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Difference Cordiality of Some Graphs Obtained from Double Alternate Snake Graphs

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Abstract

Let G be a (p,q) graph. Let $f: V(G) \rightarrow \{1,2,\ldots,p\}$ be a function. For each edge uv, assign the label $|f(u) - f(v)|$. f is called a difference cordial if f is a one to one map and $|e_f(0) - e_f(1)| \leq 1$ where $e_f(1)$ and $e_f(0)$ denote the number of edges labeled with 1 and not labeled with 1 respectively. A graph with admits a difference cordial labeling is called a difference cordial graph. In this paper, we investigate the difference cordial labeling behavior of $DA(T_n) \odot K_1$, $DA(T_n) \odot K_2$, $DA(T_n) \odot 2K_1$, $DA(Q_n) \odot K_1$, $DA(Q_n) \odot K_2$ and $DA(Q_n) \odot 2K_1$where $DA(T_n)$ and $DA(Q_n)$ are double alternate triangular snake and double alternate quadrilateral snakes respectively.

Keywords: Corona, Triangular snake, Quadrilateral snake, Complete graph.

2010 Subject Classification: 05C78

Introduction:

In this paper we have considered only simple and undirected graph. Let $G = (V, E)$ be a (p,q) graph. The number $|V|$ is called the order of G and the number $|E|$ is called the size of G. The notion of difference cordial labeling has been introduced by R. Ponraj, S. Sathish Narayanan, R. Kala in [3]. In [3, 4, 5, 6, 7] difference cordial labeling behavior of several graphs like path, cycle, complete graph, complete bipartite graph, bistar, wheel, web, sunflower graph, pyramid, grid, Mongolian tent, n-cube, $G \odot P_n$, $G \odot mK_1$ $(m = 1,2,3)$ where G is either unicycle or tree, crown $C_n \odot K_1$, comb $P_n \odot K_1$, $P_n \odot C_m$, $C_n \odot C_m$, $W_n \odot K_2$, $W_n \odot 2K_1$, $L_n \odot K_1$, $L_n \odot 2K_1$, $L_n \odot K_2$, $DT_n \odot K_1$, $DT_n \odot K_2$, $DT_n \odot 2K_1$, $DQ_n \odot K_1$, $DQ_n \odot K_2$, $DQ_n \odot 2K_1$ and some more standard graphs have been investigated. In this paper we are going to investigate the difference cordial labeling behavior of $DA(T_n) \odot K_1$, $DA(T_n) \odot K_2$, $DA(T_n) \odot 2K_1$, $DA(Q_n) \odot K_1$, $DA(Q_n) \odot K_2$ and $DA(Q_n) \odot 2K_1$where $DA(T_n)$ and $DA(Q_n)$ are double alternate triangular snake and double alternate quadrilateral snakes respectively. Let x be any real number. Then $\lfloor x \rfloor$ stands for the largest integer less than or equal to x and $\lceil x \rceil$ stands for the smallest integer greater than or equal to x. Terms and definitions not defined here are used in the sense of Harary [2].

Difference Cordial Labeling

Definition 2.1:

Let G be a (p, q) graph. Let f be a map from $V(G)$ to $\{1, 2, \dots, p\}$. For each edge uv, assign the label $|f(u) - f(v)|$. f is called difference cordial labeling if f is $1-1$ and $|e_f(0) - e_f(1)| \leq 1$ where $e_f(1)$ and $e_f(0)$ denote the number of edges labeled with 1 and not labeled with 1 respectively. A graph with a difference cordial labeling is called a difference cordial graph.

The corona of G with H, $G \odot H$ is the graph obtained by taking one copy of G and p copies of H and joining the i$^{\text{th}}$ vertex of G with an edge to every vertex in the i$^{\text{th}}$ copy of H.

A double alternate triangular snake $DA(T_n)$ consists of two alternate triangular snakes that have a common path. That is, a double alternate triangular snake is obtained from a path $u_1, u_2 \dots u_n$ by joining u_i and u_{i+1} (alternatively) to two new vertices v_i and w_i.

Theorem 2.2: $DA(T_n) \odot K_1$ is difference cordial.

Proof: Case (i): The two triangles starts from u_1 and ends with u_n.

Let $V(DA(T_n) \odot K_1) = V(DA(T_n)) \cup \{u_i' : 1 \leq i \leq n\} \cup \left\{v_i', w_i' : 1 \leq i \leq \frac{n}{2}\right\}$ and $E(DA(T_n) \odot K_1) = E(DA(T_n)) \cup \{u_i u_i' : 1 \leq i \leq n\} \cup \left\{v_i v_i', w_i\, w_i' : 1 \leq i \leq \frac{n}{2}\right\}$.

