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Study of Some Mappings in Bitopological Space

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Abstract

In this paper, the concept of bitopological precontinuity,p-locally closed,(i,j)-locally closed mapping, algebra of pairwise strongly continuous mapping, pairwise compact maps are investigated and discussed several characterization and properties of pairwise precontinuous, pairwise sp-continuous mapping, pairwise perfect maps, pairwise-strongly continuous mapping in bitoplogical space.

Keywords: Bitopological spaces, mappings, subspace, closure and interior.

1.INTRODUCTION

In 1963 J.C. Kelly [13] initiated the study of bitopological spaces. A non-empty set X equipped with two topologies τ_1 and τ_2 called a bitopological spaces. The concept of continuity in topological spaces was extended to bitopological spaces by Pervin [20]. A function f :$(X, \tau_1, \tau_2) \rightarrow (Y, \mu_1, \mu_2)$ is said to be pairwise continuous (resp. pairwise open, pairwise closed, a pairwise (homeomorphism) if the induced function f :$(X, \tau_1) \rightarrow (Y, \mu_1)$ and f : $(X, \tau_2) \rightarrow (Y,\mu_2)$ are continuous (resp. open, closed, homeomorphism). It has been found true by various authors that several properties which are preserved by continuous mappings remain inveriant under much less restrictive types of mappings in topological spaces. Levine [16, 17] introduced the concept of strongly continuous mappings in topological spaces. These mappings were also considered by Cullen [5]. A detailed study of these mappings was done by Arya and Gupta [1]. Rao and Felix [4] introduced the concept of pairwise strongly continuous mappings and studied their properties. They extended some results pertaining to topological spaces (with single topology) to bitopological spaces.

Levine [16,17]introduced the concept of strongly continuous mappings in topological spaces. These mappings were also considered by Cullen [5]. A detailed study of these mappings was done by Arya and Gupta [1]. K. Chandrasekhara Rao and S. M. Felix [4] introduced the concept of pair wise strongly continuous mappings and studied their properties. They extend some results pertaining to topological spaces (with single topology) to bitopological spaces. Our aim is to review some of these mappings in this paper. In a topological space (X, τ), a set A is said to be **semi-open** if there exist a open set U such that $U \subset A \subset cl(U)$, where cl(U) denotes the closure of U in X. Obviously, every open set is semi-open but the converse is not necessarily true. The family of semi-open sets is closed under arbitrary unions but not under finite intersections. The complement of a semi-open set is called **semi-closed**.

2. PRELIMINARY

Here,we have given a brief account of definitions and notations which we need in this paper.

Throughout this paper,(X, τ_1, τ_2) and (Y, μ_1, μ_2) always mean bitopological spaces. For a subset A of X, τ_i -cl(A) resp. τ_i -int(A) denotes the closure (resp. interior) of A with respect to τ_i for i = 1, 2. However, τ_i -cl(A) and τ_i - int(A) are briefly denoted by $cl_i(A)$ and $int_i(A)$ respectively, if there is no possibility of confusion. Let $Y \subset X$. Then $(Y, \tau_{1y}, \tau_{2y})$ will denote a subspace of (X, τ_1, τ_2) where $\tau_{1y} = \{G \cap Y : G \in \tau_1\}$ and $\tau_{2y} = \{G \cap Y : G \in \tau_2\}$

Definition-2.1. A subset D of X is said to be (i, j) semi-open if $D \subset \tau_j$ - cl (τ_i - int D) for $i \neq j$ and i, j = 1,2.The complement of (i, j)- semi-open set is (i, j)- semi-closed. The (i, j)-semi-closure of D is the intersection of all (j, i)- semi-closed sets of X containing D. We denote (j, i)- semi-closure of D by (j, i)- cl (D).

Definition-2.2. A subset C of X is called (i, j) – regular open if C = τ_i - int (τ_j - c1 (C)) for $i \neq j$ and i, j = 1,2.

Definition-2.3. A function f: $(X, \tau_1, \tau_2) \rightarrow (Y, \mu_1, \mu_2)$ is called pairwise semi continuous if for each μ_i - open set $V \subset Y$ $f^{-1}(V)$ is an (i, j) – semi open set (resp. (i, j)- α - set, τ_j -open set) in X, for $i \neq j$; i, j = 1,2.

Definition-2.4. A mapping f: $(X, \tau_1, \tau_2) \rightarrow (Y, \mu_1, \mu_2)$ is called a pairwise weakly continuous mapping if for each point p in X and each μ_i-open set H containing f (p), there exists a τ_i - open set G containing p such that f (G) $\subset \mu_i$ - cl (H) for $i \neq j$, i = 1,2, and j = 1, 2.

Definition-2.5. A function f: $(X, \tau_1, \tau_2) \rightarrow (Y, \mu_1, \mu_2)$ is called pairwise locally weak continuous if and only if there is an open basis B for the topology μ_i on Y such that f^{-1} (i, j) - f_r (v) is τ_i -closed in X for each V in B for $i \neq j$, i, j = 1,2.

Definition-2.6. A function f:$(X, \tau_1, \tau_2) \rightarrow (Y, \mu_1, \mu_2)$ is said to be pairwise weak Continuous if for each μ_i -open set V in Y, f^{-1}(i,j) - f_ρ (v) is τ_i-closed in Y, where (i,j) - f_r (v) = μ_j - cl (V)\ μ_i - int (v), i,j = 1,2 and $i \neq j$, (\ denotes the complement).

3.1.PAIRWISE PRECONTINUOUS AND PAIRWISE sp-CONTINUOUS MAPPINGS

Definition-3.1. 1. A subset A is said to be (i, j)-semi-open if there exists an open set U of X such that $U \subset A \subset cl(U)$.

Definition- 3.1. 2. A mapping $f : (X, \tau_1, \tau_2) \rightarrow (Y, \mu_1, \mu_2)$ termed (i,j)-semi-continuous iff for $O \in \mu_i$, $f^{-1}[O]$ is (i, j)-semi open in X for $i \neq j$ and $i, j = 1,2$. f is bitopologically semi continuous if f is (i, j)-semi- continuous for $i \neq j$ and $i, j = 1,2$.

Definition- 3.1. 3. In a bitopological space (X, τ_1, τ_2), a net $\{x_\alpha, \alpha \in D, \geq\}$ is said to be converge to a point $x \in X$, denoted by $\{x_\alpha, \alpha \in D, \geq\} \rightarrow x$, if the net is eventually in every τ_i -neighbourhood of x, i= 1, 2.

Definition-3.1. 4. In (X, τ_1, τ_2), $A \subset X$ is said to be (i, j) preopen (briefly (i, j) - p.o) iff $A \subset \tau_i\text{-int}(\tau_j\text{-cl } A)$ for $i \neq j$ and $i, j = 1, 2$.

Definition- 3.1. 5. In (X, τ_1, τ_2), $A \subset X$ is called bitopologically preopen (briefly b.p.o) iff A is (i, j) preopen for $i \neq j$ and $i, j = 1, 2$.

Definition-3.1.6. A subset A of (X, τ_1, τ_2) is said to be pairwise semi - preopen if there exists an (i, j) - preopen set U such that $U \subset A \subset cl(U)$. A subset A is said to be pairwise semi – preopen if it is (1, 2) – semi - preopen and (2, 1) - semi- preopen. The complement of pairwise semi - preopen set is called pairwise semi-preclosed.

