Global Journal of Mathematical Sciences: Theory and Practical.
ISSN 0974-3200 Volume 4, Number 4 (2012), pp. 407-420

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Generalized Alpha Continuous Mappings in Intuitionistic Fuzzy Topological Spaces

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Abstract

The purpose of this paper is to introduce and study the concepts of intuitionistic fuzzy generalized alpha continuous mappings and intuitionistic fuzzy generalized alpha irresolute mappings in intuitionistic fuzzy topological spaces.

Keywords and phrases: Intuitionistic fuzzy topology, Intuitionistic fuzzy generalized alpha continuous mappings and intuitionistic fuzzy generalized alpha irresolute mappings.

1. Introduction

Zadeh [21] introduced the concept of fuzzy sets. After that there have been a number of generalizations of this fundamental concept. Atanassov [1] introduced the notion of intuitionistic fuzzy sets. Using the notion of intuitionistic fuzzy sets, Coker [3] introduced the notion of intuitionistic fuzzy sets. In this paper we introduce the notion of intuitionistic fuzzy generalized alpha continuous mappings and intuitionistic fuzzy generalized alpha irresolute mappings and study some of their properties in intuitionistic fuzzy topological spaces.

2. Preliminaries

Definition 2.1: [1] Let X be a non empty fixed set. An intuitionistic fuzzy set (IFS in short) A in X is an object having the form

$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$

where the functions $\mu_A(x): X \to [0,1]$ and $\nu_A(x): X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \le \mu_A(x) + \nu_A(x) \le 1$ for each $x \in X$.

Denote by IFS(X), the set of all intuitionistic fuzzy sets in X.

Definition 2.2:[1] Let A and B be IFSs of the form $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X\}$ and $B = \{ \langle x, \mu_B, \nu_B(x) \rangle / x \in X\}$ Then

(a) $A \subseteq B$ if and only if $\mu_A(x) \le \mu_B(x)$ and $\nu_A(x) \ge \nu_B(x)$ for all $x \in X$

(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$

(c) $A^c = \{ \langle x, \nu_A(x), \mu_A(x \rangle / x \in X \}$

(d) $A \cap B = \{ \langle x, \mu_A(x) \wedge \mu_A(x), \nu_A(x) \vee \nu_B(x) \rangle / x \in X \}$

(e) $A \cup B = \{ \langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle / x \in X \}$

For the sake of simplicity, we shall use the notation $A = \langle x, \mu_A, \nu_A \rangle$ instead of $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$.

The intuitionistic fuzzy sets $0_\sim = \{ \langle x, 0, 1 \rangle / x \in X \}$ and $1_\sim = \{ \langle x, 1, 0 \rangle / x \in X\}$ are respectively the empty set and the whole set of X.

Definition 2.3: [3] An intuitionistic fuzzy topology (IFT in short) on X is a family τ of IFSs in X satisfying the following axioms.

(i) $0_\sim, 1_\sim \in \tau$

(ii) $G_1 \cap G_2 \in \tau$, for any $G_1, G_2 \in \tau$

(iii) $\cup G_i \in \tau$ for any family $\{ G_i / i \in J \} \subseteq \tau$.

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X.

The complement A^c of an IFOS A in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X.

Definition 2.4: [3] Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A \rangle$ be an IFS in X. Then the intuitionistic fuzzy interior and an intuitionistic fuzzy closure are defined by

$int(A) = \cup \{ G / G$ is an IFOS in X and $G \subseteq A \}$,

$cl(A) = \cap \{ K / K$ is an IFCS in X and $A \subseteq K \}$.

Note that for any IFS A in (X, τ),we have $cl(A^c) = (int(A))^c$ and $int(A^c) =$

$(\mathrm{cl}(A))^c$.

Definition 2.5: [8] An IFS $A = \langle x, \mu_A, \nu_A \rangle$ in an IFTS (X, τ) is said to be an
(i) intuitionistic fuzzy semi closed set (IFSCS in short) if $\mathrm{int}(\mathrm{cl}(A)) \subseteq A$,
(ii) intuitionistic fuzzy pre closed set (IFPCS in short) if $\mathrm{cl}(\mathrm{int}(A)) \subseteq A$,
(iii) intuitionistic fuzzy α-closed set (IFαCS in short) if $\mathrm{cl}(\mathrm{int}(\mathrm{cl}(A)) \subseteq A$,
(iv) intuitionistic fuzzy regular closed set (IFRCS in short) if $A = \mathrm{cl}(\mathrm{int}(A))$,
(v) intuitionistic fuzzy γ-closed set (IFγCS in short) if $\mathrm{cl}(\mathrm{int}(A)) \cap \mathrm{int}(\mathrm{cl}(A)) \subseteq A$.

The family of all IFCS (respectively IFSCS, IFαCS, IFRCS) of an IFTS (X, τ) is denoted by IFC(X) (respectively IFSC(X), IFαC(X), IFRC(X)).

