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On Generalised ϕ-Recurrent $N(k)$-Contact Metric Manifolds

Venkatesha and S. Shashikala

Department of Mathematics, Kuvempu University
Shankaraghatta-577451, Shimoga, Karnataka, India.
E-mail: vensmath@gmail.com; Shashikalasampath@yahoo.com

Abstract

In this paper we have studied Generalised ϕ-recurrent $N(k)$-contact metric manifolds and obtained some important results.

Keywords: Contact metric manifold, Generalised ϕ-recurrent, $N(k)$-contact metric manifolds, Einstein manifold.

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Introduction

Let M be an (2n + 1)-dimensional connected Riemannian manifold with Riemannian metric g and Levi-Civita connection ∇ . M is called locally symmetric if its curvature tensor is parallel with respect to ∇. The notion of local symmetry has been weakend by many authors in different ways such as recurrent manifold by Walker[26], semi symmetric manifold by Szabo[18], pseudo-symmetric manifold by Chaki [13], and Deszcz[16], weakly symmetric manifold by Tammasy and Binh[25], and Selberg[17]. As a weaker version of local symmetry, Takahashi [23] introduced the notion of local ϕ-symmetry on a Sasakian manifold. Extending this notion of local ϕ-symmetry, Takahashi[23] and De et al [14] introduced and studied the notion of ϕ -recurrent Sasakian manifolds. Extending the notion of ϕ-recurrency, Generalised ϕ-recurrent manifolds were studied by many geometers in their papers ([1], [3], [4], [15], [21], [22]).

In the present paper we study Generalised ϕ-recurrent $N(k)$ -contact metric manifold. The paper is organized as follows :

Section 2 contains necessary basic details about contact metric manifolds, (k,μ) manifolds and $N(k)$ contact meric manifold. In Section 3, we have proved that a Generalised ϕ-recurrent $N(k)$-contact metric manifold is an η-Einstein manifold with constant coefficients. Further it is shown that in a Generalised φ-recurrent $N(k)$ −contact metric manifold the characteristic vector field ξ and the vector field ρ associated to the 1-form A are co-directional. Finally in Section 4, we have studied 3-dimensional Generalised φ-recurrent $N(k)$ −contact metric manifold and it is shown that such a manifold is of constant curvature.

Contact Metric Manifolds

A $(2n+1)$-dimensional manifold M is said to admit an almost contact structure if it admits a tensor field φ of type $(1,1)$, a vector field ξ and a 1-form η satisfying:

(2.1) (a) $\varphi^2(X) = -X + \eta(X)\,\xi$, (b) $\eta(\xi) = 1$, (c) $\eta o\phi = 0$, (d) $\varphi\xi = 0$.

An almost contact metric structure is said to be normal if the induced almost complex structure J on the product manifold M × R defined by

$$J(X, f\frac{d}{dt}) = (\varphi X - f\xi,\ \eta(X)\frac{d}{dt})$$

is integrable, where X is tangent to M, t is the coordinate of $\boldsymbol{R}$ and f is a smooth function on $M \times \boldsymbol{R}$. Let g be a compatible Riemannian metric with almost contact structure (φ, ξ, η), that is,

(2.2) $g(\varphi X, \varphi Y) = g(X, Y) - \eta(X)\eta(Y)$,

then $\boldsymbol{M}$ becomes an almost contact metric manifold equipped with an almost contact structure (φ, ξ, η, g). From (2.1) it can be easily seen that

(2.3) (a) $g(X, \varphi Y) = -g(\varphi X, Y)$, $(b)\ g(X, \xi) = \eta(X)$

for all vector fields X, Y. An almost contact metric structure becomes a contact metric structure if

(2.4) $g(X, \varphi Y) = d\eta(X, Y)$,

for all vector fields X, Y. The 1-form η is then a contact form and ξ is its characteristic vector field. We define a $(1,1)$ tensor field h by $h = \frac{1}{2}£_\xi\phi$ where £ denotes the Lie-differentiation, then h is symmetric and satisfies $h\varphi = -\varphi h$. We have $Tr.h = Tr.\varphi h = 0$ and $h\xi = 0$. Also,

(2.5) $\nabla_X\xi = -\varphi X - \varphi hX$,

holds in a contact metric manifold. A normal contact metric manifold is a sasakian manifold. An almost contact metric manifold is Sasakian if and only if