Example-3.1.7. Let X={a, b , c, d}, $\tau_1 = \{\phi, X, \{a, b\}\}$ and $\tau_2 = \{\phi, X, \{c\}\}$. Then A={a, d} is (1,2) - semi-preopen but it is neither (1, 2) - preopen nor (1, 2) - semiopen.

Example- 3.1. 8. Let X= {a, b, c, d} $\tau_1 = \{\phi, X, \{a\}, \{a, c\}\}$ and $\tau_2 = \{\phi, X, \{b\}, \{b, d\}\}$. Then A = {b} is pairwise preopen but it not τ_i -open.

Definition- 3.1.9. A function $f : (X, \tau_1, \tau_2) \rightarrow (Y, \mu_1, \mu_2)$ is called pairwise sp-continuous (resp. pairwise semi-continuous) if the inverse image of each μ_i -open set of Y is (i, j) - semi-preopen (resp. (i, j) –semiopen) in X, where $i \neq j$ and $i, j = 1, 2$.

Remark- 3.1. 10. From the definition the following implication hold:

Pairwise Continuity	$\rightarrow$	pairwise semi-continuity
$\Downarrow$		$\Downarrow$
pairwise precontinuity	$\rightarrow$	pairwise sp-continuity

these implications are not reversible.

Example-3.1.11. Let$X=Y=\{a,b, c, d\}$,$\tau_1 = \{\phi ,X, \{a\}, \{a, c\}\}$, $\tau_2=\{\phi ,X,\{b\}, \{b,d\}\}$, $\mu_1=\{\phi,Y,\{a\}, \{b\}, \{a, b\}\}$ and $\mu_2=\{\phi ,Y, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, b, c\}\}$. Let f : $(X, \tau_1, \tau_2) \to (Y, \mu_1, \mu_2)$ be the identity function. Then f is pairwise precontinuous but not pairwise semi-continuous since $\{b\}\in \mu_1$ and f^{-1} $\{b\}$ is not (1, 2) - semiopen in X.

Example- 3.1.12. Let $X=Y=\{a, b, c\}$ $\tau_1= \{\phi ,X, \{a\}, \{b\}, \{a,b\}, \{a, c\}, \{a, b, c\}\}$, $\tau_2=\{\phi ,X, \{b\}, \{c\}, \{b, c\}\}$, $\mu_1=\{\phi ,X, \{a\}, \{a, c\}\}$and $\mu_2 = \{\phi , X, \{b\}, \{b, d\}\}$. Let $f : (X, \tau_1, \tau_2) \to (Y, \mu_1 , \mu_2)$ be identity mapping. Then f is pairwise semi-continuous but not pairwise pre continuous, since $\{b, d\} \in \mu_2$ and f^{-1} ($\{b, d\}$) is not (2, 1) - preopen.

Theorem-3.1.13. For a function $f : (X, \tau_1, \tau_2) \to (Y, \mu_1 ,\mu_2)$ the following statements are equivalent :

a. f is pairwise sp-continuous;
b. The inverse image of each μ_i - closed set in Y is (i, j) semi-preclosed in X ;
c. For each $x \in X$ and each $V \in \mu_i$ containing f (x), there exists an (i, j) semi-preopen set U of X containing x such that $f(U) \subset V$;
d. (i, j) - spcl $(f^{-1}(B)) \subset f^{-1}(cl_i(B))$ for every subset B of Y ;
e. f (i, j)-spcl $(A) \subset cl_i(f(A))$ for every subset A of X. On each statement above for $i \neq j$ and i, j = 1, 2.

Definition- 3.1. 14. A function $f : (X, \tau_1, \tau_2) \to (Y, \mu_1, \mu_2)$ is called pairwise locally weak continuous if and only if there. is an open basis B for the topology μ_i on Y such that f^{-1} (i, j)-$F_r(V)$ is τ_i - closed in X for each V in B, for $i\neq j$ and i, j = 1, 2. The following theorems give decomposition of pairwise continuity in bitopological spaces.

Theorem- 3.1. 15. A function $f: (X, \tau_1, \tau_2) \to (Y, \mu_1, \mu_2)$ is pairwise continuous if and only if it is both pairwise weakly continuous and pairwise weak continuous.

3.2. P-LOCALLY CLOSED CONTINUOUS MAPPING:

In this section the concepts of p-locally closed continuous mapping in bitopological spaces and some of their properties are being studied.

Definition- 3.2. 1. A mapping $f : (X, \tau_1, \tau_2) \to (Y, \mu_1 , \mu_2)$ is called p-locally closed continuous if $f^{-1}(V) \in$ (i, j)-LC(X), for every μ_1-open set V in Y, $i\neq j$ and i, j = 1, 2.

Remark- 3.2. 2 . Every pairwise $\boldsymbol{A}$ - continuous mapping is pairwise LC-continuous.

Definition- 3.2. 3. A mapping $f : (X, \tau_1, \tau_2) \to (Y, \mu_1 , \mu_2)$ is said to be strongly p-locally closed continuous if $f^{-1}(U) \in \tau_i$ for every $U \in$ (i, j)-LC(Y), $i\neq j$ and i, j = 1, 2.

Definition- 3.2.4. A mapping $f : (X, \tau_1, \tau_2) \to (Y, \mu_1 , \mu_2)$ is called p-LC irresolute if $f^{-1}(W) \in$(i, j)-LC(X) for each $W\in$(i, j)-LC(Y), $i\neq j$ and i, j = 1, 2.

Theorem- 3.2.5. Let $\{Z_k/k\in K\}$ be a τ_j open cover of bitopological space X and f : X $\rightarrow$Y such that f : (Z_k) : $Z_k\rightarrow$ Y be LC irresolute for each $k\in K$ Then f is p-LC .irresolute.

Theorem- 3.2.6. Let f : Z$\rightarrow$ Y be p-LC continuous and let g :X$\rightarrow$Z be a strongly p-LC continuous mapping. Then h = f o g is p-continuous.

Theorem- 3.2.7. Let f : X $\rightarrow$Y be p-LC irresolute and let g :Z $\rightarrow$ Y be p-LC continuous. Then h = g o f is p-LC continuous.

Theorem- 3.2.8. Let f : $(X, \tau_1, \tau_2) \rightarrow (Y, \mu_1, \mu_2)$ be a p-LC continuous mapping.Then g_f : $(X, \tau_1, \tau_2) \rightarrow (X \times Y, \Omega_1, \Omega_2)$ is p-LC continuous. Where g_f denoted by $g_f(x) = (x, f(x))$, $\Omega_K= \tau_k \times \mu_k$ for k = 1, 2.

Proof: Let $V\in\mu_i$ and since f is p-LC continuous $f^{-1}(V) \in$(i, j)-LC(X). Then, for $U\in \tau_i$, $g_f^{-1}(U \times V) = U\cap f^{-1}(V) \in$(i, j)-LC (X) and g_f is p-LC continuous.

Remark-3.2.9. From definition 3.2.1,3.2. 2 and 3.2. 3, it follows immediately that we have the following implications : Strongly p-LC continuity$\rightarrow$ p-continuity $\rightarrow$p-LC irresolute$\rightarrow$ p-LC continuty.

Example- 3.2.10. Let X={a, b, c}, τ_1={ϕ,X, {a}}, τ_2={ϕ, X{a, c}} and Y={1, 2, 3}, μ_1={ϕ, Y, {1}}, μ_2={ϕ, Y, {1, 3}}. Define$f$:X$\rightarrow$ Y by f(a) =2, f(b)=1 and f (c)=3 is p-LC irresolute, p-LC continuous but f is not p-continuous nor strongly p-LC continuous.

