

## Almost Contra $\tilde{G}\alpha$ -Continuous Functions

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### Abstract

The concept of  $\tilde{g}\alpha$ -closed sets in a topological space are introduced by R. Devi et. al. [2]. In this paper, we introduce the notion of almost contra  $\tilde{g}\alpha$ -continuous functions utilizing  $\tilde{g}\alpha$ -open sets and study some of its applications.

**Keywords.**  $\tilde{g}\alpha$ -closed sets, almost contra  $\tilde{g}\alpha$ -continuous, almost contra pre-continuous and almost contra  $\gamma$ -continuous.

### Introduction and Preliminaries

In 1996, Dontchev [4] introduced the notion of contra continuity in topological spaces. Also a new class of function called contra semi-continuous function is introduced and investigated by Dontchev and Noiri [6]. The concept of  $g\alpha$ -open set was introduced by R. Devi et. al. [2]. Ekici [7] introduced and studied the notion of almost contra pre continuous functions and the concept of almost contra  $\gamma$ -continuous was introduced by A.A. Nasef [11]. In this direction, we introduce the notion of almost contra  $\tilde{g}\alpha$ -continuous functions via the concept of  $\tilde{g}\alpha$ -open set and study some of the applications of this function.

All through this paper,  $(X, \tau)$  and  $(Y, \sigma)$  stand for topological spaces with no separation axioms assumed, unless otherwise stated. Let  $A \subseteq X$ , the closure of  $A$  and the interior of  $A$  will be denoted by  $\text{cl}(A)$  and  $\text{int}(A)$  respectively.  $A$  is regular open if  $A = \text{int}(\text{cl}(A))$  and  $A$  is regular closed if its complement is regular open; equivalently  $A$  is regular closed if  $A = \text{cl}(\text{int}(A))$ , see [23].

**Definition 1.1.** A subset  $A$  of a space  $(X, \tau)$  is called a

1. semi-open set [10] if  $A \subseteq \text{cl}(\text{int}(A))$  and a semi-closed set [10] if  $\text{int}(\text{cl}(A)) \subseteq A$ ,
2.  $\alpha$ -open set [12] if  $A \subseteq \text{int}(\text{cl}(\text{int}(A)))$  and an  $\alpha$ -closed set [12] if  $\text{cl}(\text{int}(\text{cl}(A))) \subseteq A$ .

The semi-closure (resp.  $\alpha$ -closure) of a subset  $A$  of a space  $(X, \tau)$  is the intersection of all semi-closed (resp.  $\alpha$ -closed) sets that contain  $A$  and is denoted by  $scl(A)$  (resp.  $\alpha cl(A)$ ).

**Definition 1.2.** A subset  $A$  of a space  $(X, \tau)$  is called a

1.  $g$ -closed set [20,21] if  $cl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is semi-open in  $(X, \tau)$ ,
2.  $*g$ -closed set [19] if  $cl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $g$ -open in  $(X, \tau)$ ,
3.  $\#gs$ -closed set [22] if  $scl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $*g$ -open in  $(X, \tau)$ .

Let  $(X, \tau)$  be a space and let  $A$  be a subset of  $X$ .  $A$  is called  $\tilde{g}\alpha$ -closed set [2] if  $\alpha cl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $\#gs$ -open set of  $(X, \tau)$ . The complement of an  $\tilde{g}\alpha$ -closed set is called  $\tilde{g}\alpha$ -open. The family of all regular open (resp. regular closed,  $\tilde{g}\alpha$ -open and  $\tilde{g}\alpha$ -closed) sets of  $X$  is denoted by  $RO(X)$  (resp.  $RC(X)$ ,  $\tilde{g}\alpha O(X)$ ,  $\tilde{g}\alpha C(X)$ ). We set  $\tilde{g}\alpha O(X, x) = \{U : x \in U \text{ and } U \in \tilde{g}\alpha O(X)\}$ . The set  $\cup\{F \subseteq X : F \subseteq A, F \text{ is } \tilde{g}\alpha\text{-open}\}$  is called  $\tilde{g}\alpha$ -interior of  $A$  and is denoted by  $int\tilde{g}\alpha(A)$ .