Definition 2.6: [8] An IFS $A = \langle x, \mu_A, \nu_A \rangle$ in an IFTS (X, τ) is said to be an
(i) intuitionistic fuzzy semi open set (IFSOS in short) if $A \subseteq \mathrm{cl}(\mathrm{int}(A))$,
(ii) intuitionistic fuzzy pre open set (IFPOS in short) if $A \subseteq \mathrm{int}(\mathrm{cl}(A))$,
(iii) intuitionistic fuzzy α-open set (IFαOS in short) if $A \subseteq \mathrm{int}(\mathrm{cl}(\mathrm{int}(A)))$,
(iv) intuitionistic fuzzy regular open set (IFROS in short) if $A = \mathrm{int}(\mathrm{cl}(A))$,
(v) intuitionistic fuzzy γ-open set (IFγOS in short) if $A \subseteq \mathrm{int}(\mathrm{cl}(A)) \cup \mathrm{cl}(\mathrm{int}(A))$.

The family of all IFOS (respectively IFSOS, IFαOS, IFROS) of an IFTS (X, τ) is denoted by IFO(X) (respectively IFSO(X), IFαO(X), IFRO(X)).

Definition 2.7: [20] Let an IFS A of an IFTS (X, τ). Then
$\mathrm{scl}(A) = \cap\{ K / K$ is an IFSCS in X and $A \subseteq K \}$.
$\mathrm{sint}(A) = \cup\{ K / K$ is an IFSOS in X and $K \subseteq A\}$.

Note that for any IFS A in (X, τ), $\mathrm{scl}(A^c) = (\mathrm{sint}(A))^c$ and $\mathrm{sint}(A^c) = (\mathrm{scl}(A))^c$

Definition 2.8: [17] An IFS A of an IFTS (X, τ) is an
(i) intuitionistic fuzzy generalized closed set (IFGCS in short) if $\mathrm{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in X.
(ii) intuitionistic fuzzy regular generalized closed set (IFRGCS in short) if $\mathrm{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFROS in X.

Definition 2.9: [10]An IFS A of an IFTS (X, τ) is said to be an intuitionistic fuzzy generalized α closed set (IFGαCS in short) if $\alpha\mathrm{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFαOS in X.

Result 2.10: [10] Every IFCS,IFGCS,IFRCS, IFαCS is an IFGαCS but the converses are not true in genereal.

Definition 2.11: [10] An IFS A of an IFTS (X, τ) is said to be an intuitionistic fuzzy generalized α open set (IFGαOS in short) if the complement A^c is an IFGαCS in X.

Definition 2.12:[4]Let f be a mapping from an IFTS (X, τ) into an IFTS (Y,σ).Then f is said to be intuitionistic fuzzy continuous(IF continuous in short) if $f^{-1}(B) \epsilon IFO(X)$ for every B $\epsilon \sigma$.

Definition 2.13: [8]Let f be a mapping from an IFTS (X, τ) into an IFTS (Y,σ).Then f is said to be
(i) intuitionistic fuzzy semi continuous (IFS continuous in short) if $f^{-1}(B) \epsilon\ IFSO(X)$ for every B $\epsilon \sigma$.
(ii) intuitionistic fuzzy α continuous (IFα continuous in short) if $f^{-1}(B) \epsilon IF\alpha O(X)$ for every B $\epsilon \sigma$.
(iii) intuitionistic fuzzy pre continuous (IFP continuous in short) if $f^{-1}(B) \epsilon IFPO(X)$ for every B $\epsilon \sigma$.

Result 2.14:[8] Every IF continuous mapping is an IFα continuous and every IFα continuous mapping is an IFS continuous mapping.

Definition 2.15:[6] A mapping f: (X, τ)→ (Y,σ) is called an intuitionistic γ continuous(IFγccontinuous in short) if $f^{-1}(B)$ is an $IF\gamma OS(X)$ in (X, τ)for every B $\epsilon \sigma$.

Definition 2.16:[17] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y,σ).Then f is said to be intuitionistic fuzzy generalized continuous (IFG continuous in short) if $f^{-1}(B) \epsilon IFGCS(X)$ for every IFCS B in Y.

Result 2.17:[17] Every IF continuous mapping is an IFG continuous mapping.

Definition 2.18: [14]A mapping f: (X, τ)→ (Y,σ) is called an intuitionistic fuzzy generalized semi continuous (IFGScontinuous in short) if $f^{-1}(B)$ is an $IFGSCS(X)$ in (X, τ) for every IFCS B of (Y,σ).

Definition 2.19: [14]Let f be a mapping from an IFTS (X, τ) into an IFTS (Y,σ).Then f is said to be intuitionistic fuzzy irresolute (IF irresolute in short) $f^{-1}(B) \epsilon IFCS(X)$ for every IFCS B in Y.

Definition 2.20:[14] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y,σ).Then f is said to be intuitionistic fuzzy generalized irresolute (IFG irresolute in short) $f^{-1}(B) \epsilon IFGCS(X)$ for every IFGCS B in Y.

Definition 2.21: An IFTS (X, τ) is said to be an intuitionistic fuzzy $\alpha_k T_{1/2}$(IF$\alpha_k T_{1/2}$ in short) space if every IFGαCS in X is an IFCS in X.

Definition 2.22: An IFTS (X, τ) is said to be an intuitionistic fuzzy $\alpha_l T_{1/2}$(IF$\alpha_l T_{1/2}$ in short) space if every IFGαCS in X is an IFαCS in X.

Definition 2.23:[15]The IFS $C(\alpha, \beta) = \{ x, c_\alpha, c_{1-\beta} \}$where $\alpha \in (0, 1]$, $\beta \in [0, 1)$ and $\alpha + \beta \leq 1$ is called an intuitionistic fuzzy point in X.

