects, many authors have studied free convection and mass transfer flow of a viscous fluid through porous medium. Lighthill (1954) studied the effects of free oscillations on the flow of a viscous incompressible fluid past an infinite plate. Nanda and Sharma (1963) analyzed for free convection boundary layers along a semi-infinite vertical plate. Shreekanth et al. have investigated the effects of permeability variation on free convention flow past a vertical porous wall in a porous medium when the permeability varies in time. Singh et.al. have discussed hydromegnetic free convective and mass transfer flow of a viscous stratified fluid considering variation in permeability with direction. More recently, Acharya et al have studied free convention and mass transfer in steady free convection and mass transfer in steady flow through porous medium with constant suction in the presence of magnetic field. Messiha (1966) studied two dimensional incompressible fluid flow problems along an infinite flat plate with no heat transfer between the fluid and the plate when the suction velocity normal to the plate as well as the external flow varies periodically with time. Further Raptis and Perdikis (1985) studied the unsteady two dimensional free convective flows through highly porous medium. Recently, Ahmed and Ahmed (2004) analyzed the effect of two dimensional MHD oscillatory flows along a uniformly moving infinite vertical porous plate bounded by porous medium. Ahmed (2007) were investigated the effects of unsteady free convective MHD flow through a porous medium bounded by an infinite vertical porous plate. Soundalgekar (1973 a, b) considered the free convection effects on the oscillatory flow past an infinite vertical porous plate with constant suction. Vighnesam and Soundalgekar (1998) studied the combined free and forced convection flow of water from a vertical plate with variable temperature. Sahoo et.al. (2003) have analyzed the effects of MHD unsteady free convection flow past an infinite vertical plate with constant suction and heat sink.

The study reported herein analyses the effect of time dependent suction and periodic heat transfer on unsteady mixed convection MHD flow past a vertical porous flat plate. The various governing equations of the problem under consideration are solved by using perturbation technique.

Formulation of the problem

Consider the two dimensional flow of an incompressible, electrically conducting, unsteady and mixed convection fluid. The plate herein is assumed to be infinite in length, vertical in direction and flat. The permeability and suction velocity is taken to be time dependent. In the Cartesian coordinate system, let $\bar{\mathbf{x}}$-axis be along the plate in the direction of the flow and $\bar{\mathbf{z}}$-axis normal to it. A uniform magnetic field is introduced normal to the direction of flow. We assume that the magnetic Reynolds number is very small so that the included magnetic field is neglected. The influence of the density variations is also negligible. With the above assumptions, the governing equations of continuity, momentum and energy and heat transfer are given by,

$$\frac{\partial \bar{w}}{\partial \bar{z}} = 0 \Rightarrow \bar{w} = -w_0\left(1+\varepsilon \mathrm{A} e^{i\bar{w}\bar{t}}\right) \text{ Where } w_0 > 0 \tag{1}$$

$$\frac{\partial \bar{u}}{\partial \bar{t}} + \bar{w}\frac{\partial \bar{u}}{\partial \bar{z}} = g\beta\left(\bar{T}-\bar{T}_\infty\right) + \nu\frac{\partial^2 \bar{u}}{\partial \bar{z}^2} + \frac{d\bar{U}}{d\bar{t}} - \frac{\sigma {B_0}^2 \bar{u}}{\rho} \tag{2}$$

$$\frac{\partial \bar{T}}{\partial \bar{t}} + \bar{w}\frac{\partial \bar{T}}{\partial \bar{z}} = \bar{s}\left(\bar{T}-\bar{T}_\infty\right) + \bar{\kappa}\frac{\partial^2 \bar{T}}{\partial \bar{z}^2} + \frac{\nu}{c_p}\left(\frac{\partial \bar{u}}{\partial \bar{z}}\right)^2 \tag{3}$$