**Definition 1.3.** A function  $f : (X, \tau) \rightarrow (Y, \sigma)$  is called a

1. perfectly continuous [13] if  $f^{-1}(V)$  is clopen in  $(X, \tau)$  for every open set  $V$  of  $(Y, \sigma)$ ,
2. almost continuous [15] if  $f^{-1}(V)$  is open in  $(X, \tau)$  for every regular open set  $V$  of  $(Y, \sigma)$ ,
3. contra continuous [4] if  $f^{-1}(V)$  is closed in  $(X, \tau)$  for every open set  $V$  of  $(Y, \sigma)$ ,
4. regular set connected [5] if  $f^{-1}(V)$  is clopen in  $(X, \tau)$  for every regular open set  $V$  of  $(Y, \sigma)$ ,
5. contra pre continuous [9] if  $f^{-1}(V)$  is pre-closed in  $(X, \tau)$  for every open set  $V$  of  $(Y, \sigma)$ ,
6. contra  $\gamma$ -continuous [11] if  $f^{-1}(V)$  is  $\gamma$ -closed in  $(X, \tau)$  for every open set  $V$  of  $(Y, \sigma)$ ,
7.  $\tilde{g}\alpha$ -continuous [3] if  $f^{-1}(V)$  is  $\tilde{g}\alpha$ -closed in  $(X, \tau)$  for every closed set  $V$  of  $(Y, \sigma)$ ,
8. contra  $\tilde{g}\alpha$ -continuous [1] if  $f^{-1}(V)$  is  $\tilde{g}\alpha$ -closed in  $(X, \tau)$  for every open set  $V$  of  $(Y, \sigma)$ ,
9. almost contra pre continuous [7] if  $f^{-1}(V)$  is pre-closed in  $(X, \tau)$  for every regular open set  $V$  of  $(Y, \sigma)$ ,
10. almost contra  $\gamma$  continuous [11] if  $f^{-1}(V)$  is  $\gamma$ -closed in  $(X, \tau)$  for every regular open set  $V$  of  $(Y, \sigma)$ ,

**Definition 1.4.** A space  $(X, \tau)$  is called

1. an ultranormal [18] if each pair of non-empty disjoint closed sets can be separated by disjoint clopen sets.
2. a  $\tilde{g}\alpha$ -normal [1] if each pair of non-empty disjoint closed sets can be separated by disjoint  $\tilde{g}\alpha$ -open sets.
3. a  $\tilde{g}\alpha$ -connected [1] if  $X$  is not the union of two disjoint non-empty  $\tilde{g}\alpha$ -open

subsets of  $X$ .

4.  $\tilde{g}\alpha$ - $T_1$  [1] if for each pair of distinct points  $x$  and  $y$  in  $X$ , there exists  $\tilde{g}\alpha$ -open sets  $U$  and  $V$  containing  $x$  and  $y$  respectively, such that  $y \in U$  and  $x \in V$ .
5.  $\tilde{g}\alpha$ - $T_2$  [1] if for each pair of distinct points  $x$  and  $y$  in  $X$ , there exists  $\tilde{g}\alpha$ -open sets  $U$  and  $V$  containing  $x$  and  $y$  respectively, such that  $U \cap V = \emptyset$ .
6.  $\tilde{g}\alpha T_1$  -space [3] if every  $\tilde{g}\alpha$ -closed set is closed.

### Properties of almost contra $\tilde{g}\alpha$ -continuous functions

**Definition 2.1.** A function  $f : (X, \tau) \rightarrow (Y, \sigma)$  is called almost contra  $\tilde{g}\alpha$ -continuous if  $f^{-1}(U) \in \tilde{g}\alpha C(X)$  for each  $U \in RO(Y)$ .

**Definition 2.2.** [1] A function  $f : (X, \tau) \rightarrow (Y, \sigma)$  is called almost  $\tilde{g}\alpha$ -continuous if for each  $x \in X$  and each open set  $V$  of  $Y$  containing  $f(x)$ , there exists  $U \in \tilde{g}\alpha O(X, x)$  such that  $f(U) \subseteq \text{int} \tilde{g}\alpha(\text{cl}(V))$ .

**Theorem 2.3.** Let  $(X, \tau)$  and  $(Y, \gamma)$  be topological spaces. The following statements are equivalent for a function  $f : (X, \tau) \rightarrow (Y, \gamma)$ .