(2.6) $(\nabla_X\varphi)(Y) = g(X, Y)\xi - \eta(Y)X,\quad X, Y \in TM$,

where ∇ is the Levi-Civita connection of the Riemannian metric g. A contact metric manifold $M\ (\varphi, \xi, \eta, g)$ for which ξ is a killing vector is said to be a K-contact manifold. It is well known that the tangent sphere bundle of a flat Riemannian manifold admits a contact metric structure satisfying $R(X,Y)\xi = 0$ [5]. On the other hand, on a Sasakian manifold the following holds

(2.7) $$R(X,Y)\xi = \eta(Y)X - \eta(X)Y.$$

As a generalisation of both $R(X,Y)\xi = 0$ and the Sasakian case; D.Blair, T. Koufogiorgos and B.J. Papantoniou[9] considered the (k,μ) −nullity condition on a contact metric manifold and gave several reasons for studying it. The (k,μ)- nullity distribution $N(k,\mu)$ ([6], [7]) of a Contact metric manifold M is defined by

$$N(k,\mu) : p \to Np(k,\mu)$$
$$= \{W \in TpM : R(X,Y)W = (kI + \mu h)[g(Y,W)X - g(X,W)Y]\},$$

for all $X,Y \in TM$, where $(k,\mu) \in R^2$. A contact metric manifold M with $\xi \in N(k,\mu)$ is called a (k,μ) −manifold. In particular on a (k,μ) −manifold, we have

(2.8) $$R(X,Y)\xi = k[\eta(Y)X - \eta(X)Y] + \mu[\eta(Y)hX - (\eta X)hY].$$

On a (k,μ)-manifold $k \leq 1$. If $k = 1$, the structure is Sasakian (h = 0 and μ is indeterminate) and if $k < 1$, the (k, μ)-nullity condition determines the curvature of M completely[6]. Infact, for a (k, μ)-manifold, the condition of being a Sasakian manifold, k-contact manifold, $k = 1$ and $h = 0$ are all equivalent. In a (k,μ) −manifold the following relations hold ([6], [8]) :

(2.9) $$h^2 = (k - 1)\varphi^2,\ k \leq 1$$

(2.10) $$(\nabla_X\varphi)(Y) = g(X + hX, Y)\xi - \eta(Y)(X + hX),$$

(2.11) $$R(\xi,X)Y = k[g(X,Y)\xi - \eta(Y)X] + \mu[g(hX, Y)\xi - \eta(Y)hX],$$

(2.12) $$S(X,\xi) = 2nk\eta(X),$$

(2.13) $$S(X,Y) = [2(n - 1) - n\mu]g(X,Y) + [2(n - 1) + \mu]g(hX,Y)$$
$$+ [2(1 - n) + n(2k + \mu)]\eta(X)\eta(Y),\ n \geq 1,$$

(2.14) $$r = 2n(2n - 2 + k - n\mu),$$

(2.15) $$S(\varphi X,\varphi Y) = S(X,Y) - 2nk\eta(X)\eta(Y) - 2(2n - 2 + \mu)g(hX, Y),$$

where S is the Ricci tensor of type (0, 2), Q is the Ricci-operator, that is $g(QX,Y) = S(X,Y)$ and r is the scalar curvature of the manifold. From (2.5), it follows that

(2.16) $$(\nabla_X\eta)(Y) = g(X + hX,\varphi Y).$$

Also in a (k,μ) −manifold,

(2.17) $$\eta(R(X,Y)Z) = k[g(Y,Z)\eta(X) - g(X,Z)\eta(Y)]$$
$$+ \mu[g(hY,Z)\eta(X) - g(hX,Z)\eta(Y)]$$

holds.

