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New Generalization of Topological Weak Continuity

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Abstract

New generalization of (a,s)-continuous functions called (Λ_a,s)-continuous functions using Λ_a-closed sets are introduced. Basic properties and characterizations of such functions are investigated.

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1. Introduction

In recent literature,we find many topologists have focused their research in the direction of investigating various types of generalized continuity.One of the outcomes of their research leads to the initiation of different orientations of contra-continuous functions such as contra-continuity [5], contra a-continuity [5], (a,s)-continuity [3] and so on. Recently Thivagar et al. [7] introduced a new class of sets called Λ_a-closed sets. In this paper, using the notion of Λ_a-closed sets, a new variation of contra-continuous functions which is a generalization of (a,s)-continuous functions called (Λ_a,s)-continuous functions is introduced and investigated. We also investigate the relationships among (Λ_a, s)-continuous functions, separation axioms, connectedness, compactness and normality.

2. Preliminaries

Throughout this paper (X,τ) and (Y,σ) and (Z,η) (or simply X, Y and Z) represent non-empty topological spaces on which no separation axioms are assumed. A subset A of a topological space X is called a semi-open set(resp.pre-open set, β)-open set if $A \subset cl(int(A))$ (resp. $A \subset int(cl(A))$, $A \subset cl(int(cl(A)))$. The family of all semi-open (resp.pre-open, β-open) sets of X is denoted by $SO(X)$(resp.PO(X), $\beta O(X)$). A subset A of a space X is called regular open if $A = int(cl(A))$.The complement of regular open is called the regular closed set. The family of all regular open sets (resp. regular closed) in X is denoted by RO(X)(resp.RC(X)). A subset A of a space X is called δ-closed [6] if $A = cl_\delta(A)$, where $cl_\delta(A) = \{x \in X : int(cl(U)) \cap A \neq \phi, U \in \tau$ and $x \in U\}$. The complement of δ-closed set is called δ-open set. A subset A of a topological space X is called an a-open set [5] if $A \subset int(cl(int_\delta(A)))$. The complement of an a-open set is called an a-closed set. The family of all a-open(resp.a-closed) sets of X is denoted by $aO(X)$ (resp.$aC(X)$).

Definition 2.1. A subset A of a topological space (X,τ) is said to be a Λ_a-set [7] if $A = \Lambda_a(A)$ where $\Lambda_a(A) = \cap\{G : G \in aO(X,\tau), A \subset G\}$.

Definition 2.2. A subset A of a topological space (X,τ) is called a Λ_a-closed set [7] $A = T \cap C$ where T is a Λ_a-set and C is an a-closed set. A is said to be Λ_a-open if $X - A$ is Λ_a-closed.

Definition 2.3. A function $f : (X,\tau) \to (Y,\sigma)$ is called

(i) a-continuous [5] if $f^{-1}(V)$ is a-open in X for every open set V in Y.

(ii) almost continuous [11] if $f^{-1}(V)$ is open in X for every regular open set V in Y.

(iii) almost a-continuous [3] if $f^{-1}(V)$ is a-open in X for every regular open set V in Y.

(iv) a-irresolute [5] if $f^{-1}(V)$ is a-open in X for every a-open set V in Y.

(v) R-map [2] if $f^{-1}(V)$ is regular open in X for every regular open set V in Y.

3. $(\Lambda_a$,s)-continuous functions

In this section, a new type of contra-continuity called $(\Lambda_a$,s)-continuity which is weaker than (a,s)-continuity is introduced and some of its characterizations are investigated.

Definition 3.1. A function $f : (X,\tau) \to (Y,\sigma)$ is called $(\Lambda_a$,s)-continuous if $f^{-1}(V)$ is Λ_a-closed in X for every regular open set V in Y.

Example 3.2. Let $X = \{a,b,c,d\} = Y$, $\tau = \{\phi,\{a\},\{b\}, \{a,b\},\{a,b,c\},\{a,b,d\},X\}$ and $\sigma = \{\phi,\{a\},\{b\}, \{a,b\},\{a,b,c\},Y\}$. Define $f : (X,\tau) \to (Y,\sigma)$ by $f(a) = b, f(b) = d, f(c) = a$ and $f(d) = b$. Then f is $(\Lambda_a$,s)-continuous.