Definition 2.24: [15] Two IFSs are said to be q-coincident (A q B in short) if and only if there exists an element $x \in X$ such that $\mu_A(x) > \nu_B(x)$ or $\nu_A(x) < \mu_B(x)$.

Definition 2.25: [15] For any two IFSs A and B of X, $\rceil$(A q B) iff $A \subseteq B^c$.

3. Intuitionistic fuzzy generalized alpha continuous mappings

In this section we introduce intuitionistic fuzzy generalized alpha continuous mapping and studied some of its properties.

Definition 3.1: A mapping f: (X, τ)→ (Y,σ) is called an intuitionistic fuzzy generalized alpha continuous (IFGα continuous in short) mapping if $f^{-1}(B)$ is an $IFG\alpha CS(X)$ in (X, τ) for every IFCS B of (Y,σ).

Example 3.2: Let us consider X= {a, b},Y= {u, v} and $G_1 = \langle x, (0.3,0.3), (0.7,0.6)\rangle$, $G_2 = \langle x, (0.7,0.6), (0.3,0.4)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTS on X and Y respectively. Define a mapping f: (X, τ)→ (Y,σ)byf(a) = u and f(b) = v. Then f is an IFGα continuous mapping.

Theorem 3.3: Every IF continuous mapping is an IFGα continuous mapping but not conversely.

Proof: Let f: (X, τ)→ (Y,σ) be an IF continuous mapping. Let A be an IFCS in Y. Since f is IF continuous mapping, $f^{-1}(A)$is an IFCS in X. Since every IFCS is an IFGαCS , $f^{-1}(A)$ is an IFGαCS in X. Hence f is an IFGα continuous mapping.

Example 3.4: Let us define X = {a, b}, Y = {u, v} and $G_1 = \langle x, (0.2,0.3), (0.8,0.7)\rangle$, $G_2 = \langle y, (0.4,0.4), (0.6,0.6)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTS on X and Y respectively. Define a mapping f: (X, τ)→ (Y,σ) by f(a) = u and f(b) = v. Then f is an IFGα continuous mapping but not IF continuous mapping, since ${G_2}^c = \langle y, (0.6,0.6), (0.4,0.4)\rangle$ is an IFCS in Y,but $f^{-1}({G_2}^c)$ is not an IFCS in X.

Theorem 3.5: Every IFα continuous mapping is an IFGα continuous mapping but not conversely.

Proof: Let f: (X, τ)→ (Y,σ) be an IFα continuous mapping. Let A be an IFCS in Y. Since f is IFα continuous mapping, $f^{-1}(A)$is an IFαCS in X. Since every IFαCS is an IFGαCS, $f^{-1}(A)$ is an IFGαCS in X. Hence f is an IFGα continuous mapping.

Example 3.6: Let us consider X = {a, b}, Y = {u, v}and $G_1 = \langle x, (0.2,0.3), (0.7,0.6)\rangle$, $G_2 = \langle y, (0.3,0.3), (0.5,0.6)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTS on X and Y respectively. Define a mapping f: (X, τ)→ (Y,σ) by f(a) = u and f(b) = v. Then f is an IFGα continuous mapping but not IFα continuous mapping, since ${G_2}^c$= $\langle y, (0.5,0.6), (0.3,0.3)\rangle$ is an IFCS in Y,but $f^{-1}({G_2}^c)$ is not IFαCS in X.

Theorem 3.7: Every IFGα continuous mapping is an IFαG continuous mapping but converse is not true in general.

Proof: Let f: (X, τ)→ (Y,σ) be an IFGα continuous mapping. Let A be an IFCS in Y. Since f is IFGα continuous mapping, $f^{-1}(A)$is an IFGαCS in X. Since every IFGαCS is an IFαGCS, $f^{-1}(A)$is an IFαGCS in X. Hence f is an IFαG continuous mapping.

Example 3.8: Let us considerX = {a, b}, Y = {u, v} and G_1= $\langle x, (0.8,0.8), (0.2,0.1)\rangle$, G_2= $\langle y, (0.1,0.3), (0.9,0.7)\rangle$. Then τ = $\{0_\sim, G_1, 1_\sim\}$ and σ =$\{0_\sim, G_2, 1_\sim\}$ are IFTS on X and Y respectively. Define a mapping f: (X, τ)→ (Y,σ) by f(a) = u and f(b) = v. Then f is an IFαG continuous mapping but not IFGα continuous mapping since ${G_2}^c$= $\langle y, (0.9,0.7), (0.1,0.3)\rangle$ is an IFCS in Y, but $f^{-1}({G_2}^c)$ is not an IFGαCS in X.

Theorem 3.9: Every IFGα continuous mapping is an IFSG continuous mapping but not conversely.

Proof: Let $f: (X, \tau) \to (Y, \sigma)$ be an IFGα continuous mapping. Let A be an IFCS in Y. Then by hypothesis $f^{-1}(A)$ is an IFGαCS in X. Since every IFGαCS is an IFSGCS, $f^{-1}(A)$ is an IFSGCS in X. Hence f is an IFSG continuous mapping.

