Selection Procedures for SKSP-2 Skip-Lot Plans through Relative Slopes

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Abstract

In skip-lot sampling plans, provision is made for skipping inspection of some of the lots when the quality of the submitted product is good. This paper presents selection of skip-lot plans of type SkSP-2 which use chain sampling plan (ChSP-1) as reference plan through relative slopes at various points on the OC curve, which describe the degree of steepness of the OC Curve.

Keywords:-Skip-lot Sampling Plan, Chain Sampling Plan, Acceptable Quality Level, Limiting Quality Level, Indifference Quality Level, Maximum Allowable Percent Defective Relative Slope.

Mathematical Subject Classification: 62P30/62D05

Introduction

Under skip-lot sampling inspection, samples may be drawn from only a fraction of the submitted lots. Here skipping of lots of good quality protection to the producer is ensured and economy of sample size is achieved. The main purpose of skip-lot sampling is to decrease the frequency of sampling inspection and thus reducing the total inspection costs.

A system of skip-lot sampling plans for lot-inspection was initially introduced by Dodge (1955b) and later was investigated extensively by Perry (1970). Dodge and Perry (1971) formalize the application of skip-lot sampling to the situation in which each lot to be inspected is sampled according to some lot inspection plan.

Perry (1970) has described a skip-lot plan of type SkSP-2 as one that uses a given lot-inspection plan by the method of attributes (single sampling, double sampling, sequential sampling, chain sampling, etc) called the "reference sampling plan". Perry (1970, 1973) observed that, from an overall point of view, skip lotting appears to be a

good and useful acceptance sampling procedure and might well qualify as a standard system of reduced inspection.

For situations involving costly or destructive testing and the lots are submitted in a continuous succession, it is advantageous to use SkSP-2 plan with ChSP-1 plan as reference plan, since both the plans (SkSP-2 and ChSP-1) ensure a relatively small sample size.

Operating Procedure for Skip-Lot Sampling Plan

The operating Procedure for SkSP-2 plan is listed below

- Start with normal inspection (inspecting every lot), using the reference plan.
- When i consecutive lots are accepted on normal inspection, switch to skipping inspection (inspect a fraction f of the lots)
- When a lot is rejected on skipping inspection, switch to normal inspection

Thus the values of f and i, called the skipping parameters, together with reference plan completely characterize a Type SkSP-2 plan. In general 0 < f < 1, and i is a positive integer. When f = 1, the plan degenerates into the original reference plan.

Operating Procedure of Chain Sampling Plan

Chain Sampling Plan (ChSP-1), which was introduced by Dodge (1955a), is used as reference plan in this article. Chain Sampling Plans allow significant reductions in sample size under conditions of a continuing succession of lots from a stable and trusted supplier as stated by Dodge (1955a).

The reference plan ChSP-1 is operated as follows

- For each lot, select a sample of n units and test each unit for conformance to the specified requirements.
- Accept the lot, if d (the observed number of defectives) is 0, if d > 1, reject the lot
- Accept the lot, if d 1 and if no defective units are found in the immediately preceding i_c samples of size n.

Thus a ChSP-1 plan has two parameters, namely, n, the sample size for each submitted lot and i_c the number of previous samples on which the decision of acceptance or rejection is based, Hence a SkSP-2 plan using ChSP-1 as reference plan has four parameters, namely, n, i_c , f and i.

Incoming Qualities and Relative Slopes

In this paper, four incoming quality levels, namely, Acceptable Quality Level (AQL), Limiting Quality Level (LQL), Indifference Quality Level (IQL) and Maximum Allowable Proportion Defective (MAPD) are considered along with their corresponding relative slopes on the OC curve for selection of SkSP-2 plans. AQL denoted by p_1 , is the maximum percentage or proportion of variant units in a lot or batch that, for the purposes of acceptance sampling, can be considered as a

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satisfactory process average. In this paper, AQL is considered as the fraction defective with a 0.95 probability of acceptance of lots. LQL, denoted by p_2 , is the percentage of proportion of variant units in a batch or lot that, for the purposes of acceptance to be restricted to a specified low value, usually 0.10. IQL, denoted by p_0 , is the percentage of variant units in a lot or batch that, for the purposes of acceptance sampling, the probability of acceptance to be restricted to a specific value, namely 0.50. The point (p_0 , 0.5) on the OC curve is referred to as point of control. The proportion nonconforming corresponding to the point of inflection (p^* , $P_a(p^*)$) of the OC curve is interpreted as MAPD (p^*). Fixing the AQL gives adequate protection to the producer and the interests of the consumer are protected by fixing LQL. Fixing IQL gives required protection not only to the consumer but also to the producer. MAPD is considered as standard incoming quality level for both producer and consumer.