Example- 3.2.11. Let (X, τ_1, τ_2) and (Y, μ_1, μ_2) be as in example 3.4.11. define f: X$\rightarrow$Y by f (a) =1, f(b)=2 and f(c) =3. Then f is p-continuous but not strongly p-LC continuous.

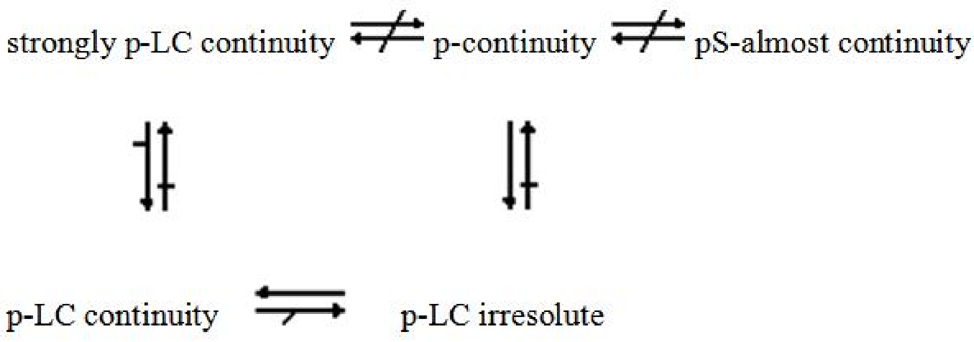
Example- 3.2.12. Let X={a,b, c}, τ_1={ϕ,X, {a, b}} τ_2=(ϕ,X, {a}, {a, b}) and Y={1, 2, 3}, μ_1= {ϕ, Y, {1}}, μ_2={ϕ, Y, {2}, {1, 3}}. Let f :X$\rightarrow$ Y be defined by f(a) =3, f(b) = 1 and f(c) =2. Then f is p-LC continuous but it is not p-LC irresolute.

Definition- 3.2.13. A mapping f : X $\rightarrow$ Y is said to be pS-Almost continuous if $f^{-1}(V) \in \tau_i$, for each (i, j)- regular open set V in Y, $i\neq j$ and i, j = 1, 2.

Theorem- 3.2.14. Let f : X $\rightarrow$Y be a strongly p-LC continuous mapping. Then is pS-almost continuous.The concept of pS-almost continuous and p-LC continuous mappings are independent which follows f is a p-LC continuous mapping but not pS--almost continuous,

Example- 3.2.17. Let X={a, b, c} τ_1={ϕ, X,{a, b}} and τ_2={ϕ, X, {a}, {a, b}} and (Y, μ_1, μ_2) as in example 3.4. 13. Define f :X$\rightarrow$Y by f(a) =2, f(b)=1 and f(c) =3.

Then f is a pS-almost continuous but not p-LC continuous.The follows digram summarizes the above results :



3.3PAIRWISE STRONGLY CONTINUOUS MAPPING

Definition- 3.3.1. A mapping $f : (X, \tau_1, \tau_2) \rightarrow (Y, \mu_1, \mu_2)$ is called pairwise strongly continuous if f (τ_1-cl A) $\subset f(A)$ and f (τ_2 -cl A) $\subset f(A)$ for every subset A of X. Here τ_1-cl (A) denotes closure of A with respect to τ_i , i = 1, 2. It is clear that $f : (X, \tau_1, \tau_2) \rightarrow (Y, \mu_1 , \mu_2)$ is pairwise strongly continuous if and only if $f : (X, \tau_1) \rightarrow (Y, \mu_1)$ and $f : (X, \tau_2) \rightarrow (Y , \mu_2)$ are strongly continuous. It follows that f is pairwise strongly continuous if $f(\tau_1$ - der A) $\subset f(A)$ for every subset A of X. Here τ_i - der A stands for the derived set of A in τ_i , i=l, 2. If $f : (X, \tau_1, \tau_2) \rightarrow (Y, \mu_1, \mu_2)$ is pairwise strongly continuous, then the map $f : (X, \tau_1, \tau_2) \rightarrow (Y, U_1, U_2)$ is also pairwise continuous for any other topologies U_1 and U_2 on Y. Hence, the knowledge of the domain of a pairwise strongly continuous map is not of much help in knowing the topologies of its range.We recall that $f: (X , \tau_1, \tau_2) \rightarrow (Y, \mu_1 , \mu_2)$ is pairwise continuous if and only if $f : (X, \tau_1) \rightarrow (Y, \mu_1)$ and $f : (X, \tau_2) \rightarrow (Y, \mu_2)$ are continuous.

Example- 3.3.2. Let X={a, b, c} with topologies τ_1 ={ ϕ , X,{a}, {b}, {a, b}}, τ_2 ={ ϕ ,X, {b}, {c}, {b, c}} Let Y={a, b, c} with topologies μ_1 = { ϕ ,Y, {a ,b}} μ_2 = { ϕ ,Y, {b}, {c}, {b, c}}. Let $f : X \rightarrow Y$ be the indentity map. Then f is pairwise continuous but not pairwise strongly continuous.

Definition- 3.3.3. A mapping $f: (X, \tau_1 , \tau_2) \rightarrow (Y, \mu_1 , \mu_2)$ is said to be closed if every τ_i-closed set of X is mapped to a μ_i-closed subset of Y, i = 1, 2.

Example- 3.3.4. Let X = {a, b, c} and let $\tau_1 = \tau_2$ discrete topology on X. Let Y =X and let μ_1 = { ϕ , Y, {a}} and μ_2 = { ϕ , Y, {b}, {b, c}}. Let $f : X \rightarrow Y$ be the indentity map. Then f is pairwise strongly continuous but not pairwise closed.

Definition- 3.3.5. A subset of (X, τ_1 , τ_2) is called $\tau_1 \tau_2$ -open if A belongs to $\tau_1 \cap \tau_2$. A subset B of X is $\tau_1 \tau_2$ -closed if its complement is $\tau_1 \tau_2$ -open.

Theorem- 3.3.5. A mapping $f : (X, \tau_1, \tau_2) \rightarrow (Y, \mu_1, \mu_2)$ is pairwise strongly continuous if and only if f^{-1} (B) is $\tau_1 \tau_2$ -closed for every subset B of Y..

Proof : Suppose that f is pairwise strongly continuous. Let B be any subset of Y. Let p be any point of τ_1 -der(f^{-1} (B)) Then f (p) belongs to f [τ_1 – der(f^{-1} (B))]. Since f is pairwise strongly continuous, we have f [τ_1- der(f^{-1} (B))] $\subset f$ (f^{-1} (B)). Also, f (f^{-1} (B)) $\subset$ B.Hence, f (p) is an element of B, whence p $\in f^{-1}$ (B). Therefore, f^{-1} (B) is τ_1-closed. Similarly,f^{-1} (B) is a τ_2 -closed set. Hence f^{-1} (B) is τ_1 τ_2 -closed.Conversely, suppose that f^{-1} (B) is τ_1 τ_2 -closed, whenever B$\subset$ Y. Let A be an arbitrary subset of X. Then A$\subset f^{-1}$ f (A) and so, τ_1 -der A$\subset$ τ_1-der. f^{-1} f(A)) $\subset f^{-1}$ (f (A)). Hence f (τ_1 -derA) $\subset f$ (A). In the same way, f (τ_2 –der A) $\subset f$ (A). Therefore, f is pairwise strongly continuous.

Corollary- 3.3.5. $f : X\rightarrow Y$ is pairwise strongly continuous if and only if f^{-1} (B) is τ_1 τ_2 -open for every subset B of Y.