Example 3.10: Let us consider X= {a, b},Y={u,v}and G_1=⟨x,(0.4, 0.4),(0.6, 0.6)⟩, G_2=⟨ y, (0.7, 0.6), (0.1, 0.1) ⟩. Then τ = $\{0_\sim, \mathrm{G}_1, 1_\sim\}$ and σ = $\{0_\sim, \mathrm{G}_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f: (X, \tau) \to (Y, \sigma)$ by f(a) = u, f(b) = v. Clearly f is an IFSG continuous mapping. Now we have $f^{-1}({G_2}^c)$ =⟨ y, (0.1, 0.1), (0.7, 0.6) ⟩. αcl$f^{-1}({G_2}^c)$=${G_1}^c \nsubseteq G_1$, which shows that 'f' is not IFGα α continuous mapping .

Theorem 3.11: Every IFGα continuous mapping is an IFGS continuous mapping but not conversely.

Proof: Let $f: (X, \tau) \to (Y, \sigma)$ be an IFGα continuous mapping. Let A be an IFCS in Y. Then by hypothesis $f^{-1}(A)$ is an IFGαCS in X. Since every IFGαCS is an IFGSCS, $f^{-1}(A)$ is an IFGSCS in X. Hence f is an IFGS continuous mapping.

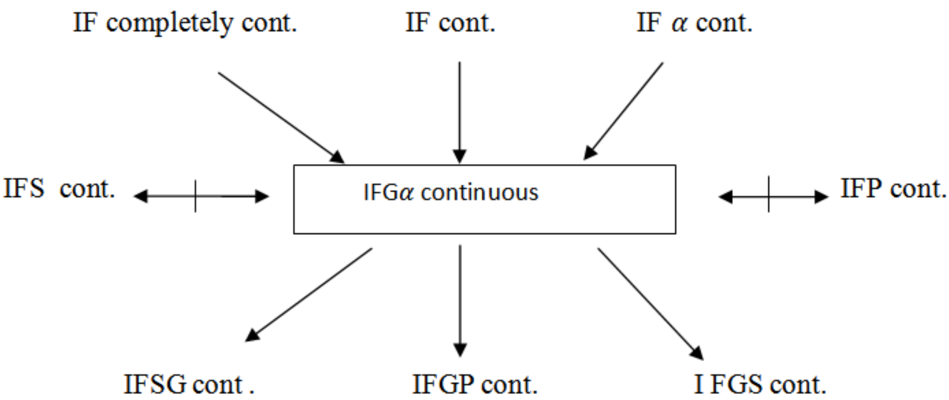
Example 3.12: Let us consider X={a, b},Y={u, v}and G_1=⟨x,(0.4, 0.3),(0.6, 0.4)⟩, G_2= ⟨ y, (0.8, 0.7), (0.2, 0.3) ⟩. Then τ = $\{0_\sim, \mathrm{G}_1, 1_\sim\}$ and σ = $\{0_\sim, \mathrm{G}_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f: (X, \tau) \to (Y, \sigma)$ by f(a) = u, f(b) = v. Clearly f is an IFGS continuous mapping. Now we have $f^{-1}({G_2}^c)$ =⟨ x, (0.2, 0.3), (0.8, 0.7)⟩. αcl$f^{-1}({G_2}^c)$ =${G_1}^c \nsubseteq G_1$,which shows that 'f' is not an IFGα continuous mapping .

Theorem 3.13: Every IFGα continuous mapping is an IFGP continuous mapping but not conversely.

Proof: Let $f:(X,\tau)\to(Y,\sigma)$ be an IFGα continuous mapping. Let A be an IFCS in Y. Then by hypothesis $f^{-1}(A)$ is an IFGαCS in X. Since every IFGαCS is an IFGPCS, $f^{-1}(A)$ is an IFGPCS in X. Hence f is an IFGP continuous mapping.

Example 3.14: Let us consider X ={a,b},Y ={u ,v}and G_1= ⟨ x,(0.2,0.3),(0.5,0.7)⟩, G_2=⟨ y, (0.8, 0.9), (0.2, 0.1) ⟩. Then $\tau=\{0_\sim, G_1, 1_\sim\}$ and $\sigma=\{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f:(X,\tau)\to(Y,\sigma)$ by f(a) = u, f(b) = v. Clearly f is an IFGP continuous mapping. Now we have $f^{-1}({G_2}^c)$=⟨x, (0.2, 0.1), (0.8, 0.9)⟩. Here αcl$f^{-1}({G_2}^c)={G_1}^c\nsubseteq G_1$, which shows that 'f' is not IFGα continuous mapping .

The following diagram implications are true:



Here cont. means Intuitionistic fuzzy continuous mapping.

Remark 3.15: An IFP continuous mapping and IFGα continuous mapping is independent of each other.

Example 3.16 : Let us consider X= {a,b},Y={u,v} and G_1=⟨x, (0.2, 0.3),(0.7,0.7)⟩, G_2=⟨ y, (0.8, 0.8), (0.2, 0.2) ⟩. Then $\tau=\{0_\sim, G_1, 1_\sim\}$ and $\sigma=\{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f:(X,\tau)\to(Y,\sigma)$ by f(a)= u, f(b) = v. Clearly f is an IFP continuous mapping. Now we have $f^{-1}({G_2}^c)$ ==⟨ x, (0.2, 0.2), (0.8, 0.8)⟩ . Here αcl $f^{-1}({G_2}^c)={G_1}^c\nsubseteq G_1$, which shows that 'f' is not an IFGα continuous mapping .