The chief features of an OC curve are its location and the relative slope (denoted by h) at that location, which describes the degree of steepness of the OC curve. Hamaker (1950) has made elaborate studies on the slope h_0 , which, along with p_0 , may be used to design any sampling plan. In a similar manner, various other sets of parameters, such as (p_1,h_1) , (p_2,h_2) and (p_*,h_*) can also be considered for selection of plans. Vedaldi (1986) has studied two principal effects of sampling inspection, which are filter and incentive effects, and has proposed a new criterion based on the AQL and LQL points on the OC curve. Suresh (1993) has presented and constructed tables for the selection of SkSP-2 with Single Sampling Plan as reference plan indexed through (p_1,h_1) and (p_2,h_2) involving incentive and filter effects. Vijayaraghavan (1990) has constructed tables and has provided selection procedures for SkSP-2 plans with single sampling plan as reference plan.

Selection procedures of SkSP-2 with ChSP-1 as reference plan will now be indicated under the conditions for application of Poisson model for OC curve. A table (Table 1) has been constructed for various parametric values indexed i_c , f, i, which can be used for selection of such plans.

Designing Plans for Given (p₁,h₁)

Column 4 and 8 of Table 1 is used to design plans for a given p_1 and h_1 . For example, for given $p_1 = 0.01$ and $h_1 = 0.12$, from Table 1 under column headed h_1 , locate the value equal to or just greater than specified h_1 , which is 0.12054. Corresponding to this h_1 , the i_c , f, i, and np_1 values associated are 3,1/3, 10 and 0.19706 respectively. From this one can obtain the sample size $n=np_1 / p_1 = 0.19706 / 0.01 = 19.106 = 20$. Thus the selected parameters for SkSP-2 plan are n=20, $i_c=3$, f=1/3 and i=10.

Designing Plans for Given (p₂,h₂)

Column 5 and 9 of Table 1 is used to design plans for a given p_2 and h_2 . For example, for given $p_2 = 0.2$ and $h_2= 2.3$, from Table 1 under column headed h_2 , locate the value equal to or just greater than specified h_2 , which is 2.32196. Corresponding to this h_2 , the i_c, f, i, and np₂ values associated are 3,2/3, 4 and 2.30522 respectively. From this one can obtain the sample size n=np₂ / p₂ = 2.30522 / 0.2 = 11.5261 = 12. Thus the selected parameters for SkSP-2 plan are n=12, i_c=3, f=2/3 and i=4.

Designing Plans for Given (p₀,h₀)

Column 6 and 10 of Table 1 is used to design plans for a given p_0 and h_0 . For example, for given $p_0 = 0.07$ and $h_0 = 0.8$, from Table 1 under column headed h_0 , locate the value equal to or just greater than specified h_0 , which is 0.80396. Corresponding to this h_0 , the i_c , f, i, and np_0 values associated are 5,1/3, 8 and 0.72003 respectively. From this one can obtain the sample size $n=np_0 / p_0 = 0.72003 / 0.07 = 10.286 = 10$. Thus the selected parameters for SkSP-2 plan are n=10, $i_c=5$, f=1/3 and i=8.

Designing Plans for Given (p*,h*)

Column 7 and 11 of Table 1 is used to design plans for a given p_* and h_* . For example, for given $p_* = 0.025$ and $h_* = 0.25$, from Table 1 under column headed h_* , locate the value equal to or just greater than specified h_* , which is 0.25008. Corresponding to this h_* , the i_c , f, i, and np_1 values associated are 5,2/3, 12 and 0.22832 respectively. From this one can obtain the sample size $n=np_* / p_* = 0.22832 / 0.025 = 9.13 = 9$. Thus the selected parameters for SkSP-2 plan are n=9, $i_c=5$, f=2/3 and i=12.