Corollary- 3.3.6. $f : X\rightarrow Y$ is pairwise strongly continuous if and only if f^{-1} (B) is both τ_1 τ_2 -open and τ_1 τ_2 -c1osed for every subset B of Y. It can be seen easily that the restriction of a pairwise strongly continuous function $f : (X, \tau_1, \tau_2) \rightarrow (Y, \mu_1, \mu_2)$ to a subset of X is also pairwise strongly continuous.

Definition- 3.3.7. A mapping $f : (X, \tau_1, \tau_2) \rightarrow (Y, \mu_1, \mu_2)$ is called a pairwise weakly continuous mapping if for each point p in X and each μ_i-open set H containing f (p), there exists a τ_1 -open set G containing p such that f (G) $\subset$ μ_i-c1(H) for i$\neq$j, i=l, 2 and j=l, 2.

Definition- 3.3.8. A mapping $f :(X, \tau) \rightarrow(Y, \mu)$ is called a weakly continuous mapping if for each point p in X and each μ -open set H containing f (p), there exists a τ -open set G containing p such that f (G) $\subset$ μ -c1(H)

Example- 3.3.9 Let X= {a, b, c,} , τ_1 ={ϕ , X, {a}, {a, c}} τ_2 ={ϕ , X, {b}, {b, c}} Let Y={a, b, c}, μ_1 ={ϕ , Y, {a}, {a, b}} μ_1 = {ϕ ,Y, {c}}and let $f : X\rightarrow Y$ be the indentity map. Then f is not pairwise weakly continuous but $f : (X, \tau_1) \rightarrow (Y, \mu_1)$ and $f : (X, \tau_2) \rightarrow (Y, \mu_2)$ are weakly continuous.

Example- 3.3.10. Let X=Y={a,b, c}, τ_1 ={ϕ , X, {a}, {a, b}, {a, c}} τ_2 ={ϕ , X, {b,}, {c}, {b, c}, {a, c}}, μ_1 ={ϕ , Y, {a}, {b, c}}, μ_2 ={ϕ , Y, {a, b}, {c}}.The indentity map $f :X\rightarrow Y$ is pairwise weakly continuous. Consequently, f is not pairwise continuous.

Theorem- 3.3.11. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \mu_1, \mu_2)$ be pairwise weakly continuous if and only if f^{-1} (μ_i-open H) $\subset$ τ_i -int [f^{-1} (μ_j-cl H)] for each μ_i-open set H in Y and for i$\neq$j with i = 1,2 and j = 1,2. Here, int(A) denotes the interior of A.

Proof: Suppose the condition holds. Let p be any point of X and let H be any μ_i-open set containing f (p). Then p $\in f^{-1}$ (μ_i-open H). By the in f^{-1} (μ_i - cl H). Hence f(q) $\in$ μ_i-cl H. Thus $f(G) \subset \mu_i$-cl H. Therefore f is pairwise weakly continuous.Conversely, suppose that f is pairwise weakly continuous. Let H be any μ_i - open set in Y and let x$\in f^{-1}$(μ_i -open H). But f is pairwise weakly continuous. So there exists a τ_1-open set G containing x such that $f(G) \subset \mu_j$ - cl H, for i$\neq$j and i, j = 1, 2. But then x $\in$ G $\subset f^{-1}$(μ_j -cl H), whence x τ_i -int [f^{-1}(μ_j -cl H]. Hence, f^{-1} (μ_i-open H) $\subset \tau_1$-int [f^{-1}(μ_j-cl H].

Definition-3.3.12. Let (X, τ_1 , τ_2) be a bitopological space. Then τ_1 is said to be regular with respect to τ_2 if each point p in X and each τ_1 - closed set F such that p $\notin$ F there is a τ_1 - open set U and τ_2-open set V disjoint from U such that p $\in$ U and F $\subset$ V . The space (X, τ_1 , τ_2) is pairwise regular if τ_1 is regular with respect to τ_2 and τ_1 is regular with respect to τ_1.

Theorem- 3.3.13. Let (Y, μ_1 , μ_2) be a pairwise regular space. Then f : (X, τ_1 , τ_2) $\rightarrow$ (Y, μ_1 , μ_2) is pairwise weakly continuous.

Proof : Suppose that f is pairwise weakly continuous. Let p be any point of X. Let f(p) belong to some μ_1 - open G with G $\subset$ Y. But Y is pairwise regular. Hence there exist a μ_i-open H in Y such that f(p) $\in \mu_i$ – open H $\subset \mu_j$-cl μ_i-open H $\subset \mu_i$ - open G. Since f is pairwise weakly continuous, there exists a τ_i-Open set A in X such that $f(A) \subset \mu_j$-cl μ_i-open H $\subset \mu_i$-open G. Hence f is pairwise continuous. The convers is obvious.

3.4 ALGEBRA OF PAIRWISE STRONGLY CONTINUOUS MAPPINGS:
In this section we discuss the algebra of pairwise strongly continuous mappings.

Theorem- 3.4.1. Let f : (X, τ_1 , τ_2) $\rightarrow$ (Y, μ_1 , μ_2)be a pairwise strongly continuous function and let g : (Y, μ_1 , μ_2) $\rightarrow$ (Z, U_1, U_2) be any pairwise map. Then their composition go f: (X, τ_1 , τ_2) $\rightarrow$ (Z, U_1, U_2) is pairwise strongly continuous.

Proof: Let A be any subset of Z. Then g^{-1} (A) is a subset of Y. Pairwise strong continuity of f implies that f^{-1} [g^{-1}(A)] is a τ_1 τ_2 –open subset of X. Hence (g o $f)^{-1}$ (A) is a τ_1 τ_2 -open subset of X. So g o f is pairwise strongly continuous.

Corollary-3.4.2. The composition of any two pairwise strongly continuous functions is again pairwise strongly continuous.

Theorem- 3.4.3. If f:(X, τ_1 , τ_2) $\rightarrow$ (Y, μ_1 , μ_2) is pairwise weakly continuous and g : (Y, μ_1 , μ_2) $\rightarrow$ (Z, U_1, U_2) is pairwise strongly continuous, then g o f : X $\rightarrow$ Z is pairwise strongly continuous.

Proof: Let A be any subset of Z. Then g^{-1} (A) is a subset of Y. pairwise strong continuity of f implies that f^{-1} [g^{-1}(A)] is a τ_1 τ_2-open subset of X. Hence (g o $f)^{-1}$ (A) is a $\tau_1\,\tau_2$-open subset of X. So g o f is pairwise strongly continuous.

Corollary-3.4.4. The composition of any two pairwise strongly continuous mappings is again pairwise strongly continuous.

Theorem-3.4.5. If $f : (X, \tau_1, \tau_2) \to (Y, \mu_1, \mu_2)$ is pairwise weakly continuous and g : $(Y, \mu_1, \mu_2) \to (Z, U_1, U_2)$ is Pairwise strongly continuous, then g o $f : X \to Z$ is pairwise strongly continuous.

Proof: Let A be an subset of Z. Pairwise strong continuity of g implies that $g^{-1}(A)$ is both $\mu_1\mu_2$-open and $\mu_1\mu_2$-closed in Y. But f is pairwise strongly continuous and $g^{-1}(A)$ is μ_i-open in Y. Hence by theorem 4.1. 14, we have f^{-1} [μ_i - open $g^{-1}(A)] \subset \tau_i$-int $f^{-1}[\mu_j$ - cl $g^{-1}(A)]$. Also $\mu_i = g^{-1}(A)$ and μ_j - cl $g^{-1}(A) = g^{-1}(A)$. Hence $f^{-1}[g^{-1}(A)] \subset \tau_i$ –int $[f^{-1}(g^{-1}(A))]$. Whence, $(g \text{ o } f)^{-1}(A)$ is τ_i-open for i=l, 2. Therefore, g o f is pairwise strongly continuous. An obvious derivation is the following result.