Example 3.17: Let us consider X={a, b},Y ={u, v} and G_1=⟨x,(0.3,0.3), (0.7, 0.7) ⟩, G_2= ⟨ y, (0.2, 0.2), (0.8, 0.5) ⟩. Then $\tau=\{0_\sim, G_1, 1_\sim\}$ and $\sigma=\{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f:(X,\tau)\to(Y,\sigma)$ by f(a) = u, f(b) = v. Clearly f is an IFGα continuous mapping. Now we have

$f^{-1}({G_2}^c)$ =⟨y, (0.8, 0.5), (0.2, 0.2)⟩, $int\, f^{-1}({G_2}^c)= G_1,\ \ clG_1 = {G_1}^c$ which shows that cl(int($f^{-1}({G_2}^c)$) $\nsubseteq f^{-1}({G_2}^c)$. Hence 'f' is not an IFP continuous mapping .

Remark 3.18: An IFS continuous mapping and IFGα continuous mapping are independent of each other.

Example 3.19: Let us consider = {a,b},Y ={u, v} and G_1=⟨ x, (0.4, 0.3), (0.6, 0.6) ⟩, G_2=⟨ y, (0.2, 0.2), (0.7, 0.8) ⟩. Then τ = $\{0_{\sim}, \mathrm{G}_1, 1_{\sim}\}$ and σ = $\{0_{\sim}, \mathrm{G}_2, 1_{\sim}\}$ are IFTs on X and Y respectively. Define a mapping $f: (X, \tau) \to (Y, \sigma)$ by f(a) = u, f(b) = v. Clearly f is an IFGα continuous mapping. Now we have $f^{-1}({G_2}^c)$=⟨x, (0.7,0.8), (0.2,0.2)⟩Let cl$f^{-1}({G_2}^c)$= 1, int(cl($f^{-1}({G_2}^c)$) =1 $\nsubseteq f^{-1}({G_2}^c)$, which shows that 'f' is not an IFS continuous mapping.

Example 3.20 : Let us consider X = {a, b},Y ={u, v} and G_1=⟨ x, (0, 0.1), (1,0.9) ⟩, G_2=⟨ y, (1, 0.9), (0, 0.1 ⟩. Then τ = $\{0_{\sim}, \mathrm{G}_1, 1_{\sim}\}$ and σ = $\{0_{\sim}, \mathrm{G}_2, 1_{\sim}\}$ are IFTs on X and Y respectively. Define a mapping $f: (X, \tau) \to (Y, \sigma)$ by f(a) = u, f(b) = v. Clearly f is an IFS continuous mapping . Now we have $f^{-1}({G_2}^c)$ = ⟨ x,(0, 0.1), (1,0.9) ⟩. Let cl$f^{-1}({G_2}^c)$ = ${G_1}^c$,int ${G_1}^c$= G_1, clG_1 = ${G_1}^c$, αcl($f^{-1}({G_2}^c)$) = ${G_1}^c \nsubseteq G_1$ which shows that 'f' is not IFGα continuous mapping.

Theorem 3.21: If *f*: (X, τ) →(Y, σ) is IFGα- continuous mapping then for each intuitionistic fuzzy point c(α,β) of X and each IFOS V of Y such that *f*(c(α, β)) ⊆ V, there exists an IFGα- open set U of X such that c(α, β) ⊆ U and *f*(U) ⊆ V.

Proof: Let c(α, β) be intuitionistic fuzzy point of X and V be an IFOS of Y such that *f*(c(α, β)) ⊆ V. Put U = $f^{\ -1}$(V). Then by hypothesis U is an IFGα- open set of X such that c(α, β) ⊆ U and f (U) =$f\,(f^{-1}$(V))⊆ V.

Theorem 3.22: If *f:* (X, τ) →(Y, σ) is an IFGα- continuous mapping then for each intuitionistic fuzzy point c(α,β) of X and each IFOS V of Y such that *f*(c(α,β))qV, there exists an IFGα- open set U of X such that c(α,β) q U and f(U) ⊆ V.

Proof: Let c(α,β) be intuitionistic fuzzy point of X and V be an IFOS of Y such that *f*(c(α,β)) q V. Put U = $f^{\ -1}$(V). Then by hypothesis U is an IFGα-open set of X such that c(α,β)q U and *f*(U) =*f*($f^{\ -1}$ (V))⊆ V.

Theorem 3.23: Let $f: (X, \tau) \to (Y, \sigma)$ be a mapping from an IFTS X into an IFTS Y. Then the following are equivalent if X is an IF$\alpha_k T_{1/2}$ space.

f is an IFGα continuous mapping

$f^{-1}(B)$ is an IFGαCS in X for every IFCS B in Y.

$cl(int(cl(f^{-1}(B))) \subseteq f^{-1}(cl(B))$ for every IFS B in Y.

Proof: (i)⟹(ii) : is obviously true.
(ii)⟹(iii) : Let B be an IFS in Y. Then cl(B) is an IFCS in Y. By hypothesis $f^{-1}(cl(B))$ is an IFGαCS in X. Since X is an IF$\alpha_k T_{1/2}$ space, $f^{-1}(cl(B))$ is an IFCS in X. Therefore cl($f^{-1}cl(B)$) = $f^{-1}(cl(B))$. Now we have
cl($int(cl(f^{-1}(B)))$) ⊆ cl($int(cl(f^{-1}(cl(B))))$) ⊆ $f^{-1}(cl(B))$.