Also the free stream velocity oscillates with time is assumed to be of the form:

$$\bar{U}(\bar{t}) = U_0\left(1+\varepsilon e^{i\bar{w}\bar{t}}\right)$$

Introducing the following dimensionless quantities:

$$z=\frac{w_0\bar{z}}{\nu},u=\frac{\bar{u}}{U_0},U=\frac{\bar{U}}{U_0},t=\bar{t}w_0^{\,2}/4\nu,\ \omega=\frac{4\bar{\omega}\nu}{w_0^{\,2}},S=\frac{4\nu\bar{S}}{w_0^{\,2}},\nu=u/\rho,T=\frac{\bar{T}-\bar{T}_\infty}{\bar{T}_w-\bar{T}_\infty},$$

$$M=\sigma B_0^{\,2}\nu/\rho w_0^{\,2},G_r=\frac{\nu g\beta\left(\bar{T}_w-\bar{T}_\infty\right)}{\bigcup_0 w_0^2},\mathrm{P}_r=\frac{\nu}{\bar{\kappa}},\ \bar{\kappa}=\kappa/\left(\rho c_p\right),E_c=\frac{\bigcup_0^2}{c_p\left(\bar{T}_w-\bar{T}_\infty\right)}$$

With the help of dimensionless quantities, the equations (2) and (3) reduce to

$$\frac{1}{4}\frac{\partial u}{\partial t}-\left(1+\varepsilon \mathrm{A}e^{iwt}\right)\frac{\partial u}{\partial z}=G_rT+\frac{\partial^2 u}{\partial z^2}+\varepsilon\frac{iw}{4}e^{iwt}-Mu \tag{4}$$

$$\mathrm{P}_r\left[\frac{1}{4}\frac{\partial T}{\partial t}-\left(1+\varepsilon \mathrm{A}e^{iwt}\right)\frac{\partial T}{\partial z}\right]=\frac{\partial^2 T}{\partial z^2}+\frac{1}{4}\mathrm{P}_r\,ST+\mathrm{P}_r\,Ec\left(\frac{\partial u}{\partial z}\right)^2 \tag{5}$$

And the non-dimensional free stream is

$$\bigcup(t)=1+\varepsilon e^{iwt} \tag{6}$$

The relevant boundary conditions in non-dimensional form are

$$\left.\begin{array}{ll} u=0,T=1+\varepsilon e^{iwt}\ at & y=0 \\ u\to\bigcup,T\to 0\quad at & y\to\infty \end{array}\right\} \tag{7}$$

Method of Solution

In order to solve the equations (4) and (5) under the boundary condition (7), we assume

$$\left.\begin{array}{l} u(z,t)=u_0(z)+\varepsilon e^{iwt}u_1(z) \\ T(z,t)=T_0(z)+\varepsilon e^{iwt}T_1(z) \end{array}\right\} \tag{8}$$

Where u_0 and T_0 are respectively the mean velocity and mean temperature.

Substituting (8) into the equations (4) and (5), equating the harmonic and non harmonic terms and neglecting ε, we get

$$u_0''+u_0'-Mu_0=-G_rT_0 \tag{9}$$

$$T_0''+\mathrm{P}_r\,T_0'+\mathrm{P}_r\,ST_0/4=-\mathrm{P}_r\,Ec(u_0')^2 \tag{10}$$

$$u_1''+u_1'-\left(M+i\omega/4\right)u_1=-G_rT_1-\frac{i\omega}{4}-Au_0' \tag{11}$$

$$T_1''+\mathrm{P}_r\,T_1'+\mathrm{P}_r(S-i\omega)T_1/4=-2\,\mathrm{P}_r\,Ecu_0'u_1' \tag{12}$$

Where the primes denote differential with respect to z.