Corollary-3.4.6. If $f : (X, \tau_1, \tau_2) \to (Y, \mu_1, \mu_2)$ is pairwise strongly continuous and g : $(Y, \mu_1, \mu_2) \to (Z, U_1, U_2)$ is pairwise strongly then their composition g o f :$X \to Z$ is pairwise strongly continuous mapping.

Theorem- 3.4.7. Let $X = \prod_{k \in I} X_k$ and let $(X_k, \tau_{1k}, \tau_{2k})$ be a bitopological space for each k in an index set I. Let $\tau_1\,\tau_2$ be the tychonoff product topologies on X corresponding to $(\tau_{1k})_{k \in I}$ and $(\tau_{2k})_{k \in I}$ respectivily. In other words, $\tau_1 = \prod \tau_{1k}$ and $\tau_2 = \prod \tau_{2k}$ Futher set $f: (X, \tau_1, \tau_2) \to \prod_{k \in I} (X_k, \tau_{1k}, \tau_{2k})$ be a pairwise strongly continuous function. Let $f_k : (X, \tau_1, \tau_2) \to (X_k, \tau_{1k}, \tau_{2k})$ be defind by $f_k(x) = x_k$ if $f(x) = (x_k)$ For each $k \in I$. Then the map f_k is pairwise strongly continuous for each $k \in I$.

Proof: Let p_k denote the projection $\prod X_k$ onto X_k . clearly $f_k = p_k$ For each $k \in I$ But f is pairwise strongly continuous. Hence it follows that f_k is pairwise strongly continuous for each k in I.

Theorem-3.4.8. Let $f_1: (X_1, \tau_{11}, \tau_{21}) \to (Y_1, \mu_{11}, \mu_{21})$ and $f_2 : (X_2, \tau_{12}, \tau_{22}) \to (Y, \mu_{12}, \mu_{22})$ be pairwise strongly continuous. Let $X = X_1 \times X_2$, $Y = Y_1 \times Y_2$. Let $\tau_i = \tau_i \times \tau_i$ and $\mu_i = \mu_i \times \mu_i$. for i = 1, 2. Suppose that $f: (X, \tau_1, \tau_2) \to (Y, \mu_1, \mu_2)$ is defined by $f(X_1, X_2) = [f_1(x_1), f_2(x_2)]$.Then f is pairwise strongly continuous.

Proof: Let $y \in Y_1 \times Y_2$. Then $y = (y_l, y_2)$ with $y_l \in Y_1$ and $y_2 \in Y_2$. We have $f^{-1}(y) = f^{-1}_1(y_1) \times f^{-1}_2(y_2)$ but $f_1 : X_l \to Y_l$ is strongly pairwise continuous. So $f^{-1}_1(y_1)$ is a $\tau_{11}\,\tau_{21}$- open subset of X_l. Similarly $f^{-1}_2(y_2)$ is a subset of $\tau_{11}\,\tau_{22}$-open subset of X_2.

Hence f^{-1} (y) is a τ_{11} τ_{12}-open subset of X . This implies that f is pairwise strongly continuous.

Definition- 3.4.9. A bitopological space (X, τ_1 , τ_2) is called a pairwise connected space if X can not be expressed as the union of two non-empty disjoint sets A and B such that (A $\cap$ τ_1- cl B) U (τ_2-cl A $\cap$ B) = ϕ,the empty set.When X can be so expressed, we write X =A/B and call this a separation of X.

Definition- 3.4.10. A subset A of (X, τ_1 , τ_2) is said to be pairwise connected if (A , τ_1/A , τ_2 /A) is pairwise connected.

Theorem- 3.4.11. Let f : (X, τ_1 , τ_2) $\rightarrow$ (Y, μ_1 , μ_2) be pairwise strongly continuous and let A be a non-empty pairwise connected subset of X. Then f (A) is a singleton.

Proof : Assume that f(A) contains more than one point. Let p $\in f$ (A). Then f (p) $\cap$ A is proper subset of A and this set is both τ_1/A, τ_2/A-open and τ_1/A, τ_2/A -closed in A. Hence A is not pairwise connected, a contradiction to our hypothesis. This contradiction shows that f(A) is a singleton.

Definition- 3.4.12. Let (X, τ_1 , τ_2) be a bitopological space. The topology τ_1 is said to be locally connected with respect to τ_2 if for each point p in X and each τ_1-open set U containing x, there is a pairwise connected τ_1 - open set G such that x $\in$ G $\subset$ U. If τ_1 is locally connected with respcet to τ_2, and τ_2 is locally connected with respect to τ_1 , then (X, τ_1 , τ_2) is called a pairwise locally connected space.

Theorem- 3.4.13. Let f : (X, τ_1 , τ_2) $\rightarrow$ (Y, μ_1 , μ_2) be a pairwise map where X is pairwise locally connected. Suppose that f(B) is a singleton, whenever B is a pairwise connected non-empty subset of X. Then f is pairwise strongly continuous.

Proof: Let p $\in$ τ_1 -der A, where A is a subset of X. But X is pairwise locally connected. So, there exists a pairwise connected τ_i -open set G in X such that p $\in$ G and G $\cap$ A $\neq$ ϕ. Now f (p) $\in$ f (G)and f (G)is a singleton. Φ = f(G $\cap$A) $\subset$ f(G)= f (p).Hence, f (p)= f (G $\cap$A). This means that f (p) $\in$ f (A). Therefore, f (τ_i -derA) $\subset$ f (A). Hence f is pairwise strongly continuous.

Definition- 3.4.14. Given a bitopological space (X, τ_1 , τ_2) we define a relation R by (p, q) $\in$ R if and only if p and q can not be separated by a separation of X. That is for each separation A/B of X either p $\in$ A and q A or p $\in$ B and q $\in$ B. It is easy to see that R is an equivalence relation on X.Let p be any point of (X, τ_1 , τ_2). The equivalence class of p with respect to R is called the quasi-component of p and is denoted by Q_p. Some times, Q_p may be designated as a pairwise component of X.

Definition- 3.4.15. A function f defined on a bitopological space (X, τ_1 , τ_2) is called pairwise quasi-constant if f is constant on each pairwise quasi-component in X.

Example-3.4.16. Let X={a, b, c}, τ_1 = { ϕ, X, {a}, {b, c}}, τ_2={ ϕ, X, {b}, {b, c}}. Let Y={p, q, r}, μ_1 = { ϕ,Y,{p}, {q, r}}, μ_2={ ϕ,Y,{q}, {q, r}}. Define f: X $\rightarrow$ Y by f(a) =p, f(b) = r = f(c). Then f is pairwise quasi-constant but not pairwise strongly continuous.

Theorem- 3.4.17. Let f: (X, τ_1 , τ_2) $\rightarrow$ (Y, μ_1 , μ_2) be pairwise strongly continuous. Then f is pairwise quasi-constant.

Proof : Let there are two points p and q of X that lie in the same quasi-component of X. Let f(p) = a $\neq$ b = f(q). Now f^{-1} (a) ∩ f^{-1}(X-a) = ϕ and f^{-1}(a) U f^{-1} (X-a) = X. Also f^{-1} (a) and f^{-1} (X-a) are both τ_1 τ_2-open. Hence f^{-1} (a) U f^{-1} (X-a) is a separation of X. Also p is in f^{-1} (a) and q is in f^{-1} (X-a).This is a contradiction to our assumption. This contradiction shows that f is pairwise quasi-constant.

