Global Journal of Mathematical Sciences: Theory and Practical.
ISSN 0974-3200 Volume 4, Number 3 (2012), pp. 313-318
© International Research Publication House
http://www.irphouse.com

"Double Inequalities for the Fox's H-function"

A.K. Ronghe, Kulwant Kaur and Arati Baurase

Department of Mathematics, S.S.L. Jain P.G. College, Vidisha, M.P., India
E-mail: dr.ashokrao@yahoo.in, akshada.ronghe@gmail.com,
kulwant.ahluwalia34@gmail.com

Abstract

In the present paper, we establish some double inequalities for h-function of one variable with the help of Nguyen Van h and Ngo Phuoc Nguyen Ngo [4] inequality.

Mathematics Subject Classification: 33D99.

Keywords: Fox's H-function Gamma function, inequalities,

Introduction

The H-function [3, sec mathai and saxena (1978)]
Braaksma (1964) is defined by,

$$= \frac{1}{2\pi\varpi} \int \frac{\underset{i=1}{\overset{d}{\pi}} \Gamma(b_j - B_j S) \underset{i=1}{\overset{u}{\pi}} \Gamma(1 - a_j + A_j S)}{\underset{j=1+d}{\overset{q}{\pi}} \Gamma(1 - b_j + B_j S) \underset{j=u+1}{\overset{p}{\pi}} (a_j - A_j S)} \tag{1.1}$$

where

1) $\varpi = \sqrt{-1}$
2) $z\,(\neq 0)$ Is complex variable
3) $z^s = \exp(s(\ln)|z| + i \arg z)$
4) An empty product is interpreted as unity,
5) *d*, u, p and q are non- negative integers satisfying $0 \le u \le p, 0 \le d \le q$ (both *d* and u are not zeroes).
6) $A_j(i = 1, ..., p)$ and $B_j(i = 1, ..., q)$ are assumed to be positive quantities,

7) $A_j (i = 1,....,d)$ and $b_j (j = 1,....,q)$ are complex numbers such that none of the poles of $A_j \Gamma(b_j - B_j S), (i = 1,...d)$ coincide with the poles of $\Gamma\ 1 - a_j$, $(i = 1,...,u)$ i.e., A_k
$(b_l + v) \neq B_{hd}(a_k - \lambda - 1)$ for $V, \lambda = 1,....., h = 1,......d, k = 1,.....u,$

In Braakama [(1964) P278]. In has been shown that the H-function make sense and define an analytic function of z in the following cases.

(1) $\delta > 0,\ z \neq 0$ where

$$\delta = \sum_{i=1}^{q} B_j - \sum_{i-1}^{b} A_j ,$$

(2) $\delta = 0$ and $0 < |z| < D^{-1}$ where,

$$\delta = \frac{\pi_{i=1}^{p} A_j^{A_j}}{\pi_{i=1}^{q} B_j^{B_j}}$$

And

(3) $|\arg(z)| < \frac{1}{2}\phi\pi$,

Where

$$\phi = \left[\sum_{1}^{u} (A_j) - \sum_{u+1}^{p} (A_j) + \sum_{1}^{d} (B_j) - \sum_{d+1}^{q} (B_j) \right] > 0$$

Double Inequality for the Fox's H-function

(1) 1st Inequality:-

$$H^{d,\ u+n}_{p+n,q+1}\left[z \left| \begin{matrix} (\alpha_i, u_i)_{i=1,n}, \{a_p . A_p\} \\ \{(b_q . B_q)\}, (-\beta - \sum_{i=1}^{n} a_i : u + \sum_{i=1}^{n} (u_i) \end{matrix} \right. \right]$$

$$\leq H^{d,\quad u+n}_{p+n,q+1}\left[z \left| \begin{matrix} (-\alpha_i, xu_i)_{i=1,n'}\{a_p.A_p\} \\ \{(b_q.B_q)\},(-\beta - \sum_{i=1}^{n} a_i : u + \sum_{i=1}^{n}(u_i)) \end{matrix} \right. \right]$$