1.  $f$  is almost contra  $\tilde{g}\alpha$ -continuous;
2.  $f^{-1}(F) \in \tilde{g}\alpha C(X)$  for every  $F \in RO(Y)$ ;
3. for each  $x \in X$  and each regular open subset  $F$  in  $Y$  containing  $f(x)$ , there exists a  $\tilde{g}\alpha$  closed set  $U$  in  $X$  containing  $x$  such that  $f(U) \subseteq F$ ;
4. for each  $x \in X$  and each regular closed subset  $V$  in  $Y$  non-containing  $f(x)$ , there exists a  $\tilde{g}\alpha$  open set  $K$  in  $X$  non-containing  $x$  such that  $f^{-1}(V) \subseteq K$ ;
5.  $f^{-1}(\text{int}(\text{cl}(G))) \in \tilde{g}\alpha C(X)$  for every open subset  $G$  of  $Y$ ;
6.  $f^{-1}(\text{cl}(\text{int}(F))) \in \tilde{g}\alpha O(X)$  for every closed subset  $F$  of  $Y$ .

**Proof.**

(1)  $\Leftrightarrow$  (2) Let  $F \in RO(Y)$ . Then  $Y - F \in RC(Y)$ . By (1)  $f^{-1}(Y - F) = X - f^{-1}(F) \in \tilde{g}\alpha O(X)$ . We have  $f^{-1}(F) \in \tilde{g}\alpha C(X)$ . Reverse can be obtained similarly.

(2)  $\Rightarrow$  (3) Let  $F$  be any regular open set in  $Y$  containing  $f(x)$ . By (2),  $f^{-1}(F) \in \tilde{g}\alpha C(X)$  and  $x \in f^{-1}(F)$ . Take  $U = f^{-1}(F)$ . Then  $f(U) \subseteq F$ .

(3)  $\Rightarrow$  (2) Let  $F$  be any regular open set of  $Y$  and  $x \in f^{-1}(F)$ . From (3), there exists  $\tilde{g}\alpha$ -closed set  $U_x$  in  $X$  containing  $x$  such that  $U \subseteq f^{-1}(F)$ . We have  $f^{-1}(F) = \cup_{x \in f^{-1}(F)} U_x$ . Thus  $f^{-1}(F)$  is  $\tilde{g}\alpha$ -closed.

(3)  $\Leftrightarrow$  (4) Let  $V$  be any regular closed in  $Y$  not containing  $f(x)$ . Then  $Y - V$  is a regular open set containing  $f(x)$ . By (3), there exists  $\tilde{g}\alpha$ -closed set  $U$  in  $X$  containing  $x$  such that  $f(U) \subseteq Y - V$ . Hence,  $U \subseteq f^{-1}(Y - V) \subseteq X - f^{-1}(V)$  and then  $f^{-1}(V) \subseteq X - U$ . Take  $H = X - U$ . We obtain that  $H$  is a  $\tilde{g}\alpha$ -open set in  $X$  non containing  $x$ . The converse can be solve easily.

(2)  $\Leftrightarrow$  (5) Let  $G$  be open subset of  $Y$ . Since  $\text{int}(\text{cl}(G))$  is regular open, then by (2), it follows that  $f^{-1}(\text{int}(\text{cl}(G))) \in \tilde{g}\alpha C(X)$ . The converse can be shown easily.

(1)  $\Leftrightarrow$  (6) It can be obtained similar as (2)  $\Leftrightarrow$  (5).

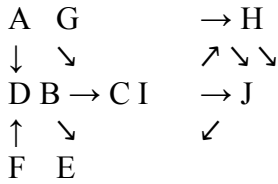
**Theorem 2.4.** Every regular set connected is almost contra  $\tilde{g}\alpha$ -continuous.

**Proof.** Let  $V$  be any regular open in  $Y$ . Since  $f : X \rightarrow Y$  is regular set-connected,  $f^{-1}(V)$  is clopen in  $X$ . Hence  $f^{-1}(V)$  is  $\tilde{g}\alpha$ -closed and  $\tilde{g}\alpha$ -open. Therefore  $f$  is almost contra  $\tilde{g}\alpha$ -continuous.

The converse of the above Theorem need not be true by the following example.

**Example 2.5.** Let  $X = \{a, b, c\}$ ,  $\tau = \{X, \phi, \{a\}, \{a, b\}, \{a, c\}\}$  and  $\sigma = \{X, \phi, \{b\}, \{c\}, \{b, c\}\}$ . Then the identity function  $f : (X, \tau) \rightarrow (X, \sigma)$  is almost contra  $\tilde{g}\alpha$ -continuous. But it is not regular set connected.

