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A Group Acceptance Sampling Plans Using Weighted Binomial On Truncated Life Tests For Marshall – Olkin Extended Distributions

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Abstract

In this paper, a group acceptance sampling plan using weighted binomial is developed for a truncated life test when the lifetime of an item follows Marshall – Olkin extended exponential and Marshall – Olkin extended Lomax distributions. The minimum number of groups required for a given group size and the acceptance number is determined when the consumer's risk and the test termination time are specified. The operating characteristic values according to various quality levels are obtained. The results are explained with examples.

Keywords: Marshall – Olkin extended exponential, Marshall – Olkin extended Lomax distribution, Group acceptance sampling using weighted binomial, consumer's risk, Operating characteristics, Producer's risk, truncated life test.

Introduction

Quality control has become one of the most important consumer decision factor in the selection among competing products and services. Understanding and improving quality is a key factor leading to business success, growth and an enhanced competitive position. There is a substantial return on investment from improved quality and from successfully employing quality as an integral part of overall business strategy. Quality improvement is the reduction of variability in processes and products. There are three major areas of statistical and engineering technology useful in quality improvement. They are i) statistical process control ii) design of experiments and iii) acceptance sampling.

Acceptance sampling is closely connected with inspection and testing of products. Inspection can occur at many points in a process. Acceptance sampling defined as the inspection and classification of a sample of units selected at random from a larger batch or lot. The ultimate decision about disposition of the lot usually occurs at two points; incoming raw materials or components or final production. Thus a specific plan that states the sample size or sizes to be used and the associated acceptance and non-acceptance criteria is an acceptance sampling plan.

In most acceptance sampling plans for a truncated life test, determining the sample size from a lot under consideration is the major issue. In the usual sampling plan, it is implicitly assumed that only a single item is put in a tester. However testers accommodating a multiple number of items at a time are used in practise because testing those items simultaneously. Items in a tester can be regarded as a group and the number of items in a group is called as group size. The acceptance sampling plan based on this group of items is called group acceptance sampling plan.

The quality of the product is tested on the basis of few items taken from an infinite lot. The statistical test can be stated as: Let μ be the true average life and μ_0 be the specified average life of a product. Based on the failure data, we want to test the hypothesis H_0: $\mu \geq \mu_0$ against H_1: $\mu < \mu_0$. A lot is considered as good if $\mu \geq \mu_0$ and bad if $\mu < \mu_0$. This hypothesis is tested using the acceptance sampling scheme as: In a life test experiment, a sample of size n selected from a lot of products is put on the test. The experiment is terminated at a pre – assigned time t_0. when we set acceptance number as c, H_0 is rejected if more than c failures are recorded before time t_0 and H_0 is accepted if there are c or fewer failures before t_0. Probability of rejection of good lot is called the producer's risk and probability of accepting a bad lot is known as consumer's risk. If the confidence level is p*, then the consumers risk will be $\beta = 1-p^*$. A well acceptance sampling plan minimizes both the risks.

Many authors have discussed acceptance sampling based on truncated life tests. Aslam M., and Jun C.H. (2009) have studied a group acceptance sampling plans for truncated life tests based on the inverse Rayleigh and log-logistic distributions. Ghitany M. E., Al-Awadhi F. A. and Alkhalfan L. A. (2007) have studied Marshal-Olkin extended Lomax distribution and its application to censored data. Mughal A. R., Aslam M., Hussain J., Rehman A., (2010) have studied economic reliability group acceptance sampling plans for lifetimes following a Marshall – Olkin extended distribution. Radhakrishnan R. and Alagirisamy K. (2011) have studied on construction of group acceptance sampling plan using weighted binomial distribution. Sobel M. and Tischendrof J. A. (1959) have studied acceptance sampling with new life test objectives. Srinivasa Rao G. (2010) has studied group acceptance sampling plans for truncated life tests for Marshall-Olkin extended Lomax distribution. And again Srinivasa Rao G., with Ghitany M. E. and Kantam R. R. L. have studied acceptance sampling plans for Marshall-Olkin extended Lomax distribution. Sudamani Ramaswamy A. R. and Priyah Anburajan in 2012 have discussed on a Hybrid group acceptance sampling plans for lifetimes based on Marshall – Olkin extended Lomax distribution.