Eq. cont. to next page

$$\leq H^{d,\quad u+n}_{p,q+1}\left[z \left| \begin{matrix} \{a_p.A_p\} \\ \{(b_q.B_q)\},(1-\beta, u) \end{matrix} \right. \right]$$

Provided; $x \in [0,1)$,

$$\operatorname{Re}\left[\beta + \min_{1\leq j\leq d} u\left(\frac{b_j}{B_i}\right)\right] > 1,$$

$$\operatorname{Re}\left[\alpha_i + \min_{1\leq j\leq d} u_i\left(\frac{b_j}{B_i}\right)\right] > 0,$$

$$|\arg(z)|_l < \frac{1}{2}\phi\pi,$$

for j=1,......*l*, i=1,……n and $n\in N$, (2.1)

(2) 2nd Inequality

$$H^{d+n,\quad u}_{p+1,q+n}\left[z \left| \begin{matrix} \{(a_p.A_p)\},(1+a_i,u_i)_{i=1,n} \\ \left(\beta + \sum_{i=1}^{n} a_i ; u + \sum_{i=1}^{n} u_i\right)\{(b_q.B_q)\} \end{matrix} \right. \right]$$

$$\leq H^{d+n,\quad u}_{p+1,\, q+n}\left[z \left| \begin{matrix} \{(a_p.A_p)\},(1+a_i x, u_i x)_{i=1,n} \\ \left(\beta + \sum_{i=1}^{n} (a_i).x ; u + \sum_{i=1}^{n} u_i\right) x,\{(b_q.B_q)\} \end{matrix} \right. \right]$$

$$\leq H^{d,u}_{p+1,\,q}\left[z \left| \begin{matrix} \{(a_p.A_p)\} \\ (\beta, u),\{(b_q.B_q)\} \end{matrix} \right. \right]$$

Eq. cont. to next page

Provided: - $x \in [0,1)$,

$$\mathrm{Re}\left[a_i - \min_{1\le j\le d} u_i\left(\frac{b_j}{B_j}\right)\right] > 0$$

$$\mathrm{Re}\left[\beta - \min_{1\le j\le d} u\left(\frac{b_j}{B_j}\right)\right] > 0,$$

$$|\arg(z)| < \frac{1}{2}\phi\pi,$$

For, k = 1,……d, i = 1,….,n and $n \in N$, (2.2)

(3) 3rd Inequality

$$H^{d,\ u+n}_{p+n+1,\ q}\left[z\left|\begin{matrix}(-a_i, u_i)_{i=1,n}\{(a_p.A_p)\} \\ \left(\beta+\sum_{i=1}^{n} a_i' u+\sum_{i=1}^{n} u_i\right),\{(b_q.B_q)\},\left(-\beta-\sum_{i=1}^{n} a' u_i+\sum_{i=1}^{n}(u_i)\right)\end{matrix}\right.\right]$$

$$\le H^{d,\ u+n}_{p+n+1,\ q}\left[z\left|\begin{matrix}(-a_i x, x u_i)_{i=1,n},\{(a_p.A_p)\} \\ \beta+x\sum_{i=1}^{n} a_i'; x\left(u+\sum_{i=1}^{n} u_i\right),\{(b_q.B_q)\},\left(-\beta+x\sum_{i=1}^{n} u_i\right)\end{matrix}\right.\right]$$

$$\le H^{d,u}_{p+1,\ q}\left[z\left|\begin{matrix}\{(a_p.A_p)\} \\ (\beta, u),\{(b_q.B_q)\}\end{matrix}\right.\right]$$

Provided: - $x \in [0,1)$,

$$\mathrm{Re}\left[\beta - \min_{1\le j\le d} u\left(\frac{b_j}{B_j}\right)\right] > 1,$$

Eq. cont. to next page.

$$\mathrm{Re}\left[a_i + \min_{1\le j\le d} u_i\left(\frac{b_j}{B_j}\right)\right] > 0,$$

$$|\arg(z)| < \frac{1}{2}\phi\pi,$$

$\beta \geq 1, a_i > 0$, for k= 1,…….,d. i=1,….n, and $n \in N$, (2.3)

