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EOQ Model for Weibull Deteriorating Items with Imperfect Quality and Time Varying Holding Cost under Permissable Delay in Payments

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Abstract

Many times it happens that units ordered are not of 100% good quality. A deterministic inventory model with imperfect quality is developed when the deterioration rate is Weibull distribution. Here it is assumed that holding cost is time dependent. The model has been framed to study the items whose deterioration rate increase with time under permissible delay in payments with imperfect quality. Numerical example is taken to support the model.

Keywords: Inventory, Weibull deterioration, Imperfect items, Permissible delay

AMS Subject Classification: 90B05

Introduction

Many existing inventory models in the literature assume that items can be stored indefinitely to meet the future demand. However, certain types of commodities either deteriorate or become obsolete in the course of time and hence are unstable. Therefore, if the rate of deterioration is not sufficiently low, its impact on modeling of such an inventory system cannot be ignored.

Inventory problems for deterioration items have been studied extensively by many researchers from time to time. Research in this area started with the work of Whitin [17], who considered fashion goods deteriorating at the end of prescribed storage period. Ghar and Schrader [4] developed an inventory model with a constant rate of deterioration. Shah and Jaiswal [13] considered an order level inventory model for items deteriorating at a constant rate. Raafat [11] provided a comprehensive survey on continuous deteriorating inventory models where the deterioration is considered as a function of on-hand inventory.

The classical EOQ model assumes that the retailer must be paid for the items as soon as the items were received. But in practice, for attracting more retailers, the supplier offers a delay period called trade credit period to the retailer. So the permissible delay is a common phenomenon in retailing where a supplier permits retailer a fixed time period to settle the total amount purchased. By this way, the retailer can earn interest on accumulated revenue received during the period of permissible delay. Goyal [5] developed an EOQ model under the conditions of permissible delay in payments. Mandal and Phaujdar [9] extended this issue by considering the interest earned from the sales revenue. Goyal's model was extended by Aggarwal and Jaggi [1] by considering the point that if the credit period is less than the cycle length, the customer continues to accumulate revenue and earn interest on it for the rest of the period in the cycle, from the stock remaining beyond the credit period. Teng [15] amended Goyal's [5] model by identifying the difference between unit price and unit cost. Chang et al. [2] established EOQ model with deteriorating items under supplier trade credits linked to order quantity. Shah [14] derived an inventory model by assuming constant rate of deterioration of units in inventory, time value of money under the conditions of permissible delay in payments. Huang [6] developed EOQ model in which the supplier offers a partially permissible delay in payments when the order quantity is smaller than the predetermined quantity. Ouyang et al. [10] have considered trade credit linked to order quantity for deteriorating items. Tripathy and Mishra [16] developed an inventory model for Weibull deteriorating items when delay in payments is allowed to retailer to settle the account against the purchases made by the retailer. Mahata [8] developed an EOQ model for deteriorating items with instantaneous replenishment, exponential decay rate and a time varying linear demand without shortages under permissible delay in payments.

Cheng [3] developed a model of imperfect production quantity by establishing relationship between demand dependent unit production cost and imperfect production processes. Salman and Jaber [12] developed an inventory model in which items received are of defective quality and after 100% screening, imperfect items are withdrawn from the inventory and sold at a discounted price. Jaggi et al. [7] developed an inventory model for deteriorating items with imperfect quality under permissible delay in payment.

In this paper we have developed an inventory model for Weibull deteriorating items with imperfect quality under permissible delay in payment. Shortages are not allowed. We have found optimal total profit, optimal total cost, optimal order quantity for the model. Sensitivity analysis is also carried out to observe the effect on optimal solution.

Notations and Assumptions

Following notations and assumptions are used for developing the model:

Notations

R : Rate of demand

d : defective items (%)

λ:	Screening rate
Q:	Order quantity
I(t):	Inventory level at time t
$\alpha\beta t^{\beta-1}$:	Deterioration rate, $0<\alpha<1$ and $\beta>0$.
SR :	Sales revenue
A :	Ordering cost
z :	Screening cost per unit
p	Selling price per unit
p_d	Price of defective items per unit
c	Purchasing cost per unit
I_p	Interest paid per unit per unit time
I_e	Interest earned per unit per unit time
t_1	Screening time
T	Inventory cycle length
M	Permissible delay in settling the accounts
h(t):	x+yt, Variable holding cost
$\pi_1(T)$:	Total profit for case I , ($t_1 \le M \le T$)
$\pi_2(T)$:	Total profit for case II, ($t_1 \le T \le M$)
$A\pi_1(T)$:	Average total profit for case I , ($t_1 \le M \le T$)
$A\pi_2(T)$:	Average total profit for case II , ($t_1 \le T \le M$)