Here we apply GASP on the truncated life tests when a lifetime of the product assumed to follow a Generalized Rayleigh distribution. In this type of tests, determining the sample size is equivalent to determining the number of groups.

Cumulative Distributive Function

Marshall – Olkin extended Lomax Distribution

The cumulative distribution function (cdf) of the Marshall – Olkin extended Lomax distribution is given by

$$F(t,\sigma)=\frac{(1+t/\sigma)^{\theta}-1}{(1+t/\sigma)^{\theta}-\bar{\gamma}},\bar{\gamma}=1-\gamma \tag{1}$$

where σ is a scale parameter and γ is the shape parameter.

Marshall – Olkin extended Exponential Distribution

The cumulative distribution function (cdf) of the Marshall – Olkin extended exponential distribution is given by

$$F(t,\sigma)=\frac{1-e^{-t/\sigma}}{1-\bar{\gamma}e^{-t/\sigma}},\bar{\gamma}=1-\gamma \tag{2}$$

where σ is a scale parameter and γ is the shape parameter

If some other parameters are involved, then they are assumed to be known, for an example, if shape parameter of a distribution is unknown it is very difficult to design the acceptance sampling plan. In quality control analysis, the scale parameter is often called the quality parameter or characteristics parameter. Therefore it is assumed that the distribution function depends on time only through the ratio of t/ σ.

The failure probability of an item by time t_0 is given by

$$p = F(t_{0\,:}\sigma) \tag{3}$$

The quality of an item is usually represented by its true mean lifetime although some other options such as median lifetime are sometimes used. Let us assume that the true mean μ can be represented by the scale parameter. In addition, it is convenient to specify the test time as a multiple of the specified life so that $a\mu_0$ and the quality of an item as a ratio of the true mean to the specified life (μ/μ_0).

Then we can rewrite (3) as a function of ‘a’ (termination time) and the ratio μ/μ_0.

$$p = F(a\,\mu_0 : \mu/\mu_0) \tag{4}$$

Here when the underlying distribution is the Marshall – Olkin extended Lomax distribution

$$p = \frac{\left[1+1.5708a/(\mu/\mu_0)\right]^{\theta} - 1}{\left[1+1.5708a/(\mu/\mu_0)\right]^{\theta} - \bar{\gamma}}, \bar{\gamma} = 1-\gamma \tag{5}$$

When the underlying distribution is the Marshall – Olkin extended exponential distribution

$$p = \frac{1-e^{-\frac{1.5708a}{\mu/\mu_0}}}{1-\bar{\gamma}e^{-\frac{1.5708a}{\mu/\mu_0}}}, \bar{\gamma} = 1-\gamma \tag{6}$$

Design of the proposed sampling plan:
Procedure:

- Select the number of groups g and allocate predefined r items to each group so that the sample size for a lot will be n = gr.
- Select the acceptance number c for a group and specify the experiment time t_0.
- Perform the experiment for the g groups simultaneously and record the number of failures for each group.
- Accept the lot if at most c failures occur in each of all groups by the experiment time.
- Terminate the experiment as soon as more than c failures occur in any group and reject the lot.

We are interested in determining the number of groups g, whereas the various values of acceptance number c and the termination time t_0 are assumed to be specified. Since it is convenient to set the termination time as a multiple of the specified life μ_0, we will consider $t_0=a\mu_0$ for a specified constant a (time multiplier).

The lot acceptance probability will be

$$L(p) = \left(\sum_{i=1}^{c} \binom{r-1}{i-1} p^{i-1} (1-p)^{r-1} \right)^{g} \tag{7}$$

where p is the probability that an item in a group fails before the termination time $t_0 = a\mu_0$.

The minimum number of groups required can be determined by considering the consumer's risk when the true median life equals the specified median life ($\mu = \mu_0$) (worst case) by means of the following inequality:

$$L(p_0) \le \beta \tag{8}$$

where p_0 is the failure probability at $\mu = \mu_0$. Here minimum group size (g) is obtained using (5) and (6) in (9) at worst case.

Operating Characteristic functions

The probability of acceptance can be regarded as a function of the deviation of the specified value μ_0 of the median from its true value μ. This function is called Operating Characteristic (OC) function of the sampling plan. Once the minimum sample size is obtained, one may be interested to find the probability of acceptance of a lot when the quality (or reliability) of the product is sufficiently good. As mentioned earlier, the product is considered to be good if $\mu \geq \mu_0$. Te probabilities of acceptance are displayed in Table 3 and 4 for various values of the median ratios μ/μ_0, producer's risks β and time multiplier a.