**Remark 2.6.** The following diagram shows the relationships established between almost contra  $\tilde{g}\alpha$ -continuous functions and some other functions.



**Notation 2.7.** A = perfectly continuous, B = contra continuous, C = contra  $\tilde{g}\alpha$ -continuous, D = regular set connected, E = almost contra  $\tilde{g}\alpha$ -continuous, F = almost S-continuous, G = contra pre-continuous, H = almost contra precontinuous, I = contra  $\gamma$ -continuous, J = almost contra  $\gamma$ -continuous.

These implications are not reversible as shown by the examples stated below.

**Example 2.8.** Let  $X = \{a, b, c\}$ ,  $\tau = \{X, \phi, \{a\}, \{a, b\}, \{a, c\}\}$  and  $\sigma = \{X, \phi, \{a\}, \{a, b\}\}$ . Then the identity function  $f : (X, \tau) \rightarrow (X, \sigma)$  is regular set connected, but not perfectly continuous.

**Example 2.9.** Let  $X = \{a, b, c\}$ ,  $\tau = \{X, \phi, \{a\}, \{a, b\}, \{a, c\}\}$  and  $\sigma = \{X, \phi, \{a\}, \{a, b\}\}$ . Then the identity function  $f : (X, \tau) \rightarrow (X, \sigma)$  is almost contra  $\tilde{g}\alpha$ -continuous, but not contra  $\tilde{g}\alpha$ -continuous.

**Example 2.10.** Let  $X = \{a, b, c\}$ ,  $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$  and  $Y = \{1, 2\}$  be the Sierpinski space with the topology  $\sigma = \{Y, \phi, \{1\}\}$ . Let  $f : (X, \tau) \rightarrow (Y, \sigma)$  be defined by  $f(a) = 1, f(b) = f(c) = 2$ . Then  $f$  is contra  $\gamma$ -continuous, but not contra pre-continuous.

**Example 2.11.** Let  $X = \{a, b, c\} = Y$ ,  $\tau = \{X, \phi, \{a\}, \{b, c\}\}$  and  $\sigma = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$ . Let  $f : (X, \tau) \rightarrow (Y, \sigma)$  be defined by  $f(a) = b, f(b) = a, f(c) = c$ . Then  $f$  is almost contra  $\gamma$ -continuous, but not almost contra pre-continuous.

**Theorem 2.12.** If a function  $f : X \rightarrow Y$  is almost contra  $\tilde{g}\alpha$ -continuous, almost continuous and  $X$  is  $\tilde{g}\alpha T_1$ -space, then  $f$  is regular set connected.

**Proof.** Let  $V \in RO(Y)$ . Since  $f$  is almost contra  $\tilde{g}\alpha$ -continuous and almost continuous, then  $f^{-1}(V)$  is  $\tilde{g}\alpha$ -closed and open. Since  $X$  is  $\tilde{g}\alpha T_1$ -space,  $f^{-1}(V)$  is closed and open. Hence  $f^{-1}(V)$  is clopen. We obtained that  $f$  is regular set connected.

**Theorem 2.13.** If  $f : X \rightarrow Y$  is almost contra  $\tilde{g}\alpha$ -continuous surjection and  $X$  is  $\tilde{g}\alpha$ -connected, then  $Y$  is connected.

**Proof.** Suppose that  $Y$  is not connected space. There exists non-empty disjoint open sets  $V_1$  and  $V_2$  such that  $Y = V_1 \cup V_2$ . Therefore,  $V_1$  and  $V_2$  are clopen in  $Y$ . Since  $f$  is almost contra  $\tilde{g}\alpha$ -continuous,  $f^{-1}(V_1)$  and  $f^{-1}(V_2)$  are  $\tilde{g}\alpha$ -open in  $X$ . Moreover,  $f^{-1}(V_1)$  and  $f^{-1}(V_2)$  are non-empty disjoint and  $X = f^{-1}(V_1) \cup f^{-1}(V_2)$ . This shows that  $X$  is not  $\tilde{g}\alpha$ -connected. This contradicts that  $Y$  is not connected assumed. Hence  $Y$  is connected.

**Theorem 2.14.** If  $f : X \rightarrow Y$  is a almost contra  $\tilde{g}\alpha$ -continuous, closed injection and  $Y$  is ultranormal, then  $X$  is  $\tilde{g}\alpha$ -normal.