Theorem 3.3. The following are equivalent for a function $f : (X, \tau) \to (Y, \sigma)$:

(i) f is (Λ_a, s)-continuous.

(ii) the inverse image of every regular closed set of X is Λ_a-open.

(iii) for each $x \in X$ and for each regular closed set V of Y containing $f(x)$, there exists a Λ_a-open set U of X containing x such that $f(U) \subset V$.

(iv) for each $x \in X$ and for each regular open set V of Y not containing $f(x)$, there exists a Λ_a-closed set K of X not containing x such that $f^{-1}(V) \subset K$.

Proof.

(i) $\Leftrightarrow$ (ii) Suppose f is (Λ_a,s)-continuous. Let V be a regular closed set of Y. Then Y-V is regular open in Y. By (i), $f^{-1}(Y - V) = X - f^{-1}(V)$ is Λ_a-closed in X which implies $f^{-1}(V)$ is Λ_a-open in X. Thus (ii) holds. Similarly we can prove (ii) $\Rightarrow$ (i).

(ii) $\Rightarrow$ (iii) Let $x \in X$ and V be a regular closed set of Y containing f(x). By (ii), $f^{-1}(V)$ is Λ_a-open in X containing x. Take $U = f^{-1}(V)$. Then U is a Λ_a-open set in X containing x and $f(U) \subset V$. Thus (iii) holds.

(iii) $\Rightarrow$ (ii) Let V be a regular closed set of Y and $x \in f^{-1}(V)$. Then $f(x) \in V$. By(iii), there exists a Λ_a-open set U_x containing x such that $f(U_x) \subset V$ which implies $U_x \subset f^{-1}(V)$. Hence $f^{-1}(V) = \cup\{U_x : x \in f^{-1}(V)\}$. Since arbitrary union of Λ_a-open set is Λ_a-open [7], $f^{-1}(V)$ is Λ_a-open in X. Thus (ii) holds.

(iii) $\Rightarrow$ (iv) Let $x \in X$ and V be a regular open set of Y not containing f(x). Then $Y - V$ is a regular closed set in Y containing f(x). By(iii), there exists a Λ_a-open set U in X containing x such that $f(U) \subset Y - V$. Then $U \subset f^{-1}(Y - V) = X - f^{-1}(V)$. Let $K = X - U$. Then K is a Λ_a-closed set not containing x such that $f^{-1}(V) \subset K$. Thus (iv) holds.

(iv) $\Rightarrow$ (iii) Let $x \in X$ and V be a regular closed set of Y containing f(x). Then $Y - V$ is a regular open set in Y not containing f(x). By (iv), there exists a Λ_a-closed set K in X not containing x such that $f^{-1}(Y - V) \subset K$. Then $X - f^{-1}(V) \subset K$ which implies $f(X - K) \subset V$. Let $U = X - K$. Then U is a Λ_a-open set in X containing x such that $f(U) \subset V$. Thus (iii) holds. ∎

Theorem 3.4. The following are equivalent for a function $f : (X, \tau) \to (Y, \sigma)$:

(i) f is (Λ_a, s)-continuous.

(ii) $f^{-1}(int(cl(G))$ is Λ_a-closed in X for every open set V of Y.

(iii) $f^{-1}(cl(int(F))$ is Λ_a-open in X for every closed set F of Y.

(iv) $f^{-1}(cl(U))$ is Λ_a-open in X for every $U \in \beta O(Y)$.

(v) $f^{-1}(cl(U))$ is Λ_a-open in X for every $U \in SO(Y)$.

(vi) $f^{-1}(int(cl(U))$ is Λ_a-closed in X for every $U \in PO(Y)$.

Proof.

(i) $\Rightarrow$ (ii) Let G be an open set in Y. Then int(cl(G)) is regular open in Y. By (i), $f^{-1}(int(cl(G))$is Λ_a-closed in X.