Notations

g	-	Number of groups
r	-	Number of items in a group
n	-	Sample size
c	-	Acceptance number
t_0	-	Termination time
a	-	Test termination time multiplier
γ	-	Shape parameter
σ	-	Scale parameter
α	-	Producer's risk
β	-	Consumer's risk
p	-	Failure probability
L(p)	-	Probability of acceptance
μ	-	Mean life
μ_0	-	Specified life

Description of tables and examples

Based on various test values of consumer's risk and the test termination time multiplier, the number of groups of GASP is found using (7) and (8). Suppose that we want to develop an economic reliability group sampling plan to test if the median is greater than 1,000 hours based on a testing time of 700 hours and using testers equipped with 6 items each. It is assumed that c = 2 and β = 0.1. This gives the termination multiplier a = 0.7. If the life time follows Marshall – Olkin extended Exponential distribution, from Table 1 the design parameters can be written as (g, c) = (4, 2). We will implement the above sampling plan as, draw the first sample of size n = 24 items and put to 6 testers, if not more than 2 failures occur during 700 hours in any one of the groups, we accept the lot, otherwise reject it. If suppose the life time follows Marshall – Olkin extended Lomax distribution, from Table 2 the design parameters can be written as (g, c) = (2, 2). We will implement the above sampling plan as, draw the first sample of size n = 12 items and put to 6 testers, if not more than 2 failures occur during 700 hours in any one of the groups we accept the lot, otherwise reject it. For this proposed sampling plan if r = 4, β = 0.25 and the life time follows Marshall – Olkin extended Exponential distribution, the probability of acceptance is 0.998179 when the true mean is 10,000 hrs from Table 3 and if the life time follows Marshall – Olkin extended Lomax distribution the probability of acceptance is 0.994397 when the true mean is 10,000 hrs from Table 4.

Comparatively the probability of acceptance of both the distributions is equally good. When the the probability of acceptance is compared Marshall – Olkin extended Lomax distribution has less probability of acceptance than the Marshall – Olkin extended exponential distribution. Thus Marshall – Olkin extended exponential distribution is comparatively good for the group acceptance sampling plan with weighted binomial. When comparing the probability of acceptance of Srinivasa Rao[6], for the same, it is 0.9917 for Marshall – Olkin extended Lomax distribution for group sampling plans. It is observed that the lot acceptance probability increases as the mean ratio increases and the number of groups tends to increase as the test duration decreases and is shown in Figure 1.

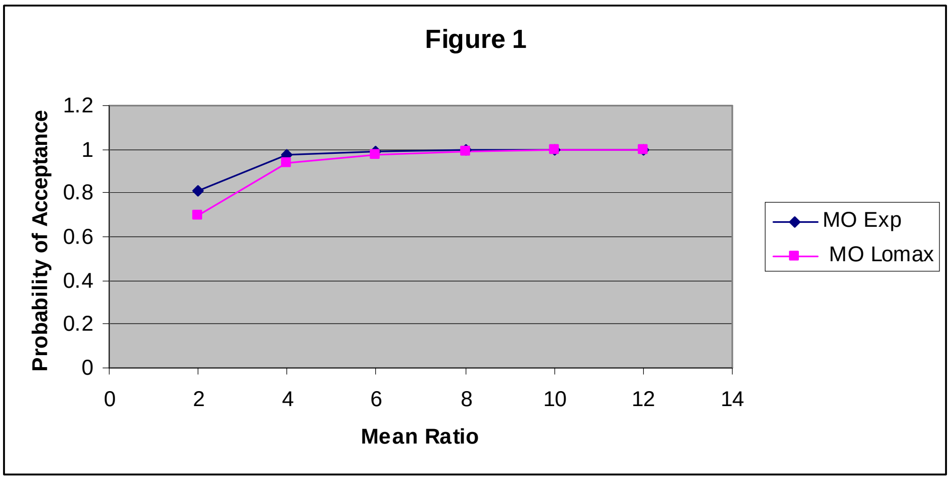

Figure 1. OC curve for Probability of acceptance against mean ratio

Conclusion

In this paper, a group acceptance sampling plan from a truncated life test is proposed in the case of a Marshall – Olkin extended exponential and Marshall – Olkin extended Lomax distributions. The number of groups and the acceptance number are determined when the consumer's risk (β) and the other plan parameters are specified. It is observed that the minimum number of groups required decreases as the test termination time multiplier increases. Moreover, the operating characteristic function increases disproportionately when the quality improves. This group acceptance sampling plan with weighted binomial can be used when a multiple number of items are tested simultaneously. Clearly, such a tester would be beneficial in terms of test time and test cost.