The k-nullity distribution $N(k)$ of a Riemannian manifold M [9] is defined by

$$N(k): p \rightarrow Np(k) = \{Z \in TpM : R(X,Y)Z = g(Y,Z)X - g(X,Z)Y\},$$

k being a constant. If the characteristic vector field $\xi \in N(k)$, then we call a contact metric manifold an $N(k)-$contact metric manifold [10] . If $k = 1$, then $N(k)-$contact metric manifold is Sasakian and if $k = 0$, then $N(k)$-contact metric manifold is locally isometric to the product $E^{n+1} \times S^n(4)$ for $n > 1$ and flat for $n = 1$. If $k < 1$, the scalar curvature is $r = 2n(2n-2+k)$. If $\mu = 0$, then $(k,\mu)-$contact metric manifold reduces to a $N(k)-$contact metric manifold. In a $N(k)-$contact metric manifold the following relations hold :

(2.18) $$h^2 = (k-1)\varphi^2, \ k \leq 1$$

(2.19) $$(\nabla_x \varphi)(Y) = g(X+hX, Y)\xi - \eta(Y)(X+hX),$$

(2.20) $$R(\xi, X)Y = k[g(X,Y)\xi - \eta(Y)X],$$

(2.21) $$S(X,\xi) = 2nk\eta(X),$$

(2.22) $$S(X,Y) = [2(n-1]g(X,Y) + [2(n-1)]g(hX,Y)$$

(2.23) $$+[2(1-n)+2nk]\eta(X)\eta(Y), \ n \geq 1$$

(2.24) $$r = 2n(2n-2+k),$$

(2.25) $$S(\varphi X, \varphi Y) = S(X,Y) - 2nk\eta(X)\eta(Y) - 4(n-1)g(hX, Y),$$

(2.26) $$(\nabla_x \eta)(Y) = g(X+hx, \varphi Y),$$

(2.27) $$R(X,Y)\xi = k[\eta(Y)X - \eta(X)Y],$$

(2.28) $$\eta(R(X,Y)Z) = k[g(Y,Z)\eta(X) - g(X,Z)\eta(Y)$$

Generalized φ-recurrent $N(k)-$contact metric manifolds

Definition 3.1. A $N(k)-$contact metric manifold is said to be locally φ-symmetric if the relation

$$\varphi^2((\nabla_w R)(X,Y)Z) = 0,$$

holds for all vector fields X, Y, Z, W orthogonal to ξ.

Definition 3.2. A $N(k)-$contact metric manifold is said to be φ -recurrent if and only if there exists a non-zero 1-form A such that

$$\varphi^2((\nabla_w R)(X,Y)Z) = A(W)R(X,Y)Z,$$

for all vector fields X, Y, Z and W. Here X, Y, Z and W are arbitrary vector fields which are not necessarily orthogonal to ξ.

Definition 3.3. A $N(k)$ −contact metric manifold is said to be generalised φ-recurrent if and only if there exists a non-zero 1-form A such that

(3.1) $$\varphi^2((\nabla_w R)(X,Y)Z) = A(W)R(X,Y)Z + B(W)[g(Y,Z)X - g(X,Z)Y],$$

for all vector fields X, Y, Z and W. Here X, Y, Z and W are arbitrary vector fields which are not necessarily orthogonal to ξ.

Definition 3.4. A Contact manifold is said to be η -Einstein if the Ricci tensor S of type (0, 2) satisfies the condition

(3.2) $$S(X,Y) = ag(X,Y) + b\,\eta(X)\,\eta(Y),$$

where a and b are smooth functions on M.

Theorem 3.1. A Generalized φ-recurrent $N(k)$ −contact metric manifold is an $\eta -$ Einstein manifold with constant coefficients.

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Proof. By virtue of (2.1) (a) and (3.1) we have

(3.3) $$-((\nabla_w R)(X,Y)Z) + \eta((\nabla_w R)(X,Y)Z)\xi = A(W)R(X,Y)Z + B(W)[g(Y,Z)X - g(X,Z)Y],$$

from which it follows that

(3.4) $$-g((\nabla_w R)(X,Y)Z,U) + \eta((\nabla_w R)(X,Y)Z)\,\eta(U) = A(W)g(R(X,Y)Z,U) + B(W)[g(Y,Z)g(X,U) - g(X,Z)g(Y,U)].$$

Let $\{ei\}$, i=1, 2, ..., 2n+1 be an orthonormal basis of the tangent space at any point of the manifold. Then putting $X = U = \{ei\}$ in (3.4) and taking summation over i, $1 \leq i \leq 2n+1$, we get

(3.5) $$-(\nabla_w S)(Y,Z) + \sum_{i=1}^{2n+1} \eta\big((\nabla_w R)(e_i,Y)Z\big)\eta(e_i) = A(W)S(Y,Z) + 2nB(W)g(Y,Z).$$