Assumptions

Following are the assumptions used in the development of the model:

- The demand of the product is declining at a constant rate.
- Replenishment rate is infinite and instantaneous.
- Lead time is zero.
- Shortages are not allowed.
- Deteriorated units can neither be repaired nor replaced during the cycle time.
- During the time, the account is not settled; generated sales revenue is deposited in an interest bearing account. At the end of the credit period, the account is settled as well as the buyer pays off all units sold and starts paying for the interest charges on the items in stocks.

The Model Analysis

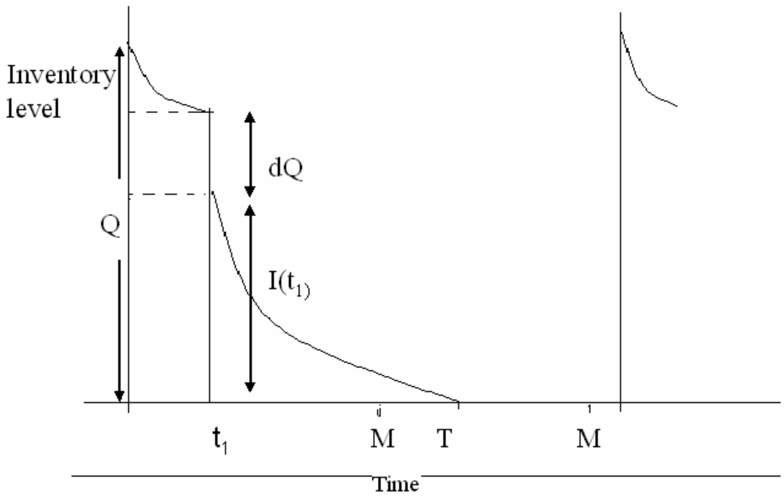
In the following situation, Q items are produced at the beginning of the period. Each lot having d % of defective items. The nature of the inventory level is shown in the given figure, where screening process is done for all the received quantity at the rate of λ units per unit time which is greater than demand rate R for the time period 0 to t_1. During the screening process the demand occurs parallel to the screening process and is fulfilled from the goods which are found to be of perfect quality by the screening process. The defective items are sold immediately after the screening process at time t_1 as a single batch at a discounted price. After the screening process at time t_1 the inventory level will be $I(t_1)$ and at time T, inventory level will become zero due to demand and partially due to deterioration.

Also here

$$t_1 = \frac{Q}{\lambda} \tag{1}$$

Defective percentage (d) is restricted to

$$d \le [1 - \frac{R}{\lambda}] \tag{2}$$



Let I(t) be the inventory at time t $(0 \le t \le T)$.

The differential equation which describes the instantaneous states of I(t) over the period (0,T) is given by

$$\frac{dI(t)}{dt} + \alpha\beta\, t^{\beta-1} I(t) = -R, \qquad 0 \le t \le T. \tag{3}$$

with the boundary conditions t = 0, I(0) = Q.

The solution of equation (3) using boundary conditions is:

$$I(t) = -R\left[t - \frac{\alpha\beta}{(\beta+1)} t^{\beta+1}\right] + \left(1 - \alpha\, t^{\beta}\right)Q. \tag{4}$$

Inventory level at t_1, including the defective items is

$$I(t_1) = -R\left[t_1 - \frac{\alpha\beta}{(\beta+1)} t_1^{\beta+1}\right] + \left(1 - \alpha t_1^{\beta}\right)Q.$$

After the screening process, the number of defective items at time t_1 is dQ.
So the effective inventory level during $t_1 \le t \le T$ is given by,

$$I(t_1) = -R\left[t_1 - \frac{\alpha\beta}{(\beta+1)} t_1^{\beta+1}\right] + \left(1 - \alpha t_1^{\beta}\right)Q - dQ,, \quad t_1 \le t \le T. \tag{5}$$

At t = T, I(t) = 0, equation (5) gives order quantity as:

$$Q = \frac{R\left[T - \frac{\alpha\beta}{\beta+1} T^{\beta+1}\right]}{\left(1 - \alpha T^{\beta} - d\right)} \tag{6}$$

The retailer's total profit during a cycle, $\pi_j(T)$, j = 1,2 consisted of the following:
$\pi_j(T)$ = Sales Revenue + Interest earned – Ordering cost – Purchasing cost
- Screening cost – Holding cost – Deterioration cost – Interest paid (7)

Individual costs are now evaluated before they are grouped together as the total profit.