Designing Plans through the Ratio of Relative Slopes

Columns 4 (or 5) and 12 of Table 1 is used to design plans for specified AQL (or LQL) with the ratio of relative slopes h_2/h_1 . For example, for given $p_1 = 0.01$, $h_1 = 0.15$, and $h_2 = 3.0$, one can obtain the ration of $h_2/h_1 = 20$. By using Table 1, under column headed h_2/h_1 , one can locate the value equal to or just greater than the desired ratio, which is 20.16793. Corresponding to this located value, one can find i_c , f, i, and np_1 values as 4, 1/3, 8 and 0.18434 respectively. From this one can obtain the sample size $n=np_1/p_1=0.18434 / 0.01 = 18$. Thus the selected parameters for plan are n=18, $i_c=4$, f=1/3 and i=8.

Conversion of Parameters

One may be interested in converting the given set of parameters into other familiar sets which provide information in the related parameters. For example when $p_1 = 0.01$, $h_1=0.11$ are specified, the other equivalent set of parameters are found using Table 1.

Corresponding to $h_1 = 0.11$, one finds tabulated $h_1=0.11003$. The np1 value associated to this h_1 is 0.17078. Now $n = np_1/p_1 = 0.17078 / 0.01 = 17$. The other associated values are $np_2 = 2.30478$, $np_0 = 0.76737$, $np_*=0.27591$, $h_2 = 2.31850$, $h_0=0.86229$, $h_*=0.27852$. Therefore $p_2=np_2/n = 2.30478 / 17 = 0.13557$, $p_0=0.76737 / 17 = 0.04514$, $p_*=0.27591 / 17 = 0.01623$. Thus, when $p_1=0.01$ and $h_1=17$, the other sets of parameters are

- a) $(p_1, p_2) = (0.01, 0.13557)$
- b) $(p_1,p_0) = (0.01, 0.04514)$
- c) $(p_1, p_*) = (0.01, 0.01623)$
- d) $(p_2,h_2) = (0.13557, 2.31850)$
- e) $(p_0,h_0) = (0.04514, 0.86226)$
- f) $(p_{*},h_{*}) = (0.01623, 0.27852)$

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Table 1: Certain Parametric Values for SkSP-2 plans with ChSP-1 Plan as Reference

 Plan

\mathbf{i}_{c}	f	i	np ₁	np ₂	np ₀	np*	h ₁	h ₂	h_0	h*	h_1/h_2
C	1		A -	A -	_	1		-	0		26.34326
		4	0.19547	2.32493	0.86422	0.47893	0.09573	2.40766	1.01330	0.46711	25.14936
											24.44774
	2/3	8	0.18839	2.32449	0.84048	0.39799	0.10031	2.40415	0.94335	0.38468	23.96720
											23.61605
											23.36218
											·
		4	0.22204	2.32536	0.88558	0.49253	0.09974	2.41118	1.08184	0.47247	24.17423
		6	0.21439	2.32450	0.85174	0.42724	0.10429	2.40426	0.99963	0.41665	23.05466
	1/2	8	0.20820	2.32449	0.84217	0.38349	0.10764	2.40415	0.95581	0.36953	22.33423
		10	0.20307	2.32449	0.83962	0.35201	0.11014	2.40415	0.93877	0.33234	21.82734
		12	0.19872	2.32449	0.83896	0.32814	0.11197	2.40415	0.93305	0.30283	21.47137
		4	0.26356	2.32623	0.92039	0.49808	0.10662	2.41815	1.19183	0.44027	22.67990
2		6	0.24883	2.32451	0.86300	0.41910	0.11394	2.40437	1.05906	0.38432	21.10187
	1/3	8	0.23762	2.32449	0.84540	0.36829	0.11920	2.40415	0.97971	0.33670	20.16929
		10	0.22869	2.32449	0.84047	0.33232	0.12303	2.40415	0.94687	0.29887	19.54159
		12	0.22135	2.32449	0.83918	0.35019	0.12581	2.40415	0.93556	0.26872	19.10977
											21.57838
											19.76250
	1/4										18.75257
											18.08584
		12	0.23789	2.32449	0.83940	0.29574	0.13634	2.40415	0.93806	0.24294	17.63399
											20.72323
											18.76830
	1/5										17.72743
		10	0.26244	2.32449	0.84214	0.318/9	0.14099	2.40415	0.96263	0.24656	17.05223
		12	0.25096	2.32449	0.83961	0.29061	0.14487	2.40415	0.94055	0.22118	16.59538
	1	*	0 12902	2 20 470	076716	0.22120	0 00000	2 21050	0.05007	0 22050	25 7(720
	1 2/3										25.76739
											24.67370
3											23.97691
											23.49614
											23.14677
		12	0.15/22	2.30478	0.70727	0.29556	0.10127	2.31830	0.80113	0.2991/	23.89478
	1/2	Λ	0 10121	2 20567	0.81260	0 41 282	0 00770	2 22520	0 00682	0 41247	23.78087
L	1/Z	4	0.19121	2.30307	0.01309	0.41282	0.09//8	2.32338	0.99082	0.41247	23.18081