(ii) $\Rightarrow$ (i) Let V be a regular open set in Y. Since every regular open set is open by (ii), $f^{-1}(int(cl(V)) = f^{-1}(V)$is Λ_a-closed in X. Hence f is (Λ_a, s)-continuous.

(i) $\Rightarrow$ (iii) Let F be a closed set in Y. Then cl(int(F)) is regular closed in Y. By (i), $f^{-1}(cl(int(F))$ is Λ_a-open in X.

(iii) $\Rightarrow$ (i) Similar to the proof of (ii) $\Rightarrow$ (i)

(i) $\Rightarrow$ (iv) Let U be a β-open set in Y. Then by theorem 2.4 of [1] cl(U) is regular closed in Y. By (i), $f^{-1}(cl(U))$ is Λ_a-open in X.

(iv) $\Rightarrow$ (v) Follows from the fact that $SO(Y) \subset \beta O(Y)$.

(v) $\Rightarrow$ (vi) Let $U \in PO(Y)$. Then $Y - int(cl(U))$ is regular closed and hence it is semi-open. We have $X - f^{-1}(int(cl(U))) = f^{-1}(Y - int(cl(U)) = f^{-1}(cl(Y - int(cl(U)))$ is Λ_a-open in X. Hence $f^{-1}(int(cl(U)))$ is Λ_a-closed in X.

(vi) $\Rightarrow$ (i) Let $U \in RO(Y)$. Then $U \in PO(Y)$ and hence $f^{-1}(U) = f^{-1}(int(cl(U))$ is Λ_a-closed in X. ■

Lemma 3.5. [11] For a subset A of a topological space (Y, σ), the following properties hold:

(i) $\alpha cl(A) = cl(A)$ for every $A \in \beta O(Y)$.

(ii) $pcl(A) = cl(A)$ for every $A \in SO(Y)$.

(iii) $scl(A) = int(cl(A))$ for every $A \in PO(Y)$.

Corollary 3.6. The following are equivalent for a function $f : (X, \tau) \to (Y, \sigma)$:

(i) f is (Λ_a, s)-continuous.

(ii) $f^{-1}(\alpha cl(U))$is Λ_a-open in X for every $U \in \beta O(Y)$.

(iii) $f^{-1}(pcl(U))$is Λ_a-open in X for every $U \in SO(Y)$.

(iv) $f^{-1}(scl(U))$is Λ_a-closed in X for every $U \in PO(Y)$.

Proof. Follows from Lemma 3.5. ■

4. The related functions with (Λ_a, s)-continuous functions

Definition 4.1. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be

(i) strongly continuous [8] if $f^{-1}(V)$ is clopen in X for every set V of Y.

(ii) almost s-continuous [10] if for each $x \in X$ and $V \in SO(Y, f(x))$, there exists an open set U in X containing x such that $f(U) \subset scl(V)$.

(iii) perfectly continuous [12] if $f^{-1}(V)$ is clopen in X for every open set V of Y.

(iv) regular set-connected[4] $f^{-1}(V)$ is clopen in X for every V $\in RO(Y)$.

(v) RC-continuous [5] if $f^{-1}(V)$ is regular closed in X for every open set V of Y.

(vi) contra R-map [3] if $f^{-1}(V)$ is regular closed in X for every regular open set V of Y.

(vii) contra-super-continuous [5] if for each $x \in X$ and each $F \in C(Y, f(x))$, there exists a regular open set U in X containing x such that $f(U) \subset F$.

(viii) almost contra-super-continuous [3] if $f^{-1}(V)$ is δ-closed in X for every regular open set V of Y.

(ix) contra a-continuous [5] if $f^{-1}(V)$ is a-closed in X for every open set V of Y.

(x) (a,s)-continuous [3] if $f^{-1}(V)$ is a-closed in X for every regular open set V of Y.