(iii)⟹(i): Let B be an IFCS in Y. By hypothesis cl($int(cl(f^{-1}(B)))$)⊆ $f^{-1}(cl(B))$= $f^{-1}(B)$. This implies $f^{-1}(B)$ is an IFαCS in X and hence $f^{-1}(B)$ is an IFGαCS in X. Therefore f is an IFGα continuous mapping.

Theorem 3.24: A mapping $f:(X,\tau) \to (Y,\sigma)$ is IFGα continuous mapping if and only if the inverse image of each IFOS in Y is an IFGαOS in X.

Proof: Let A be an IFOS in Y. This implies A^c is an IFCS in Y. Since f is an IFGαCS in X. $f^{-1}(A^c)$ = $\overline{f^{-1}}(A)$, $f^{-1}(A)$ is an IFGαOS in X.

Theorem 3.25: Let $f:(X,\tau) \to (Y,\sigma)$ be a mapping and let $f^{-1}(A)$ is an IFRCS in X for every IFCS A in Y. Then f is an IFGα continuous mapping.

Proof: Let A be an IFCS in Y. Then $f^{-1}(A)$ is an IFRCS in X. Since every IFRCS is an IFGαCS, $f^{-1}(A)$ is an IFGαCS in X. Hence f is an IFGα continuous mapping

Theorem 3.26: Let $f:(X,\tau) \to (Y,\sigma)$ be a mapping from an IFTS X into an IFTS Y. Then the following are equivalent if X is an IF$\alpha_k T_{1/2}$ space.
(i) f is an IFGα continuous mapping
(ii) $f^{-1}(A)$ is an IFGαOS in X for every IFOS A in Y.
(iii) $f^{-1}(int(A)) \subseteq int(cl\left(int(f^{-1}(A))\right)$ for every IFS A in Y

Proof: (i)⟹(ii) : is obviously true.
(ii)⟹(iii) : Let A be an IFS in Y. Then int(A) is an IFOS in Y. By hypothesis $f^{-1}(int(A))$ is an IFGαOS in X. Since X is an IF$\alpha_k T_{1/2}$ space, $f^{-1}(int(A))$ is an IFOS in X. Therefore$f^{-1}(intA) = int((f^{-1}(int(A)) \subseteq int((cl(int(f^{-1}(A))))$
(iii)⟹(i): Let A be an IFCS in Y. Then its complement A^c is an IFOS in Y. By hypothesis $f^{-1}(intA^c) \subseteq int((cl(int(f^{-1}(A^c)))))$.Hence $f^{-1}(A^c)$ is an IFαOS in X. Since every IFαOS is an IFGαOS, $f^{-1}(A^c)$ is an FGαOS in X. Hence f is an IFGα continuous mapping.

Theorem 3.27: Let $f:(X,\tau) \to (Y,\sigma)$ be an IFGα continuous mapping, then f is an IF continuous mapping if X is an IF$\alpha_k T_{1/2}$ space.

Proof: Let A be an IFCS in Y. Then $f^{-1}(A)$ is an IFGαCS in X, by hypothesis.
Since X is an IF$\alpha_k T_{1/2}$ space, $f^{-1}(A)$ is an IFCS in X. Hence f is an IF

continuous mapping.

Theorem 3.28: Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be an IFGα continuous mapping and $g:(Y,\sigma)\rightarrow(Z,\delta)$ is IF continuous mapping then $g\circ f\ :(X,\tau)\rightarrow(Z,\delta)$ is IFGα continuous mapping.

Proof: Let A be an IFCS in Z. Then $g^{-1}(A)$ is an IFCS in Y, by hypothesis. Since f is an IFGα continuous mapping, $f^{-1}(g^{-1}(A))$ is an IFGαCS in X. Since X is an IF$\alpha_k T_{1/2}$ space, $f^{-1}(g^{-1}(A))^{-1}$ is an IFCS in X. Since we know that $(g\circ f\ )^{-1}(A)=\ f^{-1}(g^{-1}(A))$ and hence $g\circ f$ is an IFGα continuous mapping.

Remark 3.29: The composition of two intuitionistic fuzzy gα– continuous mapping may not be intuitionistic fuzzy gα – continuous.

Example 3.30: Let X = {a, b}, Y= {x, y} and Z= {p, q} and intuitionistic fuzzy sets U, V and W defined as follows:
U = {< x, (0.4, 0.4), (0.6, 0.6)}
V = {< y, (0.3, 0.2), (0.4, 0.4)}
W = {< z, (0.6, 0.7), (0.3, 0.3)}

Let τ= { $0_\sim$, U, $1_\sim$}, σ = { $0_\sim$, V, $1_\sim$} and μ = {$0_\sim$, W, $1_\sim$} be intuitionistic fuzzy topologies on X , Y and Z respectively. Let the mapping *f*: (X, τ) →(Y, σ) defined by *f*(a) = x and *f*(b) = y and *g* : (Y, σ) →(Z,μ) defined by *g*(x) = p and *g*(y) = q. Then the mappings *f* and *g* are IFGα -continuous mappings but the mapping *gof* : (X, τ) →(Z, μ) is not IFGα - continuous mapping.