		(0	10450		20/	170	0.7	70004	0	25	0.45	0	100	22	22	10/1	0	<u></u>	0.4	0 1	222	4	22	<u> </u>	1/7
				18450																						
		_	_	17908	-					-								-					_			
				17458	-					-			_										_			
		12	0.	17078	2	.304	178	0.7	76737	0.	27	591	0.1	110	03	2.3	1850	0.8	862	29	0.2	2785	2	21.	071	48
		4	0.	22783	2	.306	656	0.8	34849	0.	42	157	0.	104	11	2.3	3221	1.0	096	01	0.3	3847	3	22.	402	223
		6	0.	21481	2	.304	180	0.7	79121	0.	35	567	0.	111	42	2.3	1872	0.9	977	20	0.3	3452	2	20.	810)63
	1/3		_	20492	-					-			-					-					_			
	1/5			19706																			_			
				19061	-					-			_										_			
		14	υ.	17001		.504	-/0	0.1	10137	0.	25	707	0.	123	55	2.5	1050	0.0	504	01	0.2	.477	U	10.	1).	07
		1	Δ	75616	5	207	7 / /	<u>^</u>	07667	0	12	777	0	100	20	\mathbf{r}	2000	1	174	21	0.2	2101	1	21	20/	170
		_	_	25646	-					-								-					_			
				23780	_					-								_					_			
	1/4	_	_	22419	-					-								-					_			
			_	21365	-					-								_					_			
		12	0.	20516	2	.304	178	0.7	76780	0.	25	180	0.1	133	46	2.3	1850	0.8	866	90	0.2	2264	0	17.	372	211
					-								1			1										
		4	0.	28014	- 2	.308	331	0.9	90036	0.	42	454	0.	113	90	2.3	4576	1.	239	22	0.3	3150	6	20.	595	501
	1/5	6	0.	25642	2	.304	181	0.8	31003	0.	35	071	0.1	124	70	2.3	1896	1.0	067	28	0.2	2848	4	18.	596	579
		8	0.	23961	2	.304	178	0.7	77970	0.	304	428	0.1	132	24	2.3	1851	0.9	945	47	0.2	2541	1	17.	532	253
		10	0.	22684	2	.304	178	0.7	77052	0.	27	192	0.	137	68	2.3	1850	0.8	889	44	0.2	2279	4	16.	839	980
		12	0.	21669	2	.304	178	0.7	76802	0.	24	779	0.	141	63	2.3	1850	0.8	869	19	0.2	2062	1	16.	369	987
ic	f	i		np ₁		np	2		np ₀		np)*		h_1]	h_2		h_0			h*		h	h_1/h	2
	1	*	0.	12413	-	-			-	-	-		0.0	088	76	2.3	0483	0.8	804	28	0.2	2877	4	25.	966	504
		4	0.	15050	2	.303	371	0.7	75742	0.	34	229	0.0	092	54	2.3	0427	0.3	874	10	0.3	3561	8	24.	944	134
				14745																						
	2/3			14487																						
				14266	-					-								-					_			
				14074																			_			
		12	υ.	14074	- 2	.302	212	0.1	13173	υ.	25	010	0.0	099	09	2.5	0403	0.0	505	57	0.2		5	23.	115	747
		Λ	Δ	17176	2	202	060	0.7	77010	0	26	061	0.0	005	01	<u> </u>	1140	0.0	021	00	0.3	2625	0	<u>7</u> 1	102	000
				$\frac{17170}{16562}$																						
4	1 10				_					_																
	1/2	_	_	16066	-					-								-					_			
			_	15656	-					-								_					_			
		12	0.	15309	2	.302	272	0.7	73184	0.	24	223	0.1	108	16	2.3	0483	0.8	806	45	0.2	2575	3	21.	308	398
			1		1								1					1								
				20543																			_			
	-	6	0.	19343	2	.302	274	0.7	7 <u>561</u> 8	0.	314	462	0.	109	00	2.3	0505	0.9	9 <u>13</u>	65	0.3	<u>8117</u>	2	21.	147	720
		8	0.	18434	2	.302	272	0.7	73834	0.	27	683	0.	114	28	2.3	0483	0.3	846	33	0.2	2816	8	20.	167	793
		10	0.	17712	2	.302	272	0.7	73336	0.	24	982	0.	118	15	2.3	0483	0.3	818	61	0.2	2553	2	19.	507	765
			_	17712	-					-			-					-					_			
		1																								