The equations (9) to (12) are still coupled for the variables u_0, u_1, T_0 and T_1. To solve them, it is to be noted that $E_c << 1$ for all incompressible fluid and assumed that:

$$F\left(y\right) = F_0(z) + E_c(z) + o(E_c{}^2) \tag{13}$$

Where F stands for u_0, u_1, T_0 and T_1

On using (13) into equations (9) to (12) and equating the like powers of E_c, the following equations are obtained:

$$u''_{00} + u'_{00} - \mathrm{M}u_{00} = -G_r \mathrm{T}_{00} \tag{14}$$

$$u''_{01} + u'_{01} - \mathrm{M}u_{01} = -G_r \mathrm{T}_{01} \tag{15}$$

$$u''_{10} + u'_{10} - \left(\mathrm{M} + i\omega/4\right)u_{10} = -G_r \mathrm{T}_{10} - \frac{i\omega}{4} - \mathrm{A}u'_{00} \tag{16}$$

$$u''_{11} + u'_{11} - \left(\mathrm{M} + i\omega/4\right)u_{11} = -G_r \mathrm{T}_{11} - \mathrm{A}u'_{01} \tag{17}$$

$$\mathrm{T}''_{00} + \mathrm{P}_r \mathrm{T}'_{00} + \frac{1}{4}\mathrm{P}_r S \mathrm{T}_{00} = 0 \tag{18}$$

$$\mathrm{T}''_{01} + \mathrm{P}_r \mathrm{T}'_{01} + \frac{1}{4}\mathrm{P}_r S \mathrm{T}_{01} = -\mathrm{P}r\left(u'_{00}\right)^2 \tag{19}$$

$$\mathrm{T}''_{10} + \mathrm{P}_r \mathrm{T}'_{10} + \mathrm{P}_r\left(S - i\omega\right)T_{10}/4 = 0 \tag{20}$$

$$\mathrm{T}''_{11} + \mathrm{P}_r \mathrm{T}'_{11} + \mathrm{P}_r\left(S - i\omega\right)T_{11}/4 = -2P_r\ u'_{00}u'_{10} \tag{21}$$

Subject to the boundary conditions:

$$\left.\begin{array}{l} u_{00} = 0, u_{01} = u_{10} = u_{11} = 0, \\ T_{00} = 1, T_{01} = 0, T_{10} = 1, T_{11} = 0 \quad at\ z = 0 \end{array}\right\} \tag{22}$$

$$\left.\begin{array}{l} u_{00} = 1, u_{01} = 0, u_{10} = 1, u_{11} = 0, \\ T_{00} = T_{01} = T_{10} = T_{11} = 0 \quad at\ z \to \infty \end{array}\right\} \tag{23}$$

In view of the boundary conditions (22) and (23) the solutions of the differential equations (14) to (21) are:

$$\boldsymbol{T_{00} = e^{-A_1 z}} \tag{24}$$

Where,

$$A_1 = \frac{P_r + \sqrt{P_r^2 - P_r S}}{2}, A_2 = \frac{-P_r + \sqrt{P_r^2 - P_r S}}{2}$$

$$\boldsymbol{u_{00} = e^{B_2 z} + \left(R_1 - 1\right)\bar{e}^{B_1 z} - R_1 \bar{e}^{A_1 z}} \tag{25}$$

Where,

$$B_1 = \frac{1+\sqrt{1+4M}}{2}, B_2 = \left[\frac{-1+\sqrt{1+4M}}{2}\right], R_1 = G_r\left[A_1^2 - A_1 - M\right]^{-1}$$

$$\boldsymbol{T_{01} = De^{-A_1 z} - R_2 e^{2B_2 z} - R_3 e^{-2B_1 z} - R_4 e^{-2A_1 z} + R_5 e^{(B_2 - B_1)z}}$$
$$\boldsymbol{+ R_6 e^{-(B_1 + A_1)z} - R_7 e^{(B_2 - A_1)z}} \tag{26}$$