Define $f: V(DA(T_n) \odot K_1) \to \{1, 2 \dots 4n\}$ by $f(u_{2i-1}) = 4i - 2, 1 \leq i \leq \frac{n}{2}$, $f(u_{2i}) = 4i - 1, 1 \leq i \leq \frac{n}{2}$, $f(u_{2i-1}') = 4i - 3, 1 \leq i \leq \frac{n}{2}$, $f(u_{2i}') = 4i, 1 \leq i \leq \frac{n}{2}$, $f(v_i) = 2n + 2i - 1, 1 \leq i \leq \frac{n}{2}$, $f(v_i') = 2n + 2i, 1 \leq i \leq \frac{n}{2}$, $f(w_i) = 3n + 2i - 1, 1 \leq i \leq \frac{n}{2}$, $f(w_i') = 3n + 2i, 1 \leq i \leq \frac{n}{2}$. Since $e_f(1) = \frac{5n}{2}$ and $e_f(0) = \frac{5n-2}{2}$, f is a difference cordial labeling of $DA(T_n) \odot K_1$.

Case (ii): The two triangles starts from u_2 and ends with u_{n-1}.

Define a map $f: V(DA(T_n) \odot K_1) \to \{1, 2, \dots 4n - 4\}$ by $f(u_{2i}) = 4i - 2, 1 \leq i \leq \frac{n-2}{2}$, $f(u_{2i+1}) = 4i - 1, 1 \leq i \leq \frac{n-2}{2}$, $f(u_{2i}') = 4i - 3, 1 \leq i \leq \frac{n-2}{2}$, $f(u_{2i+1}') = 4i, 1 \leq i \leq \frac{n-2}{2}$, $f(u_1) = 2n - 3$, $f(u_1') = 2n - 2$, $f(u_n) = 2n - 1$, $f(u_n') = 2n$, $f(v_i) = 2n + 2i - 1, 1 \leq i \leq \frac{n-2}{2}$, $f(v_i') = 2n + 2i, 1 \leq i \leq \frac{n-2}{2}$, $f(w_i) = 3n +$

$2i-3, 1 \le i \le \frac{n-2}{2}$, $f(w_i') = 3n + 2i - 2, 1 \le i \le \frac{n-2}{2}$. Since $e_f(1) = \frac{5n-6}{2}$ and $e_f(0) = \frac{5n-8}{2}$, f is a difference cordial labeling of $DA(T_n) \odot K_1$.

Case (iii): The two triangles starts from u_2 and ends with u_n.
Label the vertices $u_{2i}, u_{2i+1}, u_{2i}', u_{2i+1}', v_i, v_i'$ $\left(1 \le i \le \frac{n-1}{2}\right)$ as in case (ii) and define $f(u_1) = 2n-1$, $f(u_1') = 2n$, $f(w_i) = 3n + 2i - 2, 1 \le i \le \frac{n-1}{2}$, $f(w_i') = 3n + 2i - 1, 1 \le i \le \frac{n-1}{2}$. Since $e_f(1) = \frac{5n-3}{2}$ and $e_f(0) = \frac{5n-5}{2}$, f is a difference cordial labeling of $DA(T_n) \odot K_1$. ■

A difference cordial labeling of $DA(T_8) \odot K_1$ where the two triangles starts from u_1 and ends with u_8 is shown in figure 1.

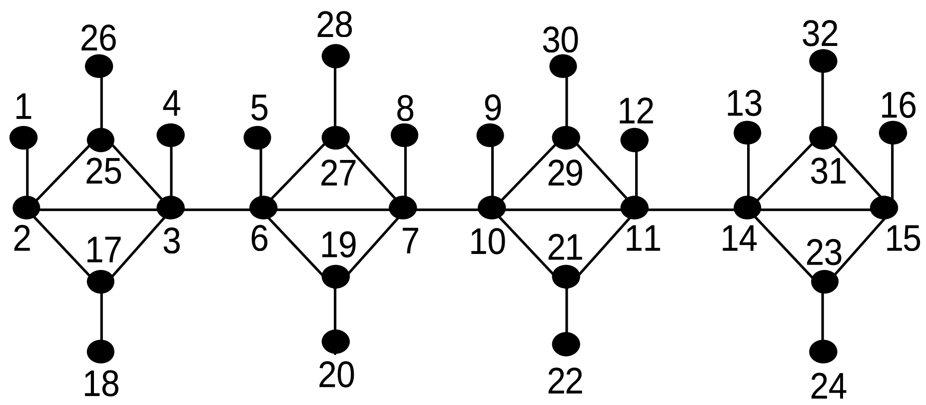

Figure 1

A difference cordial labeling of $DA(T_{10}) \odot K_1$ where the two triangles starts from u_2 and ends with u_9 is shown in figure 2.