**Proof.** Let  $F_1$  and  $F_2$  be disjoint closed subsets of  $X$ . Since  $f$  is closed injective,  $f(F_1)$  and  $f(F_2)$  are disjoint closed subsets of  $Y$ . Since  $Y$  is ultranormal,  $f(F_1)$  and  $f(F_2)$  are separated by disjoint clopen sets  $V_1$  and  $V_2$ , respectively. Hence  $F_i \subset f^{-1}(V_i)$ ,  $f^{-1}(V_i) \in \tilde{g}\alpha C(X)$  for  $i = 1, 2$  and  $f^{-1}(V_1) \cap f^{-1}(V_2) = \emptyset$ . Thus  $X$  is  $\tilde{g}\alpha$ -normal.

**Theorem 2.15.** Let  $X$  is a topological space and for each pair of distinct points  $x$  and  $y$  in  $X$  there exists a map  $f$  of  $X$  into a Urysohn topological space  $Y$  such that  $f(x) = f(y)$  and  $f$  is almost contra  $\tilde{g}\alpha$ -continuous at  $x$  and  $y$ , then  $X$  is  $\tilde{g}\alpha-T_2$ .

**Proof.** Let  $x$  and  $y$  be any distinct points in  $X$ . Then, there exists a Urysohn space  $Y$  and a function  $f : X \rightarrow Y$  such that  $f(x) = f(y)$  and  $f$  is almost contra  $\tilde{g}\alpha$ -continuous at  $x$  and  $y$ . Let  $a = f(x)$  and  $b = f(y)$ . Then  $a = b$ . Since  $Y$  is Urysohn space, there exists open sets  $V$  and  $W$  containing  $a$  and  $b$ , respectively, such that  $cl(V) \cap cl(W) = \emptyset$ . Hence,  $cl(int(V)) \cap cl(int(W)) = \emptyset$ . Since  $f$  is almost contra  $\tilde{g}\alpha$ -continuous at  $x$  and  $y$ , there exist  $\tilde{g}\alpha$ -open sets  $A$  and  $B$  containing  $a$  and  $b$ , respectively, such that  $f(A) \subseteq cl(int(V))$  and  $f(B) \subseteq cl(int(W))$ . Then  $f(A) \cap f(B) = \emptyset$ , so  $A \cap B = \emptyset$ . Hence,  $X$  is  $\tilde{g}\alpha-T_2$ .

**Corollary 2.16.** Let  $f : X \rightarrow Y$  be almost contra  $\tilde{g}\alpha$ -continuous injection. If  $Y$  is an Urysohn space, then  $X$  is  $\tilde{g}\alpha-T_2$ .

**Definition 2.17.** A space  $X$  is said to be weakly Hausdorff [17] if each element of  $X$  is an intersection of regular closed sets.

**Theorem 2.18.** If  $f : X \rightarrow Y$  is almost contra  $\tilde{g}\alpha$ -continuous injection and  $Y$  is weakly Hausdorff, then  $X$  is  $\tilde{g}\alpha-T_1$ .

**Proof.** Suppose that  $Y$  is weakly Hausdorff. For any distinct points  $x_1$  and  $x_2$  in  $X$ , there exists regular closed sets  $U$  and  $V$  in  $Y$  such that  $f(x_1) \in U$ ,  $f(x_2) \in U$ ,  $f(x_1) \in V$  and  $f(x_2) \in V$ . Since  $f$  is almost contra  $\tilde{g}\alpha$ -continuous,  $f^{-1}(U)$  and  $f^{-1}(V)$  are  $\tilde{g}\alpha$ -open subsets of  $X$  such that  $x_1 \in f^{-1}(U)$ ,  $x_2 \in f^{-1}(U)$ ,  $x_1 \in f^{-1}(V)$  and  $x_2 \in f^{-1}(V)$ . This shows that  $X$  is  $\tilde{g}\alpha$ - $T_1$ .

**Definition 2.19.** A topological space  $X$  is said to be ultra Hausdorff [18] if for each pair of distinct points  $x$  and  $y$  in  $X$  there exist clopen sets  $A$  and  $B$  containing  $x$  and  $y$ , respectively such that  $A \cap B = \emptyset$ .

**Theorem 2.20.** Let  $f : X \rightarrow Y$  be almost contra  $\tilde{g}\alpha$ -continuous injection. If  $Y$  is ultra Hausdorff space, then  $X$  is  $\tilde{g}\alpha$ - $T_2$ .