Where

$$D = R_2 + R_3 + R_4 - R_5 - R_6 + R_7$$

$$\boldsymbol{u_{01} = D_1 e^{-B_1 z} - R_8 e^{-A_1 z} + R_9 e^{2B_2 z} + R_{10} e^{-2B_1 z} + R_{11} e^{-2A1z}}$$
$$\boldsymbol{- R_{12} e^{(B_2 - B_1)z} - R_{13} e^{-(B_1 + A_1)z} + R_{14} e^{(B_2 - A_1)z}} \tag{27}$$

Where

$$D_1 = R_8 - R_9 - R_{10} - R_{11} + R_{12} + R_{13} - R_{14}$$

$$\boldsymbol{T_{10} = e^{-A_3 z}} \tag{28}$$

Where,

$$A_3 = \frac{P_r + \sqrt{P_r^2 - P_r\left(S - i\omega\right)}}{2}, \quad A_4 = \frac{-P_r + \sqrt{P_r^2 - P_r\left(S - i\omega\right)}}{2}$$

$$u_{10} = e^{B_3 z} + D_2 e^{-B_4 z} - A_5 e^{-A_3 z} + A_6 - A_7 e^{B_2 z} + A_8 e^{-B_1 z} - A_9 e^{-A_1 z} \tag{29}$$

Where

$$D_2 = \left[-1 + A_5 - A_6 + A_7 - A_8 + A_9\right]$$

$$\boldsymbol{B_3 = -\frac{1}{2} + \frac{\sqrt{4 + 4M + i\omega}}{4}, B_4 = \frac{1}{2} + \frac{\sqrt{4 + 4M + i\omega}}{4},}$$

$$\boldsymbol{A_5 = G_r\left[A_3^2 - A_3 - \left(M + \frac{i\omega}{4}\right)\right]^{-1}}$$

$$\boldsymbol{T_{11} = D_3 e^{-A_3 z} - A_{10} e^{(B_2 + B_3)z} + A_{11} e^{(B_2 - B_4)z} - A_{12} e^{(B_2 - A_3)z} - A_{13} e^{(B_2 - B_1)z} - A_{14} e^{(B_2 - A_1)z}}$$

$$+A_{15}e^{(B_3-B_1)z}-A_{16}e^{-(B_1+B_4)z}+A_{17}e^{-(B_1+A_3)z}+A_{18}e^{-(B_1+A_1)z}-A_{19}e^{(B_3-A_1)z}$$

$$+A_{20}e^{-(A_1+B_4)z}-A_{21}e^{-(A_1+A_3)z}+A_{22}e^{2B_2z}-A_{23}e^{-2B_1z}-A_{24}e^{-2A_1z} \quad (30)$$

Where,

$$D_3=[A_{10}-A_{11}+A_{12}+A_{13}+A_{14}-A_{15}+A_{16}-A_{17}$$

$$-A_{18}+A_{19}-A_{20}+A_{21}-A_{22}+A_{23}+A_{24}]$$

$$u_{11}=D_4e^{-B_4z}-A_{25}e^{-A_3z}+A_{26}e^{(B_2+B_3)z}-A_{27}e^{(B_2-B_4)z}$$

$$+A_{28}e^{(B_2-A_3)z}-A_{29}e^{(B_3-B_1)z}+A_{30}e^{-(B_1+B_4)z}-A_{31}e^{-(B_1+A_3)z}$$

$$+A_{32}e^{(B_3-A_1)z}-A_{33}e^{-(A_1+B_4)z}+A_{34}e^{-(A_1+A_3)z}+A_{35}e^{-B_1z}$$

$$-A_{36}e^{-A_1z}-A_{37}e^{2B_2z}+A_{38}e^{-2B_1z}+A_{39}e^{-2A_1z}+A_{40}e^{(B_2-B_1)z}$$

$$-A_{41}e^{-(B_1+A_1)z}+A_{42}e^{-(B_2-A_1)z} \quad (31)$$

Where,

$$D_4=[A_{25}-A_{26}+A_{27}-A_{28}+A_{29}-A_{30}-A_{34}-A_{35}+A_{36}+A_{31}-A_{32}$$

$$+A_{33}+A_{37}-A_{38}-A_{39}-A_{40}+A_{41}-A_{42}]$$

and the other constants are not presented here for the sake of brevity.