Corollary-3.4.18. If f : (X, τ_1 , τ_2) $\rightarrow$ (Y, μ_1 , μ_2) is pairwise strongly continuous, then f is constant on any component of X.

Proof : Components are contained in quasi-components of X. Hence the result follows.

Corollary-3.4.19. If (X, τ_1 , τ_2)is a pairwise connected space, then the only pairwise strongly continuous mapping are the constant mappings.

Proof : This follows from the fact that X consists of the only one component, namely, X.

Example-3.4.20. Let X={a, b, c, d}, τ_1 = {ϕ,X, {a}, {a, b}}, τ_2={ϕ,X, {c, d}}. Let Y={p, q, r, s} , μ_1 ={ϕ,Y, {p}, {r}, {p, r}}, μ_2={ϕ ,Y, {p, q, s}}. Define f:X$\rightarrow$ Y by f (a) =p = f (b), f (c)= r = f (d). Then f is pairwise quasi-constant but not pairwise weakly continuous.

Example- 3.4.21. Let X={a, b, c} τ_1 ={ ϕ ,X, {a}, {b}, {a, b}}, τ_2 ={ ϕ , {c}}. Let Y={p, q, r} μ_1 = { ϕ , Y, {p, q}}, μ_2 ={ ϕ,Y, {q}, {r}, {q, r}}. Define, f : X $\rightarrow$ Y by f (a)=p= f (c), f (b)=q. Then f is pairwise weakly continuous but not pairwise quasi-constant.

Definition- 3.4.22. A map f: (X, τ_1 , τ_2) $\rightarrow$ (Y, μ_1 , μ_2) is called a pairwise connected map if the included maps f: (X, τ_1) $\rightarrow$ (Y, μ_1), f: (X , τ_2) $\rightarrow$ (Y , μ_2) are connected map, that is the image of a connected set is connected.

Example- 3.4.23. Let X={a, b, c}, τ_1 ={ ϕ , X, {b, c}}, τ_2 ={ ϕ,X,{a}, {a, b}}. Let Y=X={a,b, c}, μ_1 ={ ϕ , Y, {b}, {b, c}} and μ_2 ={ ϕ, Y,{a}, {a, b}}. Let f : X $\rightarrow$ Y be the identity map. Then f is pairwise connected but not pairwise continuous.

Example- 3.4.24. Let X =Y = {a, b, c}, τ_1 = { ϕ,X, {b} {a, b}}, τ_2 ={ ϕ ,X, {c}, {a, c}}, μ_1 ={ ϕ , Y, {a}, {b}, {a, b}} and μ_2 ={ ϕ, Y,{a}}. Let $f : X \rightarrow Y$ be the identity map. Then f is pairwise weakly continuous but not pairwise connected.

Example-3.4.25. Let X=Y={a,b, c}, τ_1 = { ϕ ,X, {b} {a, b}}, τ_2 ={ ϕ ,X, {a}, {a, b}, {b}, {a, c}}, μ_1 = { ϕ,Y, {b}, {a, b}} and μ_2 ={ ϕ, Y,{a}, {a, c}}. Let $f : X \rightarrow Y$ be the indentity map. Then f is pairwise connected but not pairwise weakly continuous.

Definition 3.4.26. (X, τ_1 , τ_2) is said to be pairwise totally disconnected if the singletons are the only pairwise connected subsets of X.

Theorem- 3.4.27. Let (X, τ_1 , τ_2) be a pairwise locally connected space, and Let (Y, μ_1 , μ_2) be a pairwise totally disconnected space. Then $f : (X , \tau_1 , \tau_2) \rightarrow (Y, \mu_1 , \mu_2)$ is pairwise strongly continuous if and only if f is a pairwise connected map.

Proof : Suppose that f is pairwise strongly continuous. Let A be a connected subset of (X, τ_1 , τ_2). Since f is pairwise strongly continuous, then f: (X, τ_1 , τ_2) $\rightarrow$ (Y , μ_1 , μ_2) is continuous.Hence f (A) is connected in (Y , μ_1 , μ_2). Thus, f is pairwise connected.

Conversely, suppose that f is a pairwise connected map. Since Y is pairwise totally disconnected, the only pairwise connected subsets are singletons. Then f is constant on every component of X. But X is pairwise locally connected. So every component of X is $\tau_1\tau_2$ -open. Hence for every point q in Y, f^{-1} (q) is τ_1 τ_2-open. This implies that f is pairwise strongly continuous.

Corollary-3.4.28. Every pairwise weakly continuous (and hence every pairwise continuous) mapping from a pairwise connected space to a pairwise totally connected space is pairwise strongly continuous.

Definition 3.4.29. A bitopological space (X, τ_1 , τ_2) is said to be pairwise weakly locally connected if every point of X has pairwise connected neighborhood which is τ_i -open. i = 1, 2.

Example-3.4.30. Let X = {a, b, c, d}, τ_1 ={ ϕ,X , {a, b}, {a, b, d}},τ_2= { ϕ, X , {c, d}, {a, c ,d}}.Then (X, τ_1 , τ_2) is pairwise weakly locally connected but not pairwise locally connected. This example also shows that a pairwise weakly locally connected space need not be pairwise connected.

Example- 3.4.31. Let X={a, b c, d} τ_1 = { ϕ ,X, {a}, {a, b}, {a, d}, {a, b ,d}}, τ_2={ ϕ ,X, {d}, {b, d}}. Then (X, τ_1 , τ_2) is pairwise connected but not pairwise weakly locally connected.

Theorem-3.4.32. Let f: (X, τ_1 , τ_2) $\rightarrow$ (Y, μ_1 , μ_2) be a mapping from a pairwise weakly locally connected space to a space Y such that the image of every non-empty pairwise connected subset of X is a singleton.Then f is pairwise strongly continuous.

Proof: Let A be any subset of X and let $p \in \tau_i$ -der A. Since X is Pairwise weakly locally connected, there is a pairwise connected neighbourhood N_i of p. Now $p \in \tau_i$ -derA. So $N_i \cap A \neq \phi$. Therefore $f(N_i \cap A) \neq \phi$. Also $f(N_i \cap A) \subset f(N_i)=\{f(p)\}$. Hence $f(N_i \cap A)=\{f(p)\}$. Consequently, $f(p) \in f(A)$. So $f(\tau_i\text{-der } A) \subset f(A)$. Hence f is pairwise strongly continuous.

Corollary-3.4.33. If $f: (X, \tau_1, \tau_2) \rightarrow (Y, \mu_1, \mu_2)$ is a mapping from a pairwise locally connected space X to a space Y such that the image of every non-empty pairwise connected subset of X is a singleton, then f is pairwise strongly continuous.

Proof : This follows from the above theorem noting that every pairwise locally connected space is a pairwise weakly locally connected.

Theorem- 3.4.34. Let $f: (X, \tau_1, \tau_2) \rightarrow (Y, \mu_1, \mu_2)$ be a pairwise strongly continuous mapping. Then f can be extended pairwise strongly continuously to any pairwise locally connected space which contains A as a pairwise closed and open set.