Remark 4.2. The following diagram holds for a function $f : X, \tau) \to (Y, \sigma)$

$$\begin{array}{ccc}
\textit{strongly continuous} & \Rightarrow & \textit{almost s-continuous} \\
\Downarrow & & \Downarrow \\
\textit{Perfectly continuous} & \Rightarrow & \textit{regular set-connected} \\
\Downarrow & & \Downarrow \\
\textit{RC-continuous} & \Rightarrow & \textit{contra R-map} \\
\Downarrow & & \Downarrow \\
\textit{contra-super-continuous} & \Rightarrow & \textit{almost contra-super continuous} \\
\Downarrow & & \Downarrow \\
\textit{contra a-continuous} & \Rightarrow & \textit{(a,s)-continuous} \\
 & & \Downarrow \\
 & & (\Lambda_a, s)\textit{-continuous}
\end{array}$$

None of the implications is reversible as shown in the following example and in the related papers.

Example 4.3. Let $X = \{a, b, c, d\} = Y$, $\tau = \{\phi, \{c\}, \{a, d\}, \{a, c, d\}, X\}$ and $\sigma = \{\phi, \{a\}, \{b, c\}, \{a, b, c\}, Y\}$. Define a function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = c$, $f(b) = d$, $f(c) = a$ and $f(d) = b$. Then f is (Λ_a, s)-continuous, but not (a, s)-continuous since $f^{-1}(\{a\}) = \{c\}$ is not a-closed in X where {a}is regular open in Y.

Definition 4.4. A space (X, τ) is said to be a Λ_a-space if every Λ_a-closed subset of X is a-closed in X.

Definition 4.5. A space (X, τ) is said to be locally Λ_a-indiscrete if every Λ_a-open subset of X is a-closed in X.

Theorem 4.6. If (X, τ) is a Λ_a-space and $f : (X, \tau) \to (Y, \sigma)$ is (Λ_a,s)-continuous then it is (a,s)-continuous.

Proof. Let V be any regular-open subset of Y. Since f is (Λ_a, s)-continuous, $f^{-1}(V)$ is Λ_a-closed in X. Since X is a Λ_a-space, $f^{-1}(V)$ is a-closed in X which implies f is (a,s)-continuous. ∎

Theorem 4.7. If $f : (X, \tau) \to (Y, \sigma)$ is (Λ_a,s)-continuous and (X, τ) is a locally Λ_a-indiscrete space, then it is almost a-continuous.

Proof. Let V be any regular open subset of Y. Since f is (Λ_a, s)-continuous, $f^{-1}(V)$ is Λ_a-closed in X. Since X is a locally Λ_a-indiscrete space, $f^{-1}(V)$ is a-open in X which implies f is almost a-continuous. ∎

A topological space X is said to be extremely disconnected [3] if the closure of every open set of X is open in X.

Theorem 4.8. Let (Y, σ) be extremely disconnected. If $f : (X, \tau) \to (Y, \sigma)$ almost a-continuous, then it is (Λ_a,s)-continuous.

Proof. Let V be a regular closed set in Y. Since Y is extremely disconnected, by lemma 5.6 of [13], V is clopen and hence V is regular open in Y and so $f^{-1}(V)$ is a-open in X. By proposition 4.20 [7], $f^{-1}(V)$ is Λ_a-open in X. Hence f is (Λ_a,s)-continuous. ∎

Remark 4.9. The reverse implication of the above theorem need not be true as shown by the following example.

Example 4.10. Let $X = \{a, b, c, d\} = Y$, $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, X\}$ and $\sigma = \{\phi, \{a\}, \{b\}, \{a, b\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}, Y\}$. Define a function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = c$, $f(b) = d$, $f(c) = a$ and $f(d) = b$. Then f is (Λ_a, s)-continuous, but not almost a-continuous since $f^{-1}(\{b\}) = \{d\}$ is not a-open in X where {b} is regular open in Y and Y is extremely disconnected.

Theorem 4.11. Let (Y, σ) be extremely disconnected and (X, τ) is a Λ_a-space. If $f : (X, \tau) \to (Y, \sigma)$ is (Λ_a,s)-continuous, then it is almost a-continuous.