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	4 0.23194 2.30539 0.84380 0.37527 0.10650 2.32530 1.09825 0.30	
	6 0.21464 2.30275 0.76630 0.31304 0.11578 2.30516 0.95807 0.28	
1/4	8 0.20207 2.30272 0.74145 0.27298 0.12239 2.30483 0.86569 0.25	604 18.83231
	0 0.19236 2.30272 0.73421 0.24475 0.12719 2.30483 0.82511 0.23	197 18.12122
	20.18455 2.30272 0.73228 0.22355 0.13069 2.30483 0.81076 0.21	146 17.63569
	4 0.25399 2.30627 0.86826 0.37678 0.11061 2.33203 1.15944 0.27	683 21.08362
	6 0.23190 2.30276 0.77543 0.31181 0.12142 2.30527 0.99776 0.25	682 18.98652
1/5	8 0.21632 2.30272 0.74442 0.27061 0.12901 2.30484 0.88409 0.23	286 17.86532
	00.20452 2.30272 0.73505 0.24180 0.13451 2.30483 0.83180 0.21	120 17.13461
	20.19516 2.30272 0.73250 0.22029 0.13853 2.30483 0.81289 0.19	
1	* 0.11371 2.30251 0.71313 0.24077 0.08761 2.30298 0.76383 0.25	450 26.28806
	4 0.13825 2.30296 0.73962 0.30303 0.09104 2.30642 0.83094 0.31	
	6 0.13540 2.30252 0.72013 0.27738 0.09362 2.30303 0.79311 0.30	
2/3	8 0.13300 2.30251 0.71493 0.25674 0.09559 2.30298 0.77430 0.28	
_,	00.13094 2.30251 0.71358 0.24078 0.09709 2.30298 0.76722 0.26	
	20.12916 2.30251 0.71324 0.22832 0.09821 2.30298 0.76487 0.25	
	4 0.15815 2.30341 0.76198 0.32293 0.09413 2.30984 0.88671 0.32	372 24 53864
	6 0.15239 2.30252 0.72662 0.28373 0.09864 2.30309 0.82009 0.30	
1/3	8 0.14775 2.30251 0.71667 0.25546 0.10200 2.30298 0.78447 0.28	
- / -	00.14391 2.30251 0.71403 0.23446 0.10451 2.30298 0.77057 0.25	
	20.14067 2.30251 0.71335 0.21822 0.10639 2.30298 0.76590 0.23	
		21.01711
	4 0.18986 2.30430 0.79868 0.33607 0.09942 2.31666 0.97676 0.30	210 23 30246
	6 0.17853 2.30253 0.73834 0.28522 0.10671 2.30320 0.86850 0.28	
$5_{1/3}$	8 0.16997 2.30251 0.72003 0.25118 0.11200 2.30298 0.80396 0.25	
1/.	0.16319 2.30251 0.71492 0.22672 0.11590 2.30298 0.77719 0.23	
	20.15764 2.30251 0.71358 0.20815 0.11877 2.30298 0.76796 0.21	
	<u>20.15/04</u> 2.502510.715500.200150.11077[2.50290[0.70790]0.21	007 17.37032
	4 0.21500 2.30519 0.82842 0.34054 0.10379 2.32344 1.04855 0.27	280/22 28500
	6 0.19858 2.30254 0.74875 0.28468 0.11307 2.30331 0.91111 0.25	
1/		
1/4	8 0.18668 2.30251 0.72323 0.24839 0.11969 2.30298 0.82244 0.23	
	00.177522.302510.715790.222720.124532.302980.783690.21	
	2 0.17017 2.30251 0.71381 0.20341 0.12808 2.30298 0.77000 0.19	81117.98055
	4 0 226020 206070 852570 242640 107520 220171 108580 24	507 01 67100
	4 0.23603 2.30607 0.85357 0.34264 0.10752 2.33017 1.10858 0.24	
1.1	6 0.21496 2.30255 0.75813 0.28404 0.11833 2.30341 0.94924 0.23	
1/3	8 0.20017 2.30251 0.72627 0.24659 0.12596 2.30298 0.84001 0.21	
	00.189002.302510.716650.220330.131512.302980.790070.19	
	20.180162.302510.714030.200700.135582.302980.772040.18	045 16.98599