Table 1: Minimum number of groups (g) for the proposed plan in case of Marshall – Olkin extended exponential distribution

β	r	c	a					
			0.7	0.8	1.0	1.2	1.5	2.0
0.25	2	0	2	2	2	2	1	1
0.25	3	1	5	4	3	2	2	1
0.25	4	2	11	8	5	3	2	1
0.25	5	3	22	14	7	4	3	2
0.25	6	4	44	26	11	6	3	2
0.25	7	5	88	46	17	8	4	2
0.10	4	0	2	1	1	1	1	1
0.10	5	1	2	2	2	1	1	1
0.10	6	2	4	3	2	2	1	1
0.10	7	3	6	4	3	2	1	1
0.10	8	4	9	6	3	2	2	1
0.10	9	5	15	9	4	3	2	1
0.05	5	0	2	1	1	1	1	1
0.05	6	1	2	2	2	1	1	1
0.05	7	2	3	3	2	1	1	1
0.05	8	3	5	4	2	2	1	1
0.05	9	4	7	5	3	2	1	1
0.05	10	5	11	7	4	2	2	1
0.01	7	0	2	1	1	1	1	1
0.01	8	1	2	2	1	1	1	1
0.01	9	2	3	2	2	1	1	1
0.01	10	3	4	3	2	2	1	1
0.01	11	4	5	4	2	2	1	1
0.01	12	5	7	5	3	2	1	1

Table 2: Minimum number of groups (g) for the proposed plan in case of Marshall – Olkin extended Lomax distribution

β	r	c	a					
			0.7	0.8	1.0	1.2	1.5	2.0
0.25	2	0	2	2	2	1	1	1
0.25	3	1	3	3	2	2	2	1
0.25	4	2	5	4	3	3	2	2
0.25	5	3	9	7	4	3	3	2
0.25	6	4	14	10	6	4	3	2
0.25	7	5	22	15	8	6	4	3
0.10	4	0	1	1	1	1	1	1
0.10	5	1	2	2	1	1	1	1

0.10	6	2	2	2	2	1	1	1
0.10	7	3	3	2	2	2	1	1
0.10	8	4	4	3	2	2	1	1
0.10	9	5	5	4	3	2	2	1
0.05	5	0	1	1	1	1	1	1
0.05	6	1	2	1	1	1	1	1
0.05	7	2	2	2	1	1	1	1
0.05	8	3	3	2	2	1	1	1
0.05	9	4	3	3	2	2	1	1
0.05	10	5	4	3	2	2	1	1
0.01	7	0	1	1	1	1	1	1
0.01	8	1	2	1	1	1	1	1
0.01	9	2	2	2	1	1	1	1
0.01	10	3	2	2	2	1	1	1
0.01	11	4	3	2	2	1	1	1
0.01	12	5	3	3	2	2	1	1

Table 3: Operating characteristic values of the group sampling plan with c=2 for Marshall – Olkin extended exponential distribution