Definition 3.31: Let (X,τ) be an IFTS. The generalized closure (gαcl(A) in short) for any IFS A is defined as follows.
gαcl(A) = $\cap\{K\ /\ K\ is\ \ an\ IFG\alpha CS\ in\ X\ and\ A\subseteq K\}$.
If A is an $IFG\alpha CS$, then gαcl(A) = A.

Remark: It is clear that A ⊆ gαcl(A) ⊆cl (A)

Theorem 3.32: Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be an IFGα continuous mapping.
Then the following statements hold.
f(gαcl(A)) ⊆ $cl(f(A))$, for every IFS A in X.
gαcl($f^{-1}(B)$) ⊆ $f^{-1}(cl(B))$, for every IFS B in X.

Proof : (i) Let A ⊆ X. Then cl(f(A)) is an IFCS in Y. Since f is an IFGα continuous mapping, $f^{-1}(cl(f(A)))$ is an IFGαCS in X.(That is gαcl(A) ⊆ $f^{-1}(cl(f(A)))$).
Since A ⊆ $f^{-1}(f(A))$ ⊆ $f^{-1}(cl(f(A)))$ and $f^{-1}(cl(f(A)))$ is an IFGα- closed, implies gαcl(A) ⊆ $f^{-1}(cl(f(A)))$. Hence f [gαcl(A)] ⊆ $cl(f(A))$.
(ii) Replacing A by $f^{-1}(B)$ in (i), we get f(gαcl($f^{-1}(B)$)) ⊆ $cl(f(f^{-1}(B))$ ⊆ cl(B). Hence gαcl($f^{-1}(B)$) ⊆ $f^{-1}(cl(B))$, for every IFS B in Y.

Theorem 3.33: Let $f:(X,\tau)\to(Y,\sigma)$ be a mapping from an IFTS X into an IFTS Y. If X is an IFG$\alpha_l T_{1/2}$ space then f is IFGα - continuous mapping if and only if it is IFα continuous mapping.

Proof : Let f be an IFGα -continuous mapping and let A be an IFCS in Y. Then by definition, $f^{-1}(A)$ is an IFGαCS in X. Since X is an IFG$\alpha_l T_{1/2}$ space, $f^{-1}(A)$ is an IFαCS in X. Hence f is IFα- continuous mapping.

Conversely assume that f is IFα- continuous mapping by theorem 3.5, f is an IFGα-continuous mapping.

Theorem 3.34: Let $f:(X,\tau)\to(Y,\sigma)$ be a mapping from an IFTS X into an IFTS Y. Then the following statements are equivalent
(i) f is an IFGα continuous mapping
(ii) For each IFP $p(\alpha,\beta)\in X$ and every IFN A of f($p(\alpha,\beta)$), there exist an IFGαCS B such that $p(\alpha,\beta)\in B\subseteq f^{-1}(A)$.
(iii) For each IFP $p(\alpha,\beta)\in X$ and every IFN A of f($p(\alpha,\beta)$), there exist an IFGαCS B such that $p(\alpha,\beta)\in B$ and $f(B)\subseteq A$.

Proof: (i)⟹(ii) : Assume that f is an IFGα continuous mapping. Let $p(\alpha,\beta)$ be an IFP in X and A be an IFN of f($p(\alpha,\beta)$). Then by definition of IFN, there exists an IFCS C in Y, such that f($p(\alpha,\beta)$) ∈ C ⊆ A.

Taking B = $f^{-1}(C)\in$ X, Since f is an IFGα continuous mapping, $f^{-1}(C)$ is IFGαCS in X and $p(\alpha,\beta)\in B\subseteq f^{-1}\left(\mathrm{f}(p(\alpha,\beta))\right)\subseteq f^{-1}(C)=B\subseteq f^{-1}(A)$.

Hence $p(\alpha,\beta)\in B\subseteq f^{-1}(A)$.

(ii)⟹(iii) : Let $p(\alpha,\beta)$ be an IFP in X and A be an IFN of f($p(\alpha,\beta)$), such that there exists an IFGαCS B with $p(\alpha,\beta)\in B$ and $B\subseteq f^{-1}(A)$. This implies $f(B)\subseteq A$. Hence (iii) holds.

(iii)⟹(i) : Assume that (iii) holds. Let B be an IFCS in Y and take $p(\alpha,\beta)\in f^{-1}(B)$. Then f($p(\alpha,\beta)$) $\in B$.Since B is an IFCS in Y, B is an IFN of f($p(\alpha,\beta)$).

Then from (iii) there exists an IFGαCS A such that $p(\alpha,\beta)\in A$ and f(A) $\subseteq B$.

Therefore $p(\alpha,\beta)\in A\subseteq f(f^{-1}(A))\subseteq f^{-1}(B)$.That is $(\alpha,\beta)\in A\subseteq f^{-1}(B)$.

Since $p(\alpha,\beta)$ be an arbitrary point and $f^{-1}(B)$ is union of all IFP contained in $f^{-1}(B)$, by assumption $f^{-1}(B)$ is an IFGαCS. Hence f is an IFGα continuous mapping.

4.Intuitionistic fuzzy generalized alpha irresolute mappings

In this section we introduce intuitionistic fuzzy generalized alpha irresolute mappings and studied some of its properties.

Definition 4.1: A mapping $f:(X,\tau)\to(Y,\sigma)$ is called an intuitionistic fuzzy generalized alpha irresolute (IFGα irresolute) mapping if $f^{-1}(B)$ is an IFGαCS in (X,τ) for every IFGαCS B of (Y,σ).