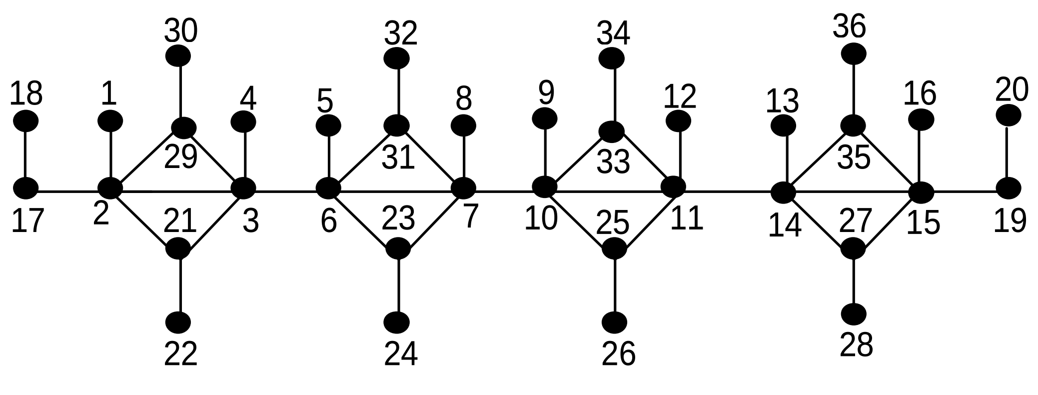

Figure 2

A difference cordial labeling of $DA(T_9) \odot K_1$ where the two triangles starts from u_2 and ends with u_9 is shown in figure 3.

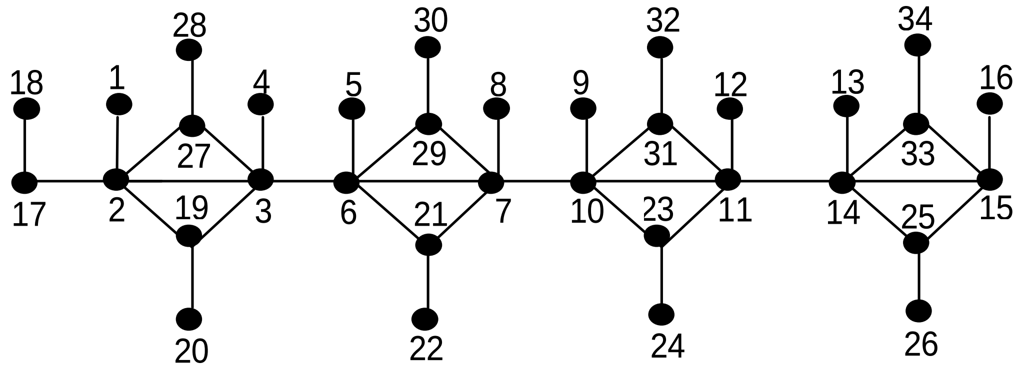

Figure 3

Theorem 2.3: $DA(T_n) \odot 2K_1$ is difference cordial.

Proof: Case (i): The two triangles starts from u_1 and ends with u_n.
Let $V(DA(T_n) \odot 2K_1) = V(DA(T_n)) \cup \{u_i', u_i'' : 1 \le i \le n\} \cup \left\{v_i', v_i'', w_i', w_i'' : 1 \le i \le \frac{n}{2}\right\}$ and $E(DA(T_n) \odot 2K_1) = E(DA(T_n)) \cup \{u_iu_i', u_iu_i'' : 1 \le i \le n\} \cup \left\{v_iv_i', v_iv_i'', w_iw_i', w_iw_i'' : 1 \le i \le \frac{n}{2}\right\}$. Define a map $f: V(DA(T_n) \odot 2K_1) \to \{1, 2 \ldots 6n\}$ by $f(u_{2i-1}) = 9i - 7, 1 \le i \le \frac{n}{2}$, $f(u_{2i}) = 9i - 1, 1 \le i \le \frac{n}{2}$, $f(u_{2i-1}') = 9i - 8, 1 \le i \le \frac{n}{2}$, $f(u_{2i}') = 9i - 2, 1 \le i \le \frac{n}{2}$, $f(u_{2i-1}'') = 9i - 6, 1 \le i \le \frac{n}{2}$, $f(u_{2i}'') = 9i, 1 \le i \le \frac{n}{2}$, $f(v_i) = 9i - 4, 1 \le i \le \frac{n}{2}$, $f(v_i') = 9i - 5, 1 \le i \le \frac{n}{2}$, $f(v_i'') = 9i - 3, 1 \le i \le \frac{n}{2}$, $f(w_i) = \frac{9n}{2} + 3i - 2, 1 \le i \le \frac{n}{2}$, $f(w_i') = \frac{9n}{2} + 3i - 1, 1 \le i \le \frac{n}{2}$, $f(w_i'') = \frac{9n}{2} + 3i, 1 \le i \le \frac{n}{2}$. Since $e_f(1) = \frac{7n}{2}$ and $e_f(0) = \frac{7n-2}{2}$, f is a difference cordial labeling of $DA(T_n) \odot 2K_1$.