**Proof.** Let  $x_1$  and  $x_2$  be any distinct points in  $X$ , then  $f(x_1) \neq f(x_2)$  and there exist clopen sets  $U$  and  $V$  containing  $f(x_1)$  and  $f(x_2)$  respectively such that  $U \cap V = \emptyset$ . Since  $f$  is almost contra  $\tilde{g}\alpha$ -continuous, then  $f^{-1}(U) \in \tilde{g}\alpha O(X)$  and  $f^{-1}(V) \in \tilde{g}\alpha O(X)$  such that  $f^{-1}(U) \cap f^{-1}(V) = \emptyset$ . Hence,  $X$  is  $\tilde{g}\alpha$ - $T_2$ .

**Theorem 2.21.** Let  $f : X \rightarrow Y$  and  $g : Y \rightarrow Z$  be functions. Then, the following properties hold:

1. If  $f$  is almost contra  $\tilde{g}\alpha$ -continuous and  $g$  is regular set connected, then  $g \circ f : X \rightarrow Z$  is almost contra- $\tilde{g}\alpha$ -continuous and almost  $\tilde{g}\alpha$ -continuous.
2. If  $f$  is almost contra  $\tilde{g}\alpha$ -continuous and  $g$  is perfect continuous, then  $g \circ f : X \rightarrow Z$  is  $\tilde{g}\alpha$ -continuous and contra  $\tilde{g}\alpha$ -continuous.
3. If  $f$  is contra  $\tilde{g}\alpha$ -continuous and  $g$  is regular set connected, then  $g \circ f : X \rightarrow Z$  is almost contra  $\tilde{g}\alpha$ -continuous and almost  $\tilde{g}\alpha$ -continuous.

**Proof.**

1. Let  $V$  be any regular open set in  $Z$ . Since  $g$  is regular set connected,  $g^{-1}(V)$  is clopen. Since  $f$  is almost contra  $\tilde{g}\alpha$ -continuous,  $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$  is  $\tilde{g}\alpha$ -open and  $\tilde{g}\alpha$ -closed. Therefore,  $g \circ f$  is almost contra  $\tilde{g}\alpha$ -continuous and almost  $\tilde{g}\alpha$ -continuous.
2. Let  $V$  be any regular open set in  $Z$ . Since  $g$  is perfectly continuous,  $g^{-1}(V)$  is clopen. Since  $f$  is almost contra  $\tilde{g}\alpha$ -continuous,  $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$  is  $\tilde{g}\alpha$ -open and  $\tilde{g}\alpha$ -closed. Therefore,  $g \circ f$  is  $\tilde{g}\alpha$ -continuous and contra  $\tilde{g}\alpha$ -continuous.
3. Let  $V$  be any regular open set in  $Z$ . Since  $g$  is regular set connected,  $g^{-1}(V)$  is clopen. Since  $f$  is contra  $\tilde{g}\alpha$ -continuous,  $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$  is  $\tilde{g}\alpha$ -open and  $\tilde{g}\alpha$ -closed. Therefore,  $g \circ f$  is almost contra  $\tilde{g}\alpha$ -continuous and almost  $\tilde{g}\alpha$ -continuous.

**Definition 2.22.** A function  $f : X \rightarrow Y$  is said to be pre  $\tilde{g}\alpha$ -open (pre  $\tilde{g}\alpha$ -closed) [1] if the image of each  $\tilde{g}\alpha$ -open set is  $\tilde{g}\alpha$ -open ( $\tilde{g}\alpha$ -closed).

**Theorem 2.23.** If  $f : X \rightarrow Y$  is a surjective pre  $\tilde{g}\alpha$ -open and  $g : Y \rightarrow Z$  is a function

such that  $g \circ f : X \rightarrow Z$  is almost contra  $\tilde{g}\alpha$ -continuous, then  $g$  is almost contra  $\tilde{g}\alpha$ -continuous.

**Proof.** Let  $V$  be any regular closed set in  $Z$ . Since  $g \circ f$  is almost contra  $\tilde{g}\alpha$ -continuous,  $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$  is  $\tilde{g}\alpha$ -open. Since  $f$  is surjective pre  $\tilde{g}\alpha$ -open,  $f(f^{-1}(g^{-1}(V))) = g^{-1}(V)$  is  $\tilde{g}\alpha$ -open. Therefore  $g$  is almost contra  $\tilde{g}\alpha$ -continuous.