The second term of (3.5) by putting $Z = \xi$ takes the form $g((\nabla_w R)(ei,Y)\,\xi,\xi)g(ei,\xi)$, which is denoted by E. In this case E vanishes. Namely we have

$$g((\nabla_w R)(ei,Y)\,\xi,\xi) = g((\nabla_w R(ei,Y)\,\xi,\xi) - g(R(\nabla_w ei,Y)\,\xi,\xi) - g(R(ei,\nabla_w Y)\xi,\xi) - g(R(ei,Y)\nabla_w \xi,\xi)$$

Using (2.3) (b) and (2.27) we obtain

$g(R(ei,Y)\,\xi,\xi)$= g($k[\eta$ ($\nabla_w Y$)ei − η (ei) ∇_w Y], ξ)

$= k[\eta(\nabla_w Y)\eta(ei)-\eta(ei)\eta(\nabla_w Y)] = 0.$

Thus we obtain

$g((\nabla_w R)(ei,Y)\xi,\xi) = g(\nabla_w R(ei,Y)\xi,\xi) - g(R(ei,Y)\nabla_w\xi,\xi).$

In view of $g(R(ei,Y)\xi,\xi) = g(R(\xi,\xi)ei,Y) = 0$, we have

$g(\nabla_w R(ei,Y)\xi, \xi) + g(R(ei,Y)\xi,\nabla_w\xi) = 0$, since $(\nabla_w g) = 0$,

which implies

$g((\nabla_w R(ei,Y)\xi,\xi) = -g(R(ei,Y)\xi,\nabla_w\xi) - g(R(ei,Y)\nabla_w\xi,\xi) = 0$

Using (2.5) and applying skew-symmetry of R we get

$g\big((\nabla_w R)(ei,Y)\xi,\xi\big) = g(R(ei,Y)\xi,\varphi W + \varphi hW)$

$+ g(R(ei,Y)(\varphi W + \varphi hW),\xi)$

$= g(R(\varphi W + \varphi hW,\xi)Y,ei) + g(R(\xi,\varphi W + \varphi hW)Y,ei).$

Hence we obtain

E=$\sum_{i=1}^{2n+1}[g(R(\varphi hW,\xi)Y,ei)g(\xi,ei)$

$+ g(R(\xi,\varphi W + \varphi hW)Y,ei)g(\xi,ei)] = 0.$

Replacing Z by ξ in (3.5) and using (2.21) we get

(3.6) $-(\nabla_w S)(Y,\xi) = 2nkA(W)\eta(Y) + 2nB(W)\eta(Y).$

Now we have

$(\nabla_w S)(Y,\xi) = \nabla_w S(Y,\xi) - S(\nabla_w Y,\xi) - S(Y,\nabla_w\xi).$

Using (2.21) and (2.5) in the above relation, it follows that

(3.7) $(\nabla_w S)(Y,\xi) = 2nk(\nabla_w\eta)(Y) + S(Y,\varphi W + \varphi hW).$

In virtue of (3.7),(2.26) and (2.3) (a) we get

(3.8) $(\nabla_w S)(Y,\xi) = -2nkg(\varphi W + \varphi hW,Y) + S(Y,\varphi W + \varphi hW).$

By (3.6) and (3.8) we have

(3.9) $2nkg(\varphi W + \varphi hW,Y) - S(Y,\varphi W + \varphi hW) = 2nkA(W)\eta(Y)$

$+ 2nB(W)\eta(Y).$

Replacing Y by φY in (3.9) and using (2.1) (d), (2.2), (2.25) we get

$2nkg(Y,W) + 2nkg(Y,hW) - S(Y,W) - S(Y,hW)$

$$+ 4(n - 1)g(Y, hW) + 4(n - 1)g(Y, h^2W) = 0,$$

since, $g(X, hY) = g(hX, Y)$.