Proof. Let V be a regular open set in Y. Since Y is extremely disconnected, by lemma 5.6 of [13], V is clopen and hence V is regular-closed in Y and so $f^{-1}(V)$ is Λ_a-open in X. Since X is a Λ_a-space, $f^{-1}(V)$ is a-open in X. Hence f is almost a-continuous. ■

Definition 4.12. A topological space (X, τ) is said to be P_Σ [15] if for any open set V of X and each $x \in V$, there exists $K \in RC(X, x)$ such that $x \in K \subset V$.

Theorem 4.13. Let $f : (X, \tau) \to (Y, \sigma)$ be a function. If f is a (Λ_a, s)-continuous and X is a Λ_a-space and Y is P_Σ, then f is a-continuous.

Proof. Let V be any open set of Y. Since Y is P_Σ, there exists a subfamily Φ of RC(Y) such that $V = \cup\{A : A \in \Phi\}$. Since f is (Λ_a, s)-continuous and X is a Λ_a-space, $f^{-1}(A)$ is a-open in X for each $A \in \Phi$ and so $f^{-1}(V)$ is a-open in X. Thus f is a-continuous. ■

Definition 4.14. A topological space (X, τ) is said to be weakly P_Σ [9] if for any $V \in RC(Y)$ and each $x \in V$,there exists $F \in RC(X, x)$ such that $x \in F \subset V$.

Theorem 4.15. Let $f : (X, \tau) \to (Y, \sigma)$ be a (Λ_a, s)-function. If Y is weakly P_Σ and X is a Λ_a-space, then f is almost a-continuous.

Proof. Let V be any regular open set of Y. Since Y is P_Σ,there exists a subfamily Φ of RC(Y) such that $V = \cup\{A : A \in \Phi\}$. Since f is (Λ_a, s)-continuous and X is a Λ_a-space, $f^{-1}(A)$ is a-open in X for each $A \in \Phi$ and so $f^{-1}(V)$ is a-open in X. Thus f is almost a-continuous. ■

Theorem 4.16. Let $f : (X, \tau) \to (Y, \sigma)$ be a function and let $g : X \to X \times Y$ be the graph function of f defined by $g(x) = (x, f(x))$ for every $x \in X$. If g is (Λ_a, s)-continuous, then f is (Λ_a, s)-continuous.

Proof. Let $V \in RC(Y)$. Then $X \times V = X \times cl(int(V) = cl(int(X)) \times cl(int(V)) = cl(int(X \times V))$. Hence $X \times V \in RC(X \times Y)$. Since g is (Λ_a, s)-continuous, $f^{-1}(V) = g^{-1}(X \times V))$ is Λ_a-open in X. Hence f is (Λ_a, s)-continuous. ■

Theorem 4.17. (Composition theorems) For two functions $f : (X, \tau) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \eta)$,let $g \circ f : (X, \tau) \to (Z, \eta)$ be a composite function. Then the following properties hold:

(i) If f is a-irresolute and g is (a,s)-continuous, then $g \circ f$ is (Λ_a, s)-continuous.

(ii) If f is (Λ_a, s)-continuous and g is a R-map, then $g \circ f$ is (Λ_a, s)-continuous.

(iii) If f is contra a-continuous and g is almost continuous, then $g \circ f$ is (Λ_a, s)-continuous.

(iv) If f is almost a-continuous and g is contra R-map, then $g \circ f$ is (Λ_a, s)-continuous.

Proof. (i)Let V be a regular open set in Z. Since g is (a,s)-continuous,$g^{-1}(V)$ is a-closed in Y. Since f is a-irresolute $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ ia a-closed in X and hence

Λ_a-closed in X. Thus $g \circ f$ is (Λ_a, s)-continuous.
Proofs of (ii)-(iv) can be obtained similarly. ∎

Theorem 4.18. Let $f : (X, \tau) \to (Y, \sigma)$ be an a-irresolute function and $g : (Y, \sigma) \to (Z, \eta)$ be a (Λ_a, s)-continuous function. If Y is a Λ_a-space then $g \circ f : (X, \tau) \to (Z, \eta)$ is (Λ_a, s)-continuous.