1. Sales Revenue = Sum of revenue generated by the demand meet during the time period (0, T) + sales of imperfect quantity items.

$$= pRT + p_d dQ \tag{8}$$

2. Ordering cost = A (9)
3. Purchasing cost = c Q (10)
4. Screening cost = z Q (11)
5. Holding cost = $\int_0^{t_1} h(t)I(t)\,dt + \int_{t_1}^{T} h(t)I(t)\,dt$

$$= \left[\begin{array}{l} Q\left[x\left(T - \frac{\alpha}{(\beta+1)}T^{\beta+1} - dT + dt_1\right) + y\left(\frac{T^2}{2} - \frac{\alpha}{(\beta+2)}T^{\beta+2} - \frac{dT^2}{2} + \frac{dt_1^2}{2}\right)\right] \\ -R\left[x\left(\frac{T^2}{2} - \frac{\alpha\beta}{(\beta+1)(\beta+2)}T^{\beta+2}\right) + y\left(\frac{T^3}{3} - \frac{\alpha\beta}{(\beta+1)(\beta+3)}T^{\beta+3}\right)\right] \end{array}\right]. \tag{12}$$

6. Deterioration cost = DC = $c\left[Q - \int_0^T R\,dt\right] = \frac{cRT\left[\frac{\alpha}{(\beta+1)}T^{\beta} + d\right]}{\left(1 - \alpha T^{\beta} - d\right)}.$ (13)

To determine the interest payable and interest earned, there will be two cases i.e.

Case I: ($t_1 \le M \le T$) and case II: ($t_1 \le T \le M$).

Case I: ($t_1 \le M \le T$): In this case the retailer can earn interest on revenue generated from the sales up to M. Although, he has to settle the accounts at M, for that he has to arrange money at some specified rate of interest in order to get his remaining stocks financed for the period M to T.

7. Interest earned per cycle has got two parts

Part-I: In the first part, one can earn interest till the time period (M),

$$= pI_e \int_0^M Rtdt = pI_e R\frac{M^2}{2} \tag{14}$$

Part-II: Second part includes the interest earned on defective items for the Time period ($M - t_1$)

$$= p_d I_e dQ(M - t_1) \tag{15}$$

Therefore, total interest earned is

$$= pI_e R\frac{M^2}{2} + p_d I_e dQ(M - t_1) \tag{16}$$

8. Interest payable per cycle for the inventory not sold after the due period M is

$$= \begin{bmatrix} cI_p R\left[\frac{1}{2}\left(T^2 \text{-} M^2\right) \text{-} \frac{\alpha\beta}{(\beta+1)(\beta+2)}\left(T^{\beta+2} \text{-} M^{\beta+2}\right)\right] \\ \text{-}cI_p Q\left[(T\text{-}M)(1\text{-}d) \text{-} \frac{\alpha}{(\beta+1)}\left(T^{\beta+1} \text{-} M^{\beta+1}\right)\right] \end{bmatrix} \tag{17}$$

Substituting values from equation (8) to (13), and equations (16) and (17) in equation (7) the total profit becomes,