Now by (2.23), (2.18) and (2.1) (a) this implies

$$S(Y, W) + 2(n - 1)g(Y, hW) - 2(n - 1)(k - 1)g(Y, W)$$
$$+ 2(n - 1)(k - 1)\eta(Y)\eta(W) = [2nk - 4(n - 1)(k - 1)]g(Y, W)$$
$$+ [2nk + 4(n - 1)]g(Y, hW) + 4(n - 1)(k - 1)\eta(Y)\eta(W),$$

which gives,

(3.10) $$S(Y, W) = 2(n + k - 1)g(Y, W) + 2(nk + n - 1)g(Y, hW)$$
$$+2(n - 1)$$
$$(k - 1)\eta(Y)\eta(W).$$

Replacing W by hW and using (2.23), (2.18) and (2.1) (a) we get from (3.10) that

$$-2kg(Y, hW) = -2nk(k - 1)g(Y, W) + 2nk(k - 1)\eta(Y)\eta(W).$$

Since we may assume that $k \neq 0$, and so

(3.11) $$g(Y, hW) = n(k - 1)g(Y, W) - n(k - 1)\eta(Y)\eta(W)$$

From (3.10) and (3.11) we get

(3.12) $$S(Y, W) = ag(Y, W) + b\,\eta(Y)\eta(W),$$

Where

$$a = 2[(n + k - 1)] + n(k - 1)(nk + n - 1),$$
$$b = 2[(n - 1)(k - 1) - n(k - 1)(nk + n - 1)]$$

are constants. So, the manifold is an η-Einstein manifold with constant coefficients. Hence the theorem is proved.

Now, from (3.3) we have

(3.13) $$(\nabla_w R)(X, Y)Z = \eta((\nabla_w R)(X, Y)Z))\xi - A(W)R(X, Y)Z$$
$$- B(W)[g(Y, Z)X - g(X, Z)Y].$$

From (3.13) and the second Bianchi identity we get

(3.14) $$A(W)\eta(R(X, Y)Z) + B(W)[g(Y, Z)\eta(X) - g(X, Z)\eta(Y)] +$$
$$A(X)\eta(R(Y, W)Z) + B(X)[g(W, Z)\eta(Y) - g(Y, Z)\eta(W)] +$$
$$A(Y)\eta(R(W, X)Z) + B(Y)[g(X, Z)\eta(W) - g(W, Z)\eta(X)] =$$
$$0$$

Using (2.28) we get from (3.14) that

(3.15) $$k[A(W)(g(Y,Z)\,\eta\,(X) - g(X,Z)\,\eta(Y\,))] + [B(W)(g(Y,Z)\,\eta\,(X) - g(X,Z)\,\eta\,(Y\,))] + k\big[A(X)\big(g(W,Z)\eta\,(Y\,) - g(Y,Z)\eta\,(W)\big)\big] + [B(X)(g(W,Z)\,\eta\,(Y\,) - g(Y,Z)\eta\,(W))] + k\big[A(Y\,)\big(g(X,Z)\eta\,(W) - g(W,Z)\eta\,(X)\big)\big] + \big[B(Y\,)\big(g(X,Z)\eta\,(W) - g(W,Z)\eta\,(X)\big)\big] = 0.$$

Putting $Y = Z = \{ei\}$ in (3.15) and taking summation over i, $1 \le i \le 2n + 1$, we get

(3.16) $$[kA(W) + B(W)]\,\eta(X) = [kA(X) + B(X)]\,\eta(W),$$

Replacing X by ξ in (3.16), it follows that

(3.17) $$[kA(W) + B(W)] = [k\,\eta(\rho_1) + \eta(\rho_2)]\,\eta(W)$$

for any vector field W, where $A(\xi) = g(\xi,\rho) = \eta\,(\rho), \rho$ being the vector field associated to the 1-form A, that is, $g(X,\rho\,) = A(X)$. Hence we state the following theorem:

Theorem 3.2. In a Generalised φ-recurrent $N(k)$ −contact metric manifold (M,g), $n > 1$, the characteristic vector field ξ and the vector field ρ associated to the 1-form A are co-directional and the 1-form A is given by (3.17).