Proof. Let V be a regular open set in Z. Since g is (Λ_a, s)-continuous, $g^{-1}(V)$is Λ_a-closed in Y. Since Y is a Λ_a-space,$g^{-1}(V)$is a-closed. Since f is a-irresolute $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is a-closed in X and by proposition 4.2 [7], Λ_a-closed in Y. Thus $g \circ f$ is (Λ_a, s)-continuous. ∎

Theorem 4.19. Let $f : (X, \tau) \to (Y, \sigma)$ be an a-irresolute function and $g : (Y, \sigma) \to (Z, \eta)$ be an almost a-continuous function. If Y is locally Λ_a-indiscrete, then $g \circ f : (X, \tau) \to (Z, \eta)$ is (Λ_a, s)-continuous.

Proof. Let V be a regular open set in Z. Since g is almost a-continuous, $g^{-1}(V)$ is a-open in Y and by proposition 4.20 [7], $g^{-1}(V)$ is Λ_a-open in Y. Since Y is a locally Λ_a-indiscrete, $g^{-1}(V)$is a-closed in Y. Since f is a-irresolute $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is a-closed in X and by proposition 4.2 [7], Λ_a-closed in X. Thus $g \circ f$ is (Λ_a, s)-continuous. ∎

Definition 4.20. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be a*-closed if the image of every a-closed set is a-closed.

Theorem 4.21. If $f : (X, \tau) \to (Y, \sigma)$ is a surjective, a*-closed function where X is a Λ_a-space and $g : (Y, \sigma) \to (Z, \eta)$ is a function such that $g \circ f : (X, \tau) \to (Z, \eta)$ is (Λ_a, s)-continuous, then g is (Λ_a, s)-continuous.

Proof. Let V be a regular open set in Z. Since $g \circ f$ is (Λ_a, s)-continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is Λ_a-closed in X. Since X is a Λ_a-space, $f^{-1}(g^{-1}(V))$ is a-closed in X. Now f is a*-closed and surjective implies $f(f^{-1}(g^{-1}(V)) = g^{-1}(V)$ is a-closed in Y. By proposition 4.2 [7], $g^{-1}(V)$ is Λ_a-closed in Y. Thus g is (Λ_a, s)-continuous. ∎

5. Applications

In this section, we relate the concept of (Λ_a, s)-continuous functions to the classes of Λ_a-compact spaces and Λ_a-connected spaces.

Definition 5.1. A topological space (X, τ) is said to be weakly Hausdorff [5] if each element of X is an intersection of regular closed sets.

Definition 5.2. A topological space (X, τ) is said to be Ultra Hausdorff [5] if for every pair of distinct points x and y, there exist disjoint clopen sets G and H containing x and y respectively.

Definition 5.3. A topological space (X, τ) is said to be $\Lambda_a - T_1$ if for every pair of distinct points x and y,there exist Λ_a-open sets G and H containing x and y respectively such that $y \notin G$ and $x \notin H$.

Definition 5.4. A topological space (X, τ) is said to be $\Lambda_a - T_2$ if for every pair of distinct points x and y, there exist disjoint Λ_a-open sets G and H containing x and y respectively.

Theorem 5.5. The following properties hold for a function $f : (X, \tau) \to (Y, \sigma)$:

(i) If f is a (Λ_a, s)-continuous injection and Y is weakly Hausdorff, then X is $\Lambda_a - T_1$.

(ii) If f is a (Λ_a, s)-continuous injection and Y is Ultra Hausdorff, then X is $\Lambda_a - T_2$.

Proof.

(i) Since Y is weakly Hausdorff, for $x \neq y$ in X, there exists $V, W \in RC(Y)$ such that $f(x) \in V, f(y) \notin V, f(y) \in W$ and $f(x) \notin W$. Since f is (Λ_a, s)-continuous, $f^{-1}(V)$ and $f^{-1}(W)$ are Λ_a-open sets in X such that $x \in f^{-1}(V), y \notin f^{-1}(V), y \in f^{-1}(W)$ and $x \notin f^{-1}(W)$. This shows that X is $\Lambda_a - T_1$.