Case (ii): The two triangles starts from u_2 and ends with u_{n-1}.
Define a map $f: V(DA(T_n) \odot 2K_1) \to \{1, 2 \ldots 6n - 6\}$ by $f(u_i) = 3i - 1, 1 \le i \le n$, $f(u_i') = 3i - 2, 1 \le i \le n$, $f(u_i'') = 3i, 1 \le i \le n$, $f(v_i) = 3n + 3i - 1, 1 \le i \le \frac{n-4}{2}$, $f(v_i') = 3n + 3i - 2, 1 \le i \le \frac{n-4}{2}$, $f(v_i'') = 3n + 3i, 1 \le i \le \frac{n-4}{2}$, $f\left(v_{\frac{n-2}{2}}\right) = \frac{9n-10}{2}$, $f\left(v_{\frac{n-2}{2}}'\right) = \frac{9n-8}{2}$, $f\left(v_{\frac{n-2}{2}}''\right) = \frac{9n-6}{2}$, $f(w_i) = \frac{9n-10}{2} + 3i, 1 \le i \le \frac{n-2}{2}$, $f(w_i') = \frac{9n-8}{2} + 3i, 1 \le i \le \frac{n-2}{2}$, $f(w_i'') = \frac{9n-6}{2} + 3i, 1 \le i \le \frac{n-2}{2}$. Since $e_f(1) = \frac{7n-8}{2}$ and $e_f(0) = \frac{7n-10}{2}$, f is a difference cordial labeling of $DA(T_n) \odot 2K_1$.

Case (iii): The two triangles starts from u_2 and ends with u_n.
Label the vertices u_i, u_i', u_i'' $(1 \le i \le n)$ and v_i, v_i', v_i'' $\left(1 \le i \le \frac{n-3}{2}\right)$ as in case (ii) and define $f\left(v_{\frac{n-1}{2}}\right) = \frac{9n-7}{2}$, $f\left(v'_{\frac{n-1}{2}}\right) = \frac{9n-5}{2}$, $f\left(v''_{\frac{n-1}{2}}\right) = \frac{9n-3}{2}$, $f(w_i) = \frac{9n-7}{2} + 3i, 1 \le i \le \frac{n-1}{2}$, $f(w_i') = \frac{9n-5}{2} + 3i, 1 \le i \le \frac{n-1}{2}$, $f(w_i'') = \frac{9n-3}{2} + 3i, 1 \le i \le \frac{n-1}{2}$. Since $e_f(1) = e_f(0) = \frac{7n-5}{2}$, f is a difference cordial labeling of $DA(T_n) \odot 2K_1$. ∎

Theorem 2.4: $DA(T_n) \odot K_2$ is difference cordial.
Proof: Case (i): The two triangles starts from u_1 and ends with u_n.
Let $V(DA(T_n) \odot K_2) = V(DA(T_n)) \cup \{u_i', u_i'': 1 \le i \le n\} \cup \left\{v_i', v_i'', w_i', w_i'': 1 \le i \le \frac{n}{2}\right\}$ and $(DA(T_n) \odot 2K_1) = E(DA(T_n)) \cup \{u_iu_i', u_iu_i'', u_i'u_i'': 1 \le i \le n\}$ $\cup$ $\left\{v_iv_i', v_iv_i'', v_i'v_i'', w_iw_i', w_iw_i'', w_i'w_i'': 1 \le i \le \frac{n}{2}\right\}$. Define a map $f: V(DA(T_n) \odot 2K_1) \to \{1, 2 \ldots 6n\}$ by $f(u_{2i-1}) = 6i - 3, 1 \le i \le \frac{n}{2}$, $f(u'_{2i-1}) = 6i - 4, 1 \le i \le \frac{n}{2}$, $f(u''_{2i-1}) = 6i - 5, 1 \le i \le \frac{n}{2}$, $f(u_{2i}) = 6i - 2, 1 \le i \le \frac{n}{2}$, $f(u'_{2i}) = 6i, 1 \le i \le \frac{n}{2}$, $f(u''_{2i}) = 6i - 1, 1 \le i \le \frac{n}{2}$, $f(v_i) = 3n + 3i - 2, 1 \le i \le \frac{n}{2}$, $f(v_i') = 3n + 3i - 1, 1 \le i \le \frac{n}{2}$, $f(v_i'') = 3n + 3i, 1 \le i \le \frac{n}{2}$, $f(w_i) = \frac{9n-4}{2} + 3i, 1 \le i \le \frac{n}{2}$, $f(w_i') = \frac{9n-2}{2} + 3i, 1 \le i \le \frac{n}{2}$, $f(w_i'') = \frac{9n}{2} + 3i, 1 \le i \le \frac{n}{2}$. Since $e_f(1) = \frac{9n}{2}$ and $e_f(0) = \frac{9n-2}{2}$, f is a difference cordial labeling of $DA(T_n) \odot K_2$.