β	r	a	g	μ/μ_0 2	4	6	8	10	12
0.25	4	0.7	11	0.807178	0.972325	0.991640	0.996452	0.998179	0.998945
0.25	4	0.8	8	0.795751	0.970142	0.990949	0.996154	0.998025	0.998855
0.25	4	1.0	5	0.764936	0.964096	0.989024	0.995323	0.997595	0.998605
0.25	4	1.2	3	0.766763	0.963308	0.988697	0.995170	0.997513	0.998556
0.25	4	1.5	2	0.725539	0.953619	0.985481	0.993759	0.996777	0.998127
0.25	4	2.0	1	0.717964	0.947818	0.983227	0.992714	0.996218	0.997795
0.10	6	0.7	4	0.589946	0.920315	0.973775	0.988423	0.993921	0.996425
0.10	6	0.8	3	0.574471	0.913997	0.971313	0.987255	0.993283	0.996041
0.10	6	1.0	2	0.528464	0.896209	0.964385	0.983965	0.991485	0.994956
0.10	6	1.2	2	0.375622	0.837526	0.941809	0.973335	0.985707	0.991485
0.10	6	1.5	1	0.445352	0.854573	0.946683	0.975199	0.986577	0.991950
0.10	6	2.0	1	0.226241	0.726956	0.890079	0.946683	0.970469	0.982031
0.05	7	0.7	3	0.513562	0.893204	0.963421	0.983540	0.991260	0.994823
0.05	7	0.8	3	0.399438	0.851204	0.947496	0.976085	0.987220	0.992399
0.05	7	1.0	2	0.358505	0.825397	0.936093	0.970361	0.983993	0.990414
0.05	7	1.2	1	0.463459	0.858929	0.947715	0.975485	0.986656	0.991964
0.05	7	1.5	1	0.290697	0.769264	0.908513	0.955748	0.975485	0.985069
0.05	7	2.0	1	0.111225	0.598753	0.820997	0.908513	0.947715	0.967519
0.01	9	0.7	3	0.253419	0.768915	0.912483	0.958825	0.977588	0.986512
0.01	9	0.8	2	0.292756	0.782960	0.916747	0.960459	0.978337	0.986903
0.01	9	1.0	2	0.138907	0.656534	0.856598	0.929291	0.960459	0.975786

0.01	9	1.2	1	0.239083	0.724319	0.884850	0.942646	0.967630	0.980030
0.01	9	1.5	1	0.109498	0.586681	0.810268	0.901058	0.942646	0.963998
0.01	9	2.0	1	0.023226	0.372702	0.663591	0.810268	0.884850	0.925526

Table 4: Operating characteristic values of the group sampling plan with c=2 for Marshall – Olkin extended exponential distribution

β	r	a	g	μ/μ_0					
				2	4	6	8	10	12
0.25	4	0.7	5	0.695783	0.934230	0.977050	0.989539	0.994397	0.996661
0.25	4	0.8	4	0.678214	0.926334	0.973727	0.987893	0.993473	0.996093
0.25	4	1.0	3	0.630374	0.905193	0.964663	0.983350	0.990906	0.994509
0.25	4	1.2	3	0.519487	0.857821	0.944226	0.973065	0.985076	0.990906
0.25	4	1.5	2	0.522822	0.844600	0.935752	0.968040	0.981962	0.988869
0.25	4	2.0	2	0.359321	0.735189	0.878031	0.935752	0.962463	0.976301
0.10	6	0.7	2	0.437562	0.825447	0.930232	0.965935	0.980987	0.988353
0.10	6	0.8	2	0.346867	0.771253	0.904312	0.952222	0.972981	0.983306
0.10	6	1.0	2	0.209806	0.656079	0.842556	0.917738	0.952222	0.969960
0.10	6	1.2	1	0.350768	0.736244	0.878210	0.935367	0.961951	0.975818
0.10	6	1.5	1	0.232354	0.624786	0.809987	0.893767	0.935367	0.957986
0.10	6	2.0	1	0.117526	0.458046	0.686292	0.809987	0.878210	0.917908
0.05	7	0.7	2	0.269657	0.720448	0.879243	0.938754	0.965047	0.978274
0.05	7	0.8	2	0.190914	0.645553	0.837999	0.915476	0.950953	0.969181
0.05	7	1.0	1	0.302735	0.707139	0.863155	0.926919	0.956805	0.972471
0.05	7	1.2	1	0.206499	0.610441	0.803463	0.890437	0.933467	0.956805
0.05	7	1.5	1	0.115458	0.476890	0.707139	0.826498	0.890437	0.926919
0.05	7	2.0	1	0.044785	0.302735	0.548806	0.707139	0.803463	0.863155
0.01	9	0.7	2	0.084486	0.500837	0.748372	0.862009	0.917543	0.947185
0.01	9	0.8	2	0.046771	0.405517	0.677219	0.815807	0.887282	0.926665
0.01	9	1.0	1	0.117078	0.502202	0.731336	0.844434	0.903220	0.936110
0.01	9	1.2	1	0.062635	0.385656	0.636802	0.777977	0.856937	0.903220
0.01	9	1.5	1	0.024647	0.251049	0.502202	0.672157	0.777977	0.844434
0.01	9	2.0	1	0.005551	0.117078	0.319933	0.502202	0.636802	0.731116

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