Construction of Tables

The expression for Pa(p) of SkSP-2 plan with parameters f and i has been derived by Perry (1973) as

 $P_{a}(p) = (f P + (1-f) P^{i}) / (f + (1-f) P^{i})$ (1)

Here $P = e^{-np} + (np) e^{-np(ic+1)}$, which is the OC function for ChSP-1 reference plan having parameters n and i_c, as derived by Dodge (1955a), under the conditions for application of Poisson model for OC curve.

For given values of i_c , f, i and $P_a(p)$, equation (1) can be solved for np by the method of iterations. The entries of the columns np_1 , np_2 and np_0 are such np values with $P_a(p) = 0.95$, 0.10 and 0.50 respectively.

The values of np_* are such values of np, which are obtained by equating the second derivative of $P_a(p)$ to 0 for given values of i_c , f, i and $P_a(p)$. The entries of the columns h_1, h_2, h_0 , and h_* are calculated through the expression.

 $h = -(p / P_a(p)) (d P_a(p) / dp), \text{ for } p = p_1, p_2, p_0 \text{ and } p_*$

The entries of column 12 are the values of the ratio of h_2 and h_1 .

References

- [1] DODGE, H,F, (1955a), "Chain Sampling Plans", Industrial Quality Control, Vol.11,No.4, pp 10-13.
- [2] DODGE, H.F. (1955b), "Skip-lot Sampling Plans", Industrial Quality Control, Vol.11, No.5, pp 3-5, (also in Journal of Quality Technology), Vol, 9, No.3, 1977, pp.143-145.
- [3] DODGE, H.F. and PERRY, R.L. (1971), "A System of Skip-Lot Plans for Lot-by-Lot Inspection", American Society for Quality Control Technical ConferenceTransactions, Chicago, Illinois, pp.469-477.
- [4] HAMAKER, H.C. (1950), "The Theory of Sampling Inspection Plans", Phillips Technical Review, Vol.11, No.9, pp.260-270.
- [5] PERRY, R.L. (1970), "A System of Skip-lot Sampling Plans for Lot Inspection", Ph.D. Thesis, Rutgers – The State University, New Brunswick, New Jersey.
- [6] PERRY, R.L. (1973), "Skip-lot Sampling Plans", Journal of Quality Technology, Vol.5, No.3. pp.123-130.
- [7] SURESH, K.K. (1993), "A Study on Acceptance Sampling using Acceptable and Limiting Quality Levels", Ph.D. Thesis, Bharathiar University, Coimbatore, India.
- [8] VEDALDI, R. (1986), "A New Criterion for the Construction of Single Sampling Inspection Plans", Rivistadi Statistica Applicata, Vol.19, No.3, pp.235-244.
- [9] VIJAYARAGHAVAN, R. (1990), "Contributions to the Study of Certain Sampling Inspection Plans by Attributes", Ph.D. Thesis, Bharathiar University, Coimbatore, India.