(ii) Since Y is Ultra Hausdorff, for $x \neq y$ in X, there exists disjoint clopen sets V,W such that containing f(x) and f(y) respectively. Since f is (Λ_a, s)-continuous, $f^{-1}(V)$ and $f^{-1}(W)$ are disjoint Λ_a-open sets in X such that $x \in f^{-1}(V)$ and $y \in f^{-1}(W)$. This shows that X is $\Lambda_a - T_2$. ■

Definition 5.6. A topological space (X, τ) is said to be

(i) Λ_a-connected [7] if X cannot be written as a union of two disjoint non-empty Λ_a-open sets.

(ii) Λ_a-ultra-connected if every two non-empty Λ_a-closed sets of X intersect.

(iii) hyperconnected [14] if every open set is dense.

Theorem 5.7. Let $f : (X, \tau) \to (Y, \sigma)$ be a (Λ_a,s)-continuous surjection. Then

(i) If X is Λ_a-connected, then Y is connected.

(ii) If X is Λ_a-ultra-connected, then Y is hyperconnected.

Proof.

(i) Suppose Y is not connected.Then there exist non-empty, disjoint open sets A and B such that $Y = A \cup B$. Also A and B are clopen sets in Y. Since f is (Λ_a,s)-continuous,$f^{-1}(A)$ and $f^{-1}(B)$ are Λ_a-open in X. Moreover $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint non-empty sets and $X = f^{-1}(A) \cup f^{-1}(B)$ which implies that X is not Λ_a-connected. This is a contradiction to the fact that X is Λ_a-connected. Hence Y is connected.

(ii) Suppose Y is not hyperconnected.Then there exists an open set V such that V is not dense in Y. Then there exists disjoint non-empty regular open sets A and B in Y namely int(cl(V)) and Y-cl(V). Since f is a (Λ_a,s)-continuous surjection, $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint non-empty Λ_a-closed sets in X. By assumption, the Λ_a-ultra-connectedness of X implies that $f^{-1}(A)$ and $f^{-1}(B)$ must intersect. By contradiction, Y is hyperconnected. ■

Definition 5.8. A topological space (X, τ) is said to be

(i) Λ_a-compact [7] if every Λ_a-open cover[7]of X has a finite subcover.

(ii) S-closed [3] if every regular closed cover of X has a finite subcover.

Theorem 5.9. Let $f : (X, \tau) \to (Y, \sigma)$ be a (Λ_a,s)-continuous surjection. If X is Λ_a-compact, then Y is S-closed.

Proof. Let $\{V_\alpha : \alpha \in I\}$ be any regular closed cover of Y. Since f is (Λ_a,s)-continuous, $\{f^{-1}(V_\alpha) : \alpha \in I\}$ is a Λ_a-open cover of X. Since X is Λ_a-compact,there exists a finite subset I_0 of I such that $X = \cup\{f^{-1}(V_\alpha) : \alpha \in I_0\}$. Since f is surjective, $Y = \cup\{V_\alpha : \alpha \in I_0\}$ and hence Y is S-closed. ■

Definition 5.10. A topological space (X, τ) is said to be

(i) Ultra normal[5]if each pair of non-empty disjoint closed sets can be separated by disjoint clopen sets.

(ii) Λ_a-normal if each pair of non-empty disjoint closed sets can be separated by disjoint Λ_a-open sets.

Theorem 5.11. If $f : (X, \tau) \to (Y, \sigma)$ is a (Λ_a,s)-continuous closed injection and Y is Ultra normal, then X is Λ_a-normal.

Proof. Let E and F be disjoint closed subsets of X. Since f is closed and injective, f(E) and f(F) are disjoint closed sets in Y. Since Y is Ultra normal, there exist disjoint clopen sets U and V of Y such that $f(E) \subset U$ and $f(F) \subset V$. Since f is (Λ_a,s)-continuous, $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint Λ_a-open sets in X such that $E \subset f^{-1}(U)$ and $F \subset f^{-1}(U)$ which shows that X is Λ_a-normal. ■

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