Proof: Let (X, τ_1, τ_2) be any pairwise locally connected space containing A as a pairwise closed and open set. Define a mapping $g : (X, \tau_1, \tau_2) \rightarrow (Y, \mu_1, \mu_2)$ as shown below : If $p \in A$, take $g(p) = f(p)$. If $p \in X\text{-}A$, and K is pairwise component of X, containing p, take g (K) = some fixed point of Y. Then g is a mapping on a pairwise locally connected space X such that the image of every non-empty pairwise connected set is a singleton. Hence g is a pairwise strongly continuous extension of f.

3.5. PAIRWISE COMPACT MAPS :

This section opens up with the study of pairwise compact maps and generalize some of their properties.

Definition 3.5.1. Let (X, τ_1, τ_2) be a bitopological space, suppose μ is a subfamily of $\tau_1 \cup \tau_2$ such that μ contains atleast one non-empty member of τ_2 and that μ covers X. Then μ is called pairwise open cover of X. if every open cover of (X, τ_1, τ_2) has a finite sub cover, then (X, τ_1, τ_2) is called a pairwise compact space. Similarly, a pairwise compact subset may be defined.

Definition 3.5.2. A map $f: (X, \tau_1, \tau_2) \rightarrow (Y, \mu_1, \mu_2)$ is said to be a pairwise compact mapping if the inverse image of every pairwise compact subset of Y is a pairwise compact subset of X.

Remark-3.5.3. A pairwise compact map need not be pairwise continuous and that a pairwise compact map need not be pairwise strongly continuous.

Theorem- 3.5.4. Every pairwise strong]y continuous map from a pairwise compact space is a pairwise compact mapping.

Proof: Let f: $(X , \tau_1 , \tau_2) \rightarrow (Y, \mu_1 , \mu_2)$ be a pairwise strongly continuous map, where X is pairwise compact. Let B be a pairwise compact subset of Y. Now f is pairwise strongly continuous. So, f^{-1} (B) is a $\tau_1 \tau_2$-closed set in X. But X is a pairwise compact. Hence f^{-1} (B) is pairwise compact. Therefore f is a pairwise compact mapping.

Definition 3.5.5. Suppose in a bitopological space. (X, τ_1 , τ_2), a subset of X is pairwise compact if and only if it is a $\tau_1 \tau_2$-closed set. Then (X, τ_1 , τ_2) is said to be pairwise c-c space.

Example- 3.5.6. Consider the space (X, τ_1 , τ_2) where X is any infinite set, τ_1 is the cofinite topology on X and τ_2 is the discrete topology on **X. Raghavan** and **Reilly[26]** have shown that this space is pairwise Hausdorff and pairwise compact. This space is also pairwise.

Theorem- 3.5.7. If f :$(X , \tau_1 , \tau_2) \rightarrow (Y, \mu_1 , \mu_2)$ be a mapping from a pairwise c-c space X to a hereditarily pairwise compact space Y, then f is pairwise strongly continuous if and only if it is pairwise compact.

Proof: Suppose f be pairwise storngly continuous and B be any pairwise subset of Y. Then f^{-1} (B) is a $\tau_1 \tau_2$-c1osed subset of X. As X is pairwise c-c, f^{-1} (B) is pairwise compact and so f is pairwise compact.Conversely let f be pairwise compact and B be any subset of Y. As Y is hereditarily pairwise compact, B is pairwise compact. Hence f^{-1} (B) is a pairwise compact subset of X. But X is pairwise c-c and so f^{-1} (B) is $\tau_1 \tau_2$-c1osed. Hence f is pairwise strongly continuous.

Theorem-3.5.8. if $f : (X , \tau_1 , \tau_2) \rightarrow (Y, \mu_1 , \mu_2)$ is pairwise strongly continuous, the image of every pairwise compact subset of X under f is a finite set.

Proof : Let A be a pairwise compact subset of X. f is pairwise strongly continuous implies f^{-1} (y) is $\tau_1 \tau_2$-open for each $y \in Y$. Then the family $\{ f^{-1} (y) / y \in f (A)\}$ is a pairwise open cover of A Hence there exist finitely many points $y_1,y_2,\ldots ,y_n \in f (A)$ such that $A \subset U\{ f^{-1} (y_k)/k = 1,2, \ldots ,n\}$.Hence $f (A) \subset \{ y_1,y_2,\ldots ,y_n \}$. i.e., f (A) is finite.

Definition 3.5.9. (X, τ_1 , τ_2) is said to be pairwise almost compact if every pairwise open cover U for X consists of a finite subfamily $\{ c_1,c_2,\ldots ,c_n \}$ such that $X = U\{ \tau_j\text{-cl } C_k/C_k \in U \cap \tau_i , i \neq j, i, j = 1, 2; k = 1, 2,\ldots,n\}$

Remark- 3.5.10. A space may be pairwise almost compact but not pairwise compact.

Remark- 3.5.11. A space may be pairwise compact but not pairwise almost compact.

Theorem- 3.5.12. Every pairwise strongly continuous image of a pairwise almost compact space is pairwise compact.

Proof : Let f :(X , τ_1 , τ_2) $\rightarrow$ (Y, μ_1 , μ_2) be a pairwise strongly continuous map of a pairwise almost compact space X onto a space Y. Let $\{U_\alpha/\alpha \in I\}$ be any pairwise open cover of Y.Then $\{f^{-1}(U_\alpha/\alpha \in I\}$ is a covering of X by τ_1 τ_2-open, τ_1 τ_2-closed sets. X is pairwise almost compact implies that there is a finite subfamily $\{f^{-1}(U_{\alpha\kappa})/k = 1,2,\ldots, n\}$ of $\{f^{-1}(U_\alpha)/ \alpha \in I\}$ which covers X. Hence $\{U_{\alpha\kappa}/k = 1,2,\ldots, n\}$ is a finite subcovering of $\{U_\alpha/\alpha \in I\}$. Therefore, Y is pairwise compact.

Definition 3.5.13. (X, τ_1 , τ_2) is said to be pairwise nearly compact if every pairwise open cover U for X consists of a finite subfamily $\{c_1,c_2,\ldots ,c_n\}$ such that X = U{ τ_i - int τ_j -cl $c_k/c_k \in U \cap \tau_i$ $i \neq j$, i, j = 1, 2 ; k = 1,2, ...,n}.

Corollary-3.5.14. Every pairwise strongly continuous image of a pairwise nearly compact space is pairwise comapact.

Proof: As every pairwise nearly compact space is pairwise almost compact. the result follows.

3.6 PAIRWISE PERFECT MAPS:

In this section we discuss pairwise perfect maps and we have been studied some properties of this maps.

Definition 3.6.1. A map f :(X , τ_1 , τ_2) $\rightarrow$ (Y, μ_1 , μ_2) is said to be pairwise perfect if (i) f is pairwise continuous; (ii) f is pairwise closed (iii) f is τ_1 - compact, τ_2 - compact and pairwise compact.

Example- 3.6.2. Let X be the real line R and τ_1 be the usual topology on X and τ_2= {ф }U {UU (x, X) / x $\in$ X, U $\in \tau_1$ }. And let Y be also the real line R and μ_1 be the usual topology on Y and μ_2= {ф}U {V∩(-∞ y) /y $\in$ Y, V $\in \mu_1$ }. Define f: X$\rightarrow$ Y be f (x) = - x for all x $\in$ X. Then f is pairwise closed and pairwise perfect but not pairwise strongly continuous.

Example-3.6.3. Let X ={a, b, c}, $\tau_1 = \tau_2$ =discrete topology on X; Y=X, μ_1 = {ф ,Y, {a}} μ_2={ф ,Y,{b}, {b, c}}. Let f :X$\rightarrow$ Y be the identity map. Then f is pairwise strongly continuous but not pairwise closed.