Separating real and imaginary parts of the velocity and temperature expression (8) and taking only the real parts, the velocity and temperature fields in terms of the fluctuating parts given by:

$u=u_0(z)+\varepsilon\,(M_r\ \cos\omega\,\text{t-}M_i\ \text{Sin}\,\omega\,\text{t})$ (32)

$T=T_0(z)+\varepsilon\,(T_r\cos\omega\,\text{t-}\ T_i\ \text{Sin}\,\omega\,\text{t})$ (33)

Hence expressions for transient velocity and temperature for $\omega t=\frac{\Pi}{2}$ are:

$$u\left(z,\frac{\Pi}{2\omega}\right)=u_0(z)-\varepsilon M_i \quad , \quad T\left(z,\frac{\Pi}{2\omega}\right)=T_0(z)-\varepsilon T_i \quad (34)$$

Skin Friction and Rate of Heat Transfer

Skin –friction coefficient at the plate can be calculated in non-dimensional form is:

$$\tau_w=\left(\frac{\partial u}{\partial z}\right)_{z=0}=u_0'(0)+\varepsilon e^{i\omega t}u_1'(0) \quad (35)$$

Splitting the equation (35) into real and imaginary parts and taking real part only

$$\tau_w = \tau_0 + \varepsilon \left| N \right| \cos\left(\omega t + \alpha\right) \tag{36}$$

$$\left|\mathrm{N}\right| = \sqrt{\mathrm{N}_r^2 + \mathrm{N}_i^2}\,, \tan\alpha = \frac{\mathrm{N}_i}{\mathrm{N}_r}\,,\ \mathrm{N}_r = \mathbf{Re}\left(\mathrm{N}\right) = \boldsymbol{M}_r'\,,\ \mathrm{N}_i = \mathbf{Im}\left(\mathrm{N}\right) = \boldsymbol{M}_i'\,.$$

Heat transfer coefficient can be calculated in non-dimensional form...

$$q_w = -\left(\frac{\partial T}{\partial z}\right)_{z=0} = T_0'\left(0\right) + \in e^{iwt} T_1'\left(0\right) \tag{37}$$

Splitting equation (37) into real and imaginary parts and taking real parts only

$$q_w = q_0 + \in \left|Q\right| \cos\left(\omega t + \beta\right),\ \left|Q\right| = \sqrt{Q_r^2 + Q_i^2}\,, \tan\beta = \frac{Q_i}{Q_r}\,,$$

$$Q_r = \mathrm{Re}\left(Q\right) = T_r', Q_i = \mathrm{Im}\left(Q\right) = T_i'\ \boldsymbol{q_0} = \boldsymbol{T_0'}\left(\mathbf{0}\right) = -\boldsymbol{A_1}\left(\mathbf{1} + \boldsymbol{E_c D}\right) + \boldsymbol{E_c}$$

$$\left[-2\boldsymbol{B_2 R_2} + 2\boldsymbol{B_1 R_3} + 2\boldsymbol{A_1 R_4} + \left(\boldsymbol{B_2} - \boldsymbol{B_1}\right)\boldsymbol{R_5} - \left(\boldsymbol{B_1} + \boldsymbol{A_1}\right)\boldsymbol{R_6} - \left(\boldsymbol{B_2} - \boldsymbol{A_1}\right)\boldsymbol{R_7}\right] \tag{38}$$

Expression for $\mathrm{N}_{r,}\mathrm{N}_{i,}Q_r$ and Q_i are not presented here for the sake of brevity

Discussion of the Result

We have solved the problem of MHD oscillatory flow in a porous plate. Now it is very difficult to study the effects of all the parameters involved in the problem. Therefore a few of them, which are comparatively important, have been selected and their influence on the flow has been discussed. Solutions for the velocity field and temperature distribution are obtained using the perturbation technique. Approximate solutions for the velocity, temperature, skin friction and rate of heat transfer have been obtained.