$$\pi_1(T) = \left[pRT + p_d dQ + pI_e R\frac{M^2}{2} + p_d I_e dQ(M\text{-}t_1)\right] \text{-} \left[A + cQ + zQ\right]$$
$$\text{-}\begin{bmatrix} + \begin{bmatrix} Q\left[x\left(T \text{-} \frac{\alpha}{(\beta+1)}T^{\beta+1} \text{ - } dT + dt_1\right) + y\left(\frac{T^2}{2} \text{ - } \frac{\alpha}{(\beta+2)}T^{\beta+2} \text{-} \frac{dT^2}{2} + \frac{dt_1^2}{2}\right)\right] \\ \text{-}R\left[x\left(\frac{T^2}{2} \text{-} \frac{\alpha\beta}{(\beta+1)(\beta+2)}T^{\beta+2}\right) + y\left(\frac{T^3}{3} \text{-} \frac{\alpha\beta}{(\beta+1)(\beta+3)}T^{\beta+3}\right)\right] + \frac{cRT\left[\frac{\alpha}{(\beta+1)}T^{\beta} + d\right]}{\left(1 - \alpha T^{\beta} - d\right)} \end{bmatrix} \\ + \begin{bmatrix} cI_p R\left[\frac{1}{2}\left(T^2 \text{-} M^2\right) \text{-} \frac{\alpha\beta}{(\beta+1)(\beta+2)}\left(T^{\beta+2} \text{-} M^{\beta+2}\right)\right] \\ \text{-}cI_p Q\left[(T \text{-} M)(1 \text{-} d) \text{-} \frac{\alpha}{(\beta+1)}\left(T^{\beta+1} \text{-} M^{\beta+1}\right)\right] \end{bmatrix} \end{bmatrix} \tag{18}$$

where $t_1 = \frac{Q}{\lambda}$ and $Q = \frac{R\left[T - \frac{\alpha\beta}{\beta+1}T^{\beta+1}\right]}{\left(1-\alpha T^{\beta} - d\right)}$.

Average total profit per unit time is

$$A\pi_1(T) = \frac{\pi_1(T)}{T}. \tag{19}$$

The optimal value of $T=T_1^*$ (say), which maximizes $A\pi_1(T)$ can be obtained by solving equation (19) by differentiating it with respect to T equate it to zero,

i.e. $$\frac{d(A\pi_1(T))}{dT} = 0 \tag{20}$$

provided it satisfies equation $\frac{d^2(A\pi_1(T))}{dT^2} \le 0$..

Case II: ($t_1 \le T \le M$):
In this case, the retailer earns interest on the sales revenue up to the permissible delay period and no interest is payable during this period for the items kept in stock. So

1. Interest earned per cycle has three parts:
Part-I: First part, one can earn interest till the time period T.

$$= pI_e \int_0^T Rtdt = pI_e R\frac{T^2}{2} \tag{21}$$

Part-II: Second part is having interest earned for the time period (M – T)

$$= pI_e RT(M\text{-}T) \tag{22}$$

Part-III: Second part includes the interest earned on defective items for the time period ($M - t_1$)

$$= p_d I_e dQ\left(M - t_1\right) \tag{23}$$

Hence, the total interest earned from equation (21), (22) and (23) is

$$\text{Interest earned} = pI_e R\frac{T^2}{2} + pI_e RT(M\text{-}T) + p_d I_e dQ\left(M\text{-}t_1\right) \tag{24}$$

9. Interest paid per cycle is zero. (25)

Substituting the values from equation (8) to (13) and (24) and (25) in equation (7), the total profit for case II becomes

$$\pi_2(T)=\left[pRT+p_d dQ+pI_e R\frac{T^2}{2}+pI_e RT(M-T)+p_d I_e dQ\left(M-t_1\right)\right]-\left[A+cQ+zQ\right]$$
$$-\left[+\left[\begin{array}{l}Q\left[a\left(T-\frac{\alpha}{(\beta+1)}T^{\beta+1}-dT+dt_1\right)+b\left(\frac{T^2}{2}-\frac{\alpha}{(\beta+2)}T^{\beta+2}-\frac{dT^2}{2}+\frac{dt_1^2}{2}\right)\right]\\ -R\left[a\left(\frac{T^2}{2}-\frac{\alpha\beta}{(\beta+1)(\beta+2)}T^{\beta+2}\right)+b\left(\frac{T^3}{3}-\frac{\alpha\beta}{(\beta+1)(\beta+3)}T^{\beta+3}\right)\right]\end{array}\right]+\frac{cRT\left[\frac{\alpha}{(\beta+1)}T^{\beta}+d\right]}{\left(1-\alpha T^{\beta}-d\right)} \atop +\left[\begin{array}{l}cI_pR\left[\frac{1}{2}\left(T^2-M^2\right)-\frac{\alpha\beta}{(\beta+1)(\beta+2)}\left(T^{\beta+2}-M^{\beta+2}\right)\right]\\ -cI_pQ\left[(T-M-d)-\frac{\alpha}{(\beta+1)}\left(T^{\beta+1}-M^{\beta+1}\right)\right]\end{array}\right]\right] \qquad (26)$$

Average total profit per unit time is $A\pi_2(T)=\frac{\pi_2(T)}{T}$. (27)

The optimal value of $T=T_1^*$ (say), which maximizes $A\pi_2(T)$ can be obtained by solving equation (26) by differentiating it with respect to T equate it to zero,

i.e. $\frac{d(A\pi_2(T))}{dT}=0$

(20) provided it satisfies equation $\frac{d^2(A\pi_2(T))}{dT^2}\le 0$..