Lemma-3.6.4. Let X be a pairwise Hausdorff space. If A and B are disjoint τ_1 - compact and τ_2-compact subsets respectively of X, then there exist a τ_1 - neighbourhood of A and a τ_2-neighbourhood of B such that they are disjoint.

Proof: For each $b \in B$, there exist disjoint τ_2-neighbourhood U(b) and τ_1 - neighbourhood $U_b(A)$. From the τ_2-open covering $\{U(b) \cap B/b \in B\}$, extract a finite subcovering $\{U(b_l) \cap B, U(b_n) \cap B\}$. Take $V = \cap_k{}^n{}_{=1} U_{bk}(A)$ and $W = U\ {}_k{}^n{}_{=1}\ U(b_k)$. Then V is a τ_1 -neighbourhood of A and W is a is τ_2-neighboured of B such that $V \cap W = \phi$

Theorem- 3.6.5. Let $f : (X, \tau_1, \tau_2) \to (Y, \mu_1, \mu_2)$ be a pairwise perfect map. If X is a pairwise Hausdorff, then so is Y.

Proof: Let y_1 and y_2 be any two distirct points of Y. Then $f^{-1}(y_1)$ and $f^{-1}(y_2)$ are disjoint τ_1 –compact and τ_2-compact sets respectively hence by the above Lemma, there exists this disjoint τ_1 -neighbourhood U_1 of $f^{-1}(y_1)$ and τ_2-neighbourhood U_2 of $f^{-1}(y_2)$.Because f is pairwise closed, there exist a μ_1 -open set V_1 and a μ_2 -open set V_2 in Y such that $f^{-1}(y_1) \subset f^{-1}(V_1) \subset U_1$ and $f^{-1}(y_2) \subset f^{-1}(V_2) \subset U_2$. Consequently, there exist a μ_1 - neighbourhood V_1 of y_1 and a μ_2 - neighbourhood V_2 of y_2 such that $V_1 \cap V_2 = \phi$. Hence Y is pairwise Haudorff.

Lemma- 3.6.6. Let X be a pairwise regular space and A be a τ_i - compact subset of X, i = 1, 2. Then, for each τ_i - neighbourhood U of A, there exists a τ_i -open set W such that $A \subset W \subset \tau_j$ - $clW \subset U$, i, j = 1, 2 ; $i \neq j$

Proof : For each $a \in A$, there is a τ_i -neighbourhood V(a) such that τ_j -cl $V(a) \subset U$. Extracting a finite τ_i -open covering of A gives that $A \subset U\ V(a_k) \subset U\ \tau_i$ cl $V(a_k)$. Let $W = U\ V(a_k)$. Then W is τ_i -open. But then τ_j -cl $W = \tau_j$-cl$(U\ V(a_k)) = U\ \tau_j$ - cl $V(a_k)$. Hence $A \subset W \subset \tau_j$ - cl $W \subset U$.

Theorem-3.6.7. Let $f : (X, \tau_1, \tau_2) \to (Y, \mu_1, \mu_2)$ be a pairwise perfect map. If X is pairwise regular, then Y is also pairwise regular.

Proof: Given a μ_i -open set V and a point $y \in Y$, $f^{-1}(y) \in f^{-1}(V)$ in X. But X is pairwise regular and so there is a set τ_i-open set U such that, by the above lemma $f^{-1}(y) \in U\ \tau_j$ – cl $U \subset f^{-1}(V)$. Since f is a τ_i -closed map, there is a μ_i -neighbourhood W of y such that $f^{-1}(y) \subset f^{-1}(W) \subset U$. But $W \subset f(\tau_j$ -cl $U) \subset V$. Since $f(\tau_j$ -cl U) is μ_j-closed, $y \in W \subset \mu_j$ - cl $W \subset f(\tau_j$-clU$) \subset V$. Hence Y is pairwise regular.

Remark 3.6.8. Every pairwise continuous map between pairwise compact, pairwise Hausdorff spaces is pairwise perfect. This result is true if pairwise continuous is replaced by pairwise strongly continuous as every pairwise strongly continuous map is pairwise continuous.

REFERENCES

[1] Arya, S.P. and Gupta, R. On strongly continuous mappings.Kyungpook Math J. 14 (1974),131-145.

[2] Birsan,T.Surles espaces bitoplolgiques Complement requlicrs,An. St. Univ. lasi. Mathematical xvi fase (1970),129-134.

[3] S.Bose Semi-open sets, semi-continuity and semi-open mappings in bitopological spaces. Bull. Cal. Math. Soc. 73 (1981),no. 4. 237-246.

[4] K.Chandrasekhara Rao and S.M.Felix.Pairwise strongly continuous functions Proc. Nat. Acad. Sci. India 62 (A) II (1992).

[5] Cullen, H.F Complete continuity for functions, Amer.Math. Monthly 68 (1961), 165-168.

[6] Dutta, M.C.Projective bitopological spaces II.J.Austral. Math. Soc. 14 (1972), 119-128.

[7] Fletcher, P.Hoyle III,H.B. and Ratty, C.W.The copparison of Topologies. Duke Math. Jour. (1969), 325-337.

[8] M.Ganster and I.Reilly Int. J.Math. Math. Sci. 12 (1990),417-424.

[9] M.jelic.Feebly p-continuous mappings. Supp. Rendiconti Cir. Mat. Dipalermo 24 (1990) 387-396.

[10] M.jelic On pairwise LC- continuous mappings.Indian J.Pure App. Math. 22 (1) (1991) 55-59.

[11] A.Kar and Bitopological preopen sets, P.Bhattacharyya precontinuity and preopen mappings. Ind. J. of Mathmatics 34(1992), 295-309.

[12] H.Kawashima On the bitopological product of minimal Hausdorff spaces. Ture Math. 1 (1965), 62-64.

[13] J.C.Kelly Bitopological spaces. . Proc. London. Math Soc. (3) 13 (1963), 71-89.

[14] F.H.Khedr.S.M Al.Areef. Precontinuity and semi-precontinuity bitopological spaces. Indian. J. Pure Appl. Math. 23 (9) (1992) 625-633.September (1992).

[15] S.Sampath kumar On pairwise A -continuous functions.J.Indian Acad. Math. Vol.16 No.1. (1994), 50-55.

[16] Levine, N.Strong continuity in bitopological spaces.Amer. Math. Monthly, 67 (1960), 269-296.

[17] S.N.Maheswari Semi open sets and semi-continuous R.Prasad functions in bitopological spaces. Math. Notae 26 (1977-78), 29-37.

[18] A.S.Mashhour,M.E,Abd, EL Monsef and S.N.EI Deeb. On precontinuous and weak precontinuous mappings. Proc. Math. Phys. Soc. Egypt 53. (1982), 47-53.

[19] Mukharjee,M.N Pairwise category properties of some pairwise minimal bitopological spaces Ann. Soc. Sci. Bruxelles Ser I. 96 No.3 (1993), 117-127.

[20] Pervin,W.J. Foundations of General Topology. Acad.Press. New-York. (1970).

[21] T.G, Raghwan,Reilly,I.L.A new bitopological Paracompactness J.Austral. Math. Soc. Ser. A. 41, No.2 (1986), 268-274.

[22] M.K.Singal and A.R. Singal. Some more separation axioms in bitopological spaces. Ann. Soc. Burxelles Ser I. 84 (1970), 207-230.

[23] J. Tong.A decomposition of continuity. Acta. Math. Hung, 48 (1986), 11-15.