Case (ii): The two triangles starts from u_2 and ends with u_{n-1}.
Define a map $f: V(DA(T_n) \odot K_2) \to \{1, 2 \ldots 6n - 6\}$ by $f(u_{2i}) = 6i - 3, 1 \le i \le \frac{n-2}{2}$, $f(u_{2i+1}) = 6i - 2, 1 \le i \le \frac{n-2}{2}$, $f(u'_{2i}) = 6i - 4, 1 \le i \le \frac{n-2}{2}$, $f(u'_{2i+1}) = 6i, 1 \le i \le \frac{n-2}{2}$, $f(u''_{2i}) = 6i - 5, 1 \le i \le \frac{n-2}{2}$, $f(u''_{2i+1}) = 6i - 1, 1 \le i \le \frac{n-2}{2}$, $f(u_1) = 3n - 5$, $f(u_1') = 3n - 4$, $f(u_1'') = 3n - 3$, $f(u_n) = 3n - 2$, $f(u_n') = 3n - 1$, $f(u_n'') = 3n$, $f(v_i) = 3n + 3i - 2, 1 \le i \le \frac{n-2}{2}$, $f(v_i') = 3n + 3i - 1, 1 \le i \le \frac{n-2}{2}$, $f(v_i'') = 3n + 3i, 1 \le i \le \frac{n-2}{2}$, $f(w_i) = \frac{9n-10}{2} + 3i, 1 \le i \le \frac{n-2}{2}$, $f(w_i') = \frac{9n-8}{2} + 3i, 1 \le i \le \frac{n-2}{2}$, $f(w_i'') = \frac{9n-6}{2} + 3i, 1 \le i \le \frac{n-2}{2}$. Since $e_f(1) = \frac{9n-10}{2}$ and $e_f(0) = \frac{9n-12}{2}$, f is a difference cordial labeling of $DA(T_n) \odot K_2$.

Case (iii): The two triangles starts from u_2 and ends with u_n.
Label the vertices $u_{2i}, u_{2i+1}, u'_{2i}, u'_{2i+1}, u''_{2i}, u''_{2i+1}, v_i, v_i', v_i''$ $\left(1 \le i \le \frac{n-1}{2}\right)$ as in case (ii) and define $f(u_1) = 3n - 2$, $f(u_1') = 3n - 1$, $f(u_1'') = 3n$, $f(w_i) = \frac{9n-7}{2} + 3i, 1 \le i \le \frac{n-1}{2}$, $f(w_i') = \frac{9n-5}{2} + 3i, 1 \le i \le \frac{n-1}{2}$, $f(w_i'') = \frac{9n-3}{2} + 3i, 1 \le i \le \frac{n-1}{2}$. Since $e_f(1) = \frac{9n-5}{2}$ and $e_f(0) = \frac{9n-7}{2}$, f is a difference cordial labeling of $DA(T_n) \odot K_2$. ∎

A double alternate quadrilateral snake $DA(Q_n)$ consists of two alternate quadrilateral snakes that have a common path. That is, a double alternate quadrilateral snake is obtained from a path $u_1, u_2 \dots u_n$ by joining u_i and u_{i+1} (alternatively) to new vertices v_i, x_i and w_i, y_i respectively and then joining v_i, w_i and x_i, y_i.

Theorem 2.5: $DA(Q_n) \odot K_1$ is difference cordial.