Theorem 4.2: Let $f: X \to Y$ be a mapping from an IFTS X into an IFTS Y. Then every IFGα- irresolute mapping is an IFGα continuous mapping

Proof: Let A be an IFCS in Y. We know that every IFCS is an IFGαCS. Therefore A is an IFGαCS in Y. Since f is an IFGα irresolute mapping, by definition $f^{-1}(A)$ is IFGαCS in X. Hence f is an IFGα continuous mapping.

Theorem 4.3 : Let $f: X \to Y$ be a mapping from an IFTS X into an IFTs Y. Then the following statements are equivalent.
(i) f is an IFGα irresolute mapping
(ii) $f^{-1}(B)$ is an IFGαOS in X for every IFGαOS B in Y.
(iii) $g\alpha\mathrm{cl}(f^{-1}(B)) \subseteq f^{-1}(g\alpha(clB))$, for every IFS B in Y.
(iv) $f^{-1}(g\alpha(intB)) \subseteq g\alpha int(f^{-1}(B)$, for every IFS B in Y.

Proof : (i)⟹(ii) . It can be proved by taking the complement of definition 4.1.
(ii)⟹(iii): Let B be any IFS in Y. Then $B \subseteq cl(B)$. Also $f^{-1}(B) \subseteq f^{-1}g\alpha(cl(B))$. Since $g\alpha(cl(B))$ is an IFGαCS in Y, $f^{-1}g\alpha(cl(B))$ is an IFGαCS in X. Therefore $g\alpha\mathrm{cl}(f^{-1}(B) \subseteq f^{-1}g\alpha(cl(B))$.
(iii)⟹(iv): Let B be any IFS in Y. Then int (B) is an IFOS in Y. Then $f^{-1}(int(B))$ is an IFGαOS in X. Since $g\alpha(int(B))$ is an IFGαOS in X, $f^{-1}(g\alpha(int(B))$ is an IFGαOS in X. Therefore $g\alpha(cl(B))$ is an IFGαCS in Y, $f^{-1}(g\alpha(cl(B)))$ is an IFGαCS in X. Therefore $g\alpha\mathrm{cl}(f^{-1}(B)) \subseteq f^{-1}(g\alpha(clB))$.
(iv)⟹(i): Let B be any IFGαOS in Y. Then $g\alpha(intB)$= B. By our assumption we have $f^{-1}(B) = f^{-1}\big(g\alpha(intB)\big) \subseteq g\alpha int f^{-1}(B))$, so $f^{-1}(B)$ is an IFGαOS in X. Hence f is an IFGα irresolute mapping.

Theorem 4.5 : Let $f: X \to Y$ be an IFGα - irresolute mapping, then f is an IF irresolute mapping if X is an IF$\alpha_k T_{1/2}$ space.

Proof : Let A be an IFCS in Y. Then A is an IFGαCS in Y. Therefore f^{-1}(A) is an IFGαCS in X, by hypothesis. Since X is an IF$\alpha_k T_{1/2}$ space, f^{-1}(A) is an IFCS in X. Hence f is an IF –irresolute mapping.

Theorem 4.6 : Let $f: X \to Y$ be an IFGα -irresolute mapping, then f is an IFα irresolute mapping if (X, τ) is an IF$\alpha_l T_{1/2}$ space.

Proof : Let B be an IFαCS in Y. Then B is an IFGαCS in Y. Since f is an IFGα – irresolute, f^{-1}(B) is an IFGαCS in X,by hypothesis. Since X is an IF$\alpha_l T_{1/2}$ space, f^{-1}(B) is an IFαCS in X. Hence f is an IFα –irresolute mapping.

Theorem 4.7: Let f : (X, τ) → (Y, σ) and g : (Y, σ) →(Z,δ) are IFGα irresolute mappings, where X,Y,Z are IFTS. Then $g \circ f$ is an IFGα irresolute mapping.

Proof : Let A be an IFGαCS in Z. Since g is an IFGα irresolute mapping g^{-1} (A) is an IFGαCS in Y. Also since f is an IFGα - irresolute mapping , $f^{-1}(g^{-1}$ (A)) is an IFGαCS in X .$(g \circ f)^{-1}$=$f^{-1}(g^{-1}$ (A)) for each A in Z. Hence $(g \circ f)^{-1}(A)$ is an IFGαCS in X. Therefore $g°f$ is an IFGα irresolute mapping.

Theorem 4.8: Let f : (X, τ) → (Y, σ) and g : (Y, σ) →(Z,δ) are IFGα irresolute and IF continuous mappings respectively, where X,Y,Z are IFTS. Then $g \circ f$ is an IFGα continuous mapping.

Proof : Let A be any IFCS in Z. Since g is an IF continuous mapping, g^{-1} (A) is an IFGα- closed set in Y. Also since f is an IFGα- irresolute mapping , $f^{-1}(g^{-1}$ (A)) is an IFGα- closed set in X .Since $(g \circ f\,)^{-1} = f^{-1}(g^{-1}$ (A)) is an IFGαCS in X for each A in Z. Hence $(g \circ f\,)^{-1}(A)$ is an IFGαCS set in X. Therefore $g \circ f$ is an IFGα irresolute mapping.

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