Nomenclature

$(\bar{u}, \bar{w})$	Velocity components along $\bar{\mathbf{x}}$ and $\bar{\mathbf{z}}$ direction respectively,
w_0	Mean suction velocity,
u	Dimensionless velocity component,
g	Acceleration due to gravity,
B_0	Magnetic field,
$\bar{t}$	Time,

$\bar{U}$	Free stream velocity,
$\bar{T}_w$	Temperature at the plate,
$\bar{T}_\infty$	Free stream temperature.
$\bar{T}$	Fluid temperature,
T	Dimensionless temperature,
$\mathbf{Pr}$	Prandtl number,
Gr	Grashof number,
S	Sink strength,
M	Hartman number,
τ_0	Mean skin-friction,
q_0	Mean heat transfer and
E_c	Eckert number,

Greek symbols

β	Coefficient of volume expansion,
ε	Amplitude parameter,
κ	Thermal conductivity,
ρ	Density,
μ	Coefficient of viscosity,
υ	Kinematics viscosity,
ω	Frequency parameter,

Subscripts

w	Evaluated at wall conditions
∞	Evaluated at free stream conditions

References

[1] Lighthill, M.J., “The response of laminar Skin friction and heat transfer to fluctuations in the stream velocity, ” Proc. Roy. Soc. A224, 1-16 (1954)

[2] Nanda, R.S. and V.P.Sharma, “Free Convection boundary layers along a semi-infinite vertical plate, ” J. Fluid Mech. 15, 419-428(1963).

[3] S.Shreekanth, S. Venkataramana and S. Ramakrishna, Acta Ciencia Indica, 22 M (1996) 267-75.

[4] N.P. Singh, Ajay Kumar singh, M.K.Yadav and Atul Kumar Singh, J, Energy Heat Mass Transfer, 21(1999) 111-15

[5] M. Acharya. G.C. Dash and L.P. Singh, Ind. J. Pure app. Math, 31 (2000) 1-18.

[6] Messiha, S.A.S., “Laminar boundary layers in oscillatiory flow along an infinite flat plate with variable suction, ” Proc. Camb. Phil. Soc., 62, 329-37 (1966)

[7] Raptis, A.A. and C.P. Perdikis, “Oscillatory flow through a porous medium by the presence of free convective flow, ” Int. J. Engng. Sci., 23, 51-55 (1985).

[8] Ahmad, S. and N. Ahmad, “Two-dimensional MHD oscillatory flow along a uniformly moving infinite vertical porous plate bounded by porous medium, ” IJPAM, 35, 1309-1319 (2004)

[9] Ahmed, S., “Effects of unsteady free convective MHD flow through a porous medium bounded by an infinite vertical porous plate, ” Bull. Cal. Math. Soc., 99, 511-522 (2007).

[10] Soundagelkar, V.M. (1973), “Free convection effects on the oscillatory flow past an infinite vertical porous plate with constant suction. 1, ” Proc. Royal Soc. Landon A 333: 25-36.

[11] Soundagelkar, V.M. (1973), “Free convection effects on the oscillatory flow past an infinite vertical porous plate with constant suction. 11, ” Proc. Royal Soc. London A 333: 37-50.

[12] Vighnesam, N. V. and V.M. Soundagelkar, “Combined free and force convection flow of water at $\mathbf{4}^{\cdot}\boldsymbol{c}$ from a vertical plate with variable temperature, ” Ind.J. Of Engineering and Material Science, 5, 124-126 (1998).

[13] Sahoo, P.K. N. Datta and S. Biswal, “MHD unsteady free convection flow past an infinite vertical plate with constant suction and heat Sink, ” IJPAM, 34, 145-55 (2003).