Proof: Case (i): The two squares starts from u_1 and ends with u_n.
Let $V(DA(Q_n) \odot K_1) = V(DA(Q_n)) \cup \{u_i' : 1 \le i \le n\} \cup \left\{v_i', w_i', x_i', y_i' : 1 \le i \le \frac{n}{2}\right\}$ and $E(DA(Q_n) \odot K_1) \cup E(DA(Q_n)) \cup \left\{v_i v_i', w_i w_i', x_i x_i', y_i y_i' : 1 \le i \le \frac{n}{2}\right\} \cup \{u_i u_i' : 1 \le i \le n\}$. Define a map $f: V(DA(Q_n) \odot K_1) \to \{1, 2 \dots 6n\}$ by $f(u_i) = 2n + 2i - 1, 1 \le i \le n$, $f(u_i') = 2n + 2i, 1 \le i \le n$, $f(v_i) = 4i - 2, 1 \le i \le \frac{n}{2}$, $f(v_i') = 4i - 3, 1 \le i \le \frac{n}{2}$, $f(w_i) = 4i - 1, 1 \le i \le \frac{n}{2}$, $f(w_i') = 4i, 1 \le i \le \frac{n}{2}$, $f(x_i) = 4n + 4i - 3, 1 \le i \le \frac{n}{2}$, $f(x_i') = 4n + 4i - 2, 1 \le i \le \frac{n}{2}$, $f(y_i) = 4n + 4i - 1, 1 \le i \le \frac{n}{2}$, $f(y_i') = 4n + 4i, 1 \le i \le \frac{n}{2}$. Since $e_f(1) = \frac{7n}{2}$ and $e_f(0) = \frac{7n-2}{2}$, f is a difference cordial labeling of $DA(Q_n) \odot K_1$.

Case (ii): The two squares starts from u_2 and ends with u_{n-1}.
Label the vertices v_i, v_i', w_i, w_i' $\left(1 \le i \le \frac{n-2}{2}\right)$ as in case (i) and define $f(u_i) = 2n + 2i - 5, 1 \le i \le n$, $f(u_i') = 2n + 2i - 4, 1 \le i \le n$, $f(x_i) = 4n + 4i - 7, 1 \le i \le \frac{n-2}{2}$, $f(x_i') = 4n + 4i - 6, 1 \le i \le \frac{n-2}{2}$, $f(y_i) = 4n + 4i - 5, 1 \le i \le \frac{n-2}{2}$, $f(y_i') = 4n + 4i - 4, 1 \le i \le \frac{n-2}{2}$. Since $e_f(1) = \frac{7n-10}{2}$ and $e_f(0) = \frac{7n-12}{2}$, f is a difference cordial labeling of $DA(Q_n) \odot K_1$.

Case (iii): The two squares starts from u_2 and ends with u_n.
Label the vertices v_i, v_i', w_i, w_i' $\left(1 \le i \le \frac{n-1}{2}\right)$ as in case (i) and define $f(u_i) = 2n + 2i - 3, 1 \le i \le n$, $f(u_i') = 2n + 2i - 2, 1 \le i \le n$, $f(x_i) = 4n + 4i - 5, 1 \le i \le \frac{n-1}{2}$, $f(x_i') = 4n + 4i - 4, 1 \le i \le \frac{n-1}{2}$, $f(y_i) = 4n + 4i - 3, 1 \le i \le \frac{n-1}{2}$, $f(y_i') = 4n + 4i - 2, 1 \le i \le \frac{n-1}{2}$. Since $e_f(1) = \frac{7n-5}{2}$ and $e_f(0) = \frac{7n-7}{2}$, f is a difference cordial labeling of $DA(Q_n) \odot K_1$. ■

Theorem 2.6: $DA(Q_n) \odot 2K_1$ is difference cordial.

Proof: Case (i): The two squares starts from u_1 and ends with u_n.
Let $V(DA(Q_n) \odot 2K_1) = V(DA(Q_n)) \cup \left\{v_i', v_i'', w_i', w_i'', x_i', x_i'', y_i', y_i'' : 1 \le i \le \frac{n}{2}\right\} \cup \{u_i', u_i'' : 1 \le i \le n\}$ and $E(DA(Q_n) \odot 2K_1) \cup E(DA(Q_n)) \cup \{u_i u_i', u_i u_i'' : 1 \le i \le n\} \cup \left\{v_i v_i', v_i v_i'', w_i w_i', w_i w_i'', x_i x_i', x_i x_i'', y_i y_i', y_i y_i'' : 1 \le i \le \frac{n}{2}\right\}$. Define a map $f: V(DA(Q_n) \odot 2K_1) \to \{1, 2 \dots 9n\}$ by $f(u_i) = 3i - 1, 1 \le i \le n$, $f(u_i') = 3i -$