Numerical Example

Case I: Considering x = 240 units per year, y=2 units per year, A= Rs 75 units per year, c = Rs. 20 per unit, p = Rs 40 per unit, I_p = Rs 0.12 per year, I_e=0.08 per year, M = 0.01 years, α = 0.05, β= 2, z = 0.4, d = 0.02, p_d= 15, λ = 10,000 and R=2500 in appropriate unit. Then we obtained the optimal value of T=0.0156, t_1=.0016 and the optimal total profit $\pi_1(T^*)$ = Rs. 599.2365 and the optimum order quantity Q*= 40.0001.

Case II: Considering x = 240 units per year, y = 2 units per year, A= Rs 75 units per year, c = Rs. 20 per unit, p = Rs 40 per unit, I_p = Rs 0.12 per year, I_e=0.08 per year, M = 0.05 years, α = 0.05, β = 2 , z= 0.4, d = 0.02, p_d= 15, λ = 10,000 and R= 2500 in appropriate unit. Then we obtained the optimal value of T= 0.0156, t_1= 0.0039 and the optimal total profit $\pi_2(T^*)$ = Rs. 600.7309 and the optimum order quantity Q*= 39.7960.

Sensitivity Analysis

On the basis of the data given in example above we have done the sensitivity analysis by changing one parameter at a time and keeping other parameters fixed.

Sensitivity Analysis Table for Case I ($t_1 \leq M \leq T$)

R	% change	T	t_1	$\pi_1(T)$	Q*
	+40%	0.0132	0.0047	733.0232	47.1430
	+20%	0.0143	0.0044	669.4708	43.7756
	-20%	0.0175	0.0036	518.9187	35.7145
	-40%	0.0202	0.0031	429.3104	30.9185
x	+40%	0.0132	0.0034	481.1137	33.6735
	+20%	0.0143	0.0036	533.1376	36.4797
	-20%	0.0175	0.0045	685.7725	44.6431
	-40%	0.0201	0.0051	810.455	51.2758
y	+40%	0.0156	0.0039	595.7308	39.7961
	+20%	0.0156	0.0039	595.7314	39.7961
	-20%	0.0156	0.0039	595.7327	39.7961
	-40%	0.0156	0.0039	595.7333	39.7961

Sensitivity Analysis Table for Case II ($t_1 \leq T \leq M$)

R	% change	T	t_1	$\pi_2(T)$	Q*
	+40%	0.0132	0.0047	738.7858	47.1430
	+20%	0.0142	0.0043	670.2763	43.4695
	-20%	0.0175	0.0036	335.4638	35.7145
	-40%	0.0202	0.0031	433.1561	30.9185
x	+40%	0.0132	0.0034	485.3581	33.6735
	+20%	0.0143	0.0036	537.7301	36.4797
	-20%	0.0175	0.0044	687.4218	44.3879
	-40%	0.0201	0.0051	816.8342	51.2758
y	+40%	0.0156	0.0039	600.7296	39.7961
	+20%	0.0156	0.0039	600.7303	39.7961
	-20%	0.0156	0.0039	600.7315	39.7961
	-40%	0.0156	0.0039	600.7323	39.7961

Here we observe that by increasing/decreasing in R, there is corresponding increase/decrease in profit and optimum order quantity both for case I and case II respectively.

Similarly, with increase/decrease in holding cost (for value of x), there is corresponding decrease/increase in profit and optimum order quantity both for case I and case II respectively.

For y there is no change in profit and optimum order quantity both for case I and case II respectively.

Moreover, we have also verified that increase/decrease in M, α and β there is no change in profit and optimum order quantity both for case I and case II respectively.

Conclusion

In this model we have derived a profit maximization imperfect quality model for deteriorating items for a retailer when the deterioration is two parameters Weibull deterioration with declining demand under permissible delay in payments. With increase or decrease in demand there is corresponding increase and decrease in profit and optimum order quantity. Also with increase or decrease in holding cost there will be decrease or increase in corresponding profit and optimum order quantity.

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