3-dimensional Generalised φ-recurrent $N(k)$ −contact metric Manifolds

In a 3-dimensional Riemannian manifold we have

(4.1) $$R(X,Y\,)Z = g(Y,Z)QX - g(X,Z)QY + S(Y,Z)X - S(X,Z)Y + \frac{r}{2}\,[g(X,Z)Y - g(Y,Z)X],$$

where Q is the Ricci-operator, that is, $g(QX,Y) = S(X,Y)$ and r is the scalar curvature of the manifold. Now putting $Z = \xi$ in (4.1) and using (2.3) (b) and (2.21) we get

(4.2) $$R(X,Y\,)\xi = \eta\,(Y)QX - \eta\,(X)QY + 2k[\eta\,(Y)X - \eta\,(X)Y\,] + \frac{r}{2}\,[\eta(X)Y - \eta\,(Y)X]$$

Using (2.27) in (4.2), we have

(4.3) $$(k - \frac{r}{2})\;[\,\eta\,(Y)X - \eta\,(X)Y\,] = \eta(X)QY - \eta(Y)QX.$$

Putting Y = ξ in (4.3) and using (2.21), we get

(4.4) $$QX = (\frac{r}{2} - k)X + \left(3k - \frac{r}{2}\right)\eta(X)\eta(Y)$$

Therefore, it follows from (4.4) that

(4.5) $$S(X,Y) = \left(\frac{r}{2} - k\right)g(X,Y) + \left(3k - \frac{r}{2}\right)\eta(X)\eta(Y)$$

Thus from (4.1), (4.4) and (4.5), we get

(4.6) $$R(X,Y)Z = \left(\frac{r}{2} - 2k\right)[g(Y,Z)X - g(X,Z)Y]$$
$$+ (3k - \frac{r}{2})[g(Y,Z)\eta(X)\xi -$$
$$g(X,Z)\eta(Y)\xi + \eta(Y)\eta(Z)X - \eta(X)\eta(Z)Y].$$

Taking the covariant differentiation to both sides of the equation (4.6) we get

(4.7) $$(\nabla_w R)(X,Y)Z = \frac{dr(w)}{2}[g(Y,Z)X - g(X,Z)Y - g(Y,Z)\eta(X)\xi$$
$$+ g(X,Z)\eta(Y)\xi - \eta(Y)\eta(Z)X + \eta(X)\eta(Z)Y]$$
$$+ (3k - \frac{r}{2})[g(Y,Z)\eta(X) - g(X,Z)\eta(Y)]\nabla_w\xi$$
$$+ \left(3k - \frac{r}{2}\right)[\eta(Y)X - \eta(X)Y](\nabla_w\eta)(Z)$$
$$+ \left(3k - \frac{r}{2}\right)[g(Y,Z)\xi - \eta(Z)Y](\nabla_w\eta)(X)$$
$$- \left(3k - \frac{r}{2}\right)[g(X,Z)\xi - \eta(Z)X](\nabla_w\eta)(Y).$$

Noting that we may assume that all vector fields X, Y, Z and W are orthogonal to ξ and using (2.1) (b), we get

(4.8) $$(\nabla_w R)(X,Y)Z = \frac{dr(w)}{2}[g(Y,Z)X - g(X,Z)Y]$$
$$+ (3k - \frac{r}{2})[g(Y,Z)(\nabla_w\eta)(X) - g(X,Z)(\nabla_w\eta)(Y)]\xi.$$

Applying $\varphi^2$to both sides of (4.8) and using (2.1) (a) and (2.1) (c), we get

(4.9) $$\varphi^2(\nabla_w R)(X,Y)Z = \frac{dr(w)}{2}[g(X,Z)Y - g(Y,Z)X].$$

Using (3.1), the equation (4.9) reduces to

(4.10) $$A(W)R(X,Y)Z + B(W)[g(Y,Z)X - g(X,Z)Y]$$
$$= \frac{dr(w)}{2}[g(X,Z)Y - g(Y,Z)X].$$

Putting $W = \{ei\}$, where $\{ei\}$, i = 1, 2, 3, is an orthonormal basis of the tangent space at any point of the manifold and taking summation over i, 1 ≤ i ≤ 3, we obtain

(4.11) $$R(X,Y)Z = \lambda[g(X,Z)Y - g(Y,Z)X],$$

where $\lambda = [\frac{dr(ei)}{2A(ei)} - \frac{B(ei)}{A(ei)}]$ is a scalar, since A is a non-zero 1-form. Then by Schur's theorem λ will be a constant on the manifold. Thus we obtain the following theorem:

Theorem 4.3. A 3-dimensional Generalised φ -recurrent $N(k)-$contact metric manifold is of constant curvature.

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