$2, 1 \leq i \leq n$, $f(u_i'') = 3i, 1 \leq i \leq n$, $f(v_i) = 3n + 6i - 4, 1 \leq i \leq \frac{n}{2}$, $f(v_i') = 3n + 6i - 5, 1 \leq i \leq \frac{n}{2}$, $f(v_i'') = 3n + 6i - 3, 1 \leq i \leq \frac{n}{2}$, $f(w_i) = 3n + 6i - 1, 1 \leq i \leq \frac{n}{2}$, $f(w_i') = 3n + 6i - 2, 1 \leq i \leq \frac{n}{2}$, $f(w_i'') = 3n + 6i, 1 \leq i \leq \frac{n}{2}$, $f(x_i) = 6n + 6i - 5, 1 \leq i \leq \frac{n}{2}$, $f(x_i') = 6n + 6i - 4, 1 \leq i \leq \frac{n}{2}$, $f(x_i'') = 6n + 6i - 3, 1 \leq i \leq \frac{n}{2}$, $f(y_i) = 6n + 6i - 2, 1 \leq i \leq \frac{n}{2}$, $f(y_i') = 6n + 6i - 1, 1 \leq i \leq \frac{n}{2}$, $f(y_i'') = 6n + 6i, 1 \leq i \leq \frac{n}{2}$. Since $e_f(1) = 5n$ and $e_f(0) = 5n - 1$, f is a difference cordial labeling of $DA(Q_n) \odot 2K_1$.

Case (ii): The two squares starts from u_2 and ends with u_{n-1}.
Define a map $f: V(DA(Q_n) \odot 2K_1) \to \{1, 2 \ldots 9n - 12\}$ by $f(u_i) = 3n + 3i - 7, 1 \leq i \leq n - 1$, $f(u_i') = 3n + 3i - 8, 1 \leq i \leq n - 1$, $f(u_i'') = 3n + 3i - 6, 1 \leq i \leq n - 1$, $f(u_n) = 6n - 8$, $f(u_n') = 6n - 7$, $f(u_n'') = 6n - 6$, $f(v_i) = 6i - 4, 1 \leq i \leq \frac{n-2}{2}$, $f(v_i') = 6i - 5, 1 \leq i \leq \frac{n-2}{2}$, $f(v_i'') = 6i - 3, 1 \leq i \leq \frac{n-2}{2}$, $f(w_i) = 6i - 1, 1 \leq i \leq \frac{n-2}{2}$, $f(w_i') = 6i - 2, 1 \leq i \leq \frac{n-2}{2}$, $f(w_i'') = 6i, 1 \leq i \leq \frac{n-2}{2}$, $f(x_i) = 6n + 6i - 11, 1 \leq i \leq \frac{n-2}{2}$, $f(x_i') = 6n + 6i - 10, 1 \leq i \leq \frac{n-2}{2}$, $f(x_i'') = 6n + 6i - 9, 1 \leq i \leq \frac{n-2}{2}$, $f(y_i) = 6n + 6i - 8, 1 \leq i \leq \frac{n-2}{2}$, $f(y_i') = 6n + 6i - 7, 1 \leq i \leq \frac{n-2}{2}$, $f(y_i'') = 6n + 6i - 6, 1 \leq i \leq \frac{n-2}{2}$. Since $e_f(1) = 5n - 7$ and $e_f(0) = 5n - 8$, f is a difference cordial labeling of $DA(Q_n) \odot 2K_1$.

Case (iii): The two squares starts from u_2 and ends with u_n.
Label the vertices $v_i, v_i', v_i'', w_i, w_i', w_i''$ $\left(1 \leq i \leq \frac{n-1}{2}\right)$ as in case (ii) and define $f(u_{i+1}) = 3n + 3i - 4, 1 \leq i \leq n - 1$, $f(u_{i+1}') = 3n + 3i - 5, 1 \leq i \leq n - 1$, $f(u_{i+1}'') = 3n + 3i - 3, 1 \leq i \leq n - 1$, $f(u_1) = 6n - 5$, $f(u_1') = 6n - 4$, $f(u_1'') = 6n - 3$, $f(x_i) = 6n + 6i - 8, 1 \leq i \leq \frac{n-1}{2}$, $f(x_i') = 6n + 6i - 7, 1 \leq i \leq \frac{n-1}{2}$, $f(x_i'') = 6n + 6i - 6, 1 \leq i \leq \frac{n-1}{2}$, $f(y_i) = 6n + 6i - 5, 1 \leq i \leq \frac{n-1}{2}$, $f(y_i') = 6n + 6i - 4, 1 \leq i \leq \frac{n-1}{2}$, $f(y_i'') = 6n + 6i - 3, 1 \leq i \leq \frac{n-1}{2}$. Since $e_f(1) = e_f(0) = 5n - 4$, f is a difference cordial labeling of $DA(Q_n) \odot 2K_1$. ■

Theorem 2.7: $DA(Q_n) \odot K_2$ is difference cordial.