(4) 4th Inequality

$$H^{d+n,\ \ u}_{p+1,q+n}\left[z \left| \begin{matrix} \{(a_p.A_p)\},(1+a_i;u_i)_{i=1,n} \\ \{(b_q.B_q)\},\left(-\beta-\sum_{i=1}^{n} a_i;u+\sum_{i=1}^{n}(u_i)\right) \end{matrix} \right. \right]$$

$$\leq H^{d,\ \ u+n}_{p+n+1,\ q}\left[z \left| \begin{matrix} \{(a_p.A_p)\},(1+a_i x;u_i x)_{i=1,n} \\ \{(b_q.B_q)\},\left(-\beta-x\sum_{i=1}^{n} a_i;x\left(u+\sum_{i=1}^{n} u_i\right)\right) \end{matrix} \right. \right]$$

$$\leq H^{d,\ \ u+n}_{p,q+1}\left[z \left| \begin{matrix} \{a_p.A_p\} \\ \{(b_q.B_q)\},(1-\beta,u) \end{matrix} \right. \right]$$

Provided: $x \in [0,1)$,

$$\mathrm{Re}\left[a_i - \min_{1\leq j\leq d} u_i \left(\frac{b_j}{B_j}\right)\right] > 0$$

$$\mathrm{Re}\left[\beta + \min_{1\leq j\leq d} u \left(\frac{b_j}{B_j}\right)\right] > 1,$$

$$\left|\arg(z)\right| < \frac{1}{2}\phi\pi,$$

For k = 1,……..,d. i= 1,…….,n and $n \in N$, (2.4)

Proof

To establish (2.1) replacing the Fox's H-Function in the inequality as a Mellin-Barnes type of contour intregal which is permissible due to absolute convergence of involved in the process. We have,

$$= \frac{1}{2\pi\varpi}\int \frac{\underset{i=1}{\overset{d}{\pi}}\Gamma(b_j - B_j S)\underset{i=1}{\overset{u}{\pi}}\Gamma(1-a_j+A_j S)}{\underset{j=1+d}{\overset{q}{\pi}}\Gamma(1-b_j+B_j S)\underset{j=u+1}{\overset{p}{\pi}}(a_j - A_j S)} \times$$

$$\left\{\begin{array}{l} \dfrac{\left[\Gamma 1-(-\alpha_i)+u_i S)_{i=1,n}\right]}{\left[1-(-\beta-\sum\limits_{i=1}^{n}(a_i)+(u+u_i)S)\right]} \\ \le \dfrac{\Gamma\left[1-(-\alpha_i x)+u_i x S)_{i=1,n}\right]}{\Gamma\left[\beta+us+x\sum\limits_{i=1}^{n}(\alpha_i+u_i S)_{i=1,n}\right]} \\ \le \dfrac{1}{\Gamma 1-(1-\beta)+us} \end{array}\right\} z^S dS$$

Now evaluating the inner inequality with the help of result Nguyen Van Vins and Ngo Phuoc Nguyen Ngoc [4,p 1381], and interpreting the result. Thus obtained with the help of(1.1), we obtain the RHS of (2.1) .

References

[1] Braaksma, B.L.U (1964) - Asymptobic Expansion and Analytics combination for a class of Barnes intergral. Cmp. Math. 239-341

[2] C. Alsina, M.S. Toms (2005) A geometrical proof of a new inequality for the Gamma function J.Jneq. Pure, Applied Mathematics 62 Art 48.

[3] Mathi, A.M. Saxena, R.K (1978) The H-function with application in statistics and other Disciplines.

[4] Nyuyen Van Vinh and An inequality for the gamma function. Ngo Phuoc Nyuyen Ngoc (2009) International mathematical form4 no. 28 .1379-1382.

[5] Ronghe, A.K. and Application of Inequality for the I-function Kaur, kulwant (2012) vijjan. Pari. Anush. Patrika Volume 55. No.1 PP 3-10.