**Theorem 2.24.** If  $f : X \rightarrow Y$  is a surjective pre  $\tilde{g}\alpha$ -closed and  $g : Y \rightarrow Z$  is a function such that  $g \circ f : X \rightarrow Z$  is almost contra  $\tilde{g}\alpha$ -continuous, then  $g$  is almost contra  $\tilde{g}\alpha$ -continuous.

**Proof.** Let  $V$  be any regular open set in  $Z$ . Since  $g \circ f$  is almost contra  $\tilde{g}\alpha$ -continuous,  $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$  is  $\tilde{g}\alpha$ -closed. Since  $f$  is surjective pre  $\tilde{g}\alpha$ -closed,  $f(f^{-1}(g^{-1}(V))) = g^{-1}(V)$  is  $\tilde{g}\alpha$ -closed. Therefore  $g$  is almost contra  $\tilde{g}\alpha$ -continuous.

**Definition 2.25.** A space  $X$  is said to be

1.  $\tilde{g}\alpha$ -closed compact if every  $\tilde{g}\alpha$ -closed cover of  $X$  has a finite sub cover;
2. countably  $\tilde{g}\alpha$ -closed compact if every countable cover of  $X$  by  $\tilde{g}\alpha$ -closed sets has a finite sub cover;
3.  $\tilde{g}\alpha$ -closed Lindelof if every cover of  $X$  by  $\tilde{g}\alpha$ -closed sets has a countable sub cover;
4. nearly compact [16] if every regular open cover of  $X$  has a finite subcover;
5. nearly countably compact [8,14] if every countable cover of  $X$  by regular open sets has a finite subcover;
6. nearly Lindelof [7] if every cover of  $X$  by regular open sets has a countable subcover.

**Theorem 2.26.** Let  $f : X \rightarrow Y$  be an almost contra  $\tilde{g}\alpha$ -continuous surjection. The following statements hold:

1. If  $X$  is  $\tilde{g}\alpha$ -closed compact, then  $Y$  is nearly compact.
2. If  $X$  is  $\tilde{g}\alpha$ -closed Lindelof, then  $Y$  is nearly Lindelof.
3. If  $X$  is countably  $\tilde{g}\alpha$ -closed compact, then  $Y$  is nearly countably compact.

**Proof.**

1. Let  $\{V_\alpha : \alpha \in I\}$  be any regular open cover of  $Y$ . Since  $f$  is almost contra  $\tilde{g}\alpha$ -continuous, then  $\{f^{-1}(V_\alpha) : \alpha \in I\}$  is an  $\tilde{g}\alpha$ -closed cover of  $X$ . Since  $X$  is  $\tilde{g}\alpha$ -closed compact, there exists a finite subset  $I_0$  of  $I$  such that  $X = \cup\{f^{-1}(V_\alpha) : \alpha \in I_0\}$ . Thus, we have  $Y = \cup\{V_\alpha : \alpha \in I_0\}$  and  $Y$  is nearly compact.
2. Let  $\{V_\alpha : \alpha \in I\}$  be any cover of  $Y$  by regular open sets. Since  $f$  is almost contra  $\tilde{g}\alpha$ -continuous, then  $\{f^{-1}(V_\alpha) : \alpha \in I\}$  is a  $\tilde{g}\alpha$ -closed cover of  $X$ . Since  $X$  is  $\tilde{g}\alpha$ -closed Lindelof, there exists an infinite subset  $I_0$  of  $I$  such that  $X = \cup\{f^{-1}(V_\alpha) : \alpha \in I_0\}$ . Thus, we have  $Y = \cup\{V_\alpha : \alpha \in I_0\}$  and  $Y$  is nearly

Lindelof.

3. Let  $\{V_\alpha : \alpha \in I\}$  be a countable cover of  $Y$  by regular open sets. Since  $f$  is almost contra  $\tilde{g}\alpha$ -continuous, then  $\{f^{-1}(V_\alpha) : \alpha \in I\}$  is an  $\tilde{g}\alpha$ -closed cover of  $X$ . Since  $X$  is countably  $\tilde{g}\alpha$ -closed compact, there exists a finite subset  $I_0$  of  $I$  such that  $X = \cup\{f^{-1}(V_\alpha) : \alpha \in I_0\}$ . Thus, we have  $Y = \cup\{V_\alpha : \alpha \in I_0\}$  and  $Y$  is nearly countably compact.

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