Proof: Case (i): The two squares starts from u_1 and ends with u_n.
Let $V(DA(Q_n) \odot K_2) = V(DA(Q_n)) \cup \left\{v_i', v_i'', w_i', w_i'', x_i', x_i'', y_i', y_i'': 1 \leq i \leq \frac{n}{2}\right\} \cup \{u_i', u_i'': 1 \leq i \leq n\}$ and $(DA(Q_n) \odot K_2) \cup E(DA(Q_n)) \cup \{u_iu_i', u_iu_i'', u_i'u_i'': 1 \leq i \leq n\} \cup \left\{v_iv_i', v_iv_i'', v_i'v_i'', w_iw_i', w_iw_i'', w_i'w_i'', x_ix_i', x_ix_i'', x_i'x_i'', y_iy_i', y_iy_i'', y_i'y_i'': 1 \leq i \leq \frac{n}{2}\right\}$. Define a map $f: V(DA(Q_n) \odot K_2) \to \{1, 2 \ldots 9n\}$ by $f(u_{2i-1}) = 6i - 3, 1 \leq i \leq \frac{n}{2}$, $f(u_{2i-1}') = 6i - 4, 1 \leq i \leq \frac{n}{2}$, $f(u_{2i-1}'') = 6i - 5, 1 \leq i \leq \frac{n}{2}$, $f(u_{2i}) = 6i -$

$2, 1 \leq i \leq \frac{n}{2}$, $f(u'_{2i}) = 6i, 1 \leq i \leq \frac{n}{2}, f(u''_{2i}) = 6i - 1, 1 \leq i \leq \frac{n}{2}, f(v_i) = 3n + 6i - 5, 1 \leq i \leq \frac{n}{2}$, $f(v'_i) = 3n + 6i - 4, 1 \leq i \leq \frac{n}{2}, f(v''_i) = 3n + 6i - 3, 1 \leq i \leq \frac{n}{2}$, $f(w_i) = 3n + 6i - 2, 1 \leq i \leq \frac{n}{2}$, $f(w'_i) = 3n + 6i - 1, 1 \leq i \leq \frac{n}{2}, f(w''_i) = 3n + 6i, 1 \leq i \leq \frac{n}{2}$, $f(x_i) = 6n + 6i - 5, 1 \leq i \leq \frac{n}{2}$, $f(x'_i) = 6n + 6i - 4, 1 \leq i \leq \frac{n}{2}, f(x''_i) = 6n + 6i - 3, 1 \leq i \leq \frac{n}{2}$, $f(y_i) = 6n + 6i - 2, 1 \leq i \leq \frac{n}{2}$, $f(y'_i) = 6n + 6i - 1, 1 \leq i \leq \frac{n}{2}, f(y''_i) = 6n + 6i, 1 \leq i \leq \frac{n}{2}$. Since $e_f(1) = \frac{13n}{2}$ and $e_f(0) = \frac{13n-2}{2}$, f is a difference cordial labeling of $DA(Q_n) \odot K_2$.

Case (ii): The two squares starts from u_2 and ends with u_{n-1}.
Define a map $f: V(DA(Q_n) \odot K_2) \to \{1, 2 \dots 9n - 12\}$ by $f(u_i) = 3n + 3i - 8, 1 \leq i \leq n$, $f(u'_i) = 3n + 3i - 7, 1 \leq i \leq n$, $f(u''_i) = 3n + 3i - 6, 1 \leq i \leq n$, $f(v_i) = 6i - 3, 1 \leq i \leq \frac{n-2}{2}$, $f(v'_i) = 6i - 4, 1 \leq i \leq \frac{n-2}{2}$, $f(v''_i) = 6i - 5, 1 \leq i \leq \frac{n-2}{2}$, $f(w_i) = 6i - 2, 1 \leq i \leq \frac{n-2}{2}$, $f(w'_i) = 6i, 1 \leq i \leq \frac{n-2}{2}$, $f(w''_i) = 6i - 1, 1 \leq i \leq \frac{n-2}{2}$, $f(x_i) = 6n + 6i - 11, 1 \leq i \leq \frac{n-2}{2}$, $f(x'_i) = 6n + 6i - 10, 1 \leq i \leq \frac{n-2}{2}$, $f(x''_i) = 6n + 6i - 9, 1 \leq i \leq \frac{n-2}{2}$, $f(y_i) = 6n + 6i - 8, 1 \leq i \leq \frac{n-2}{2}$, $f(y'_i) = 6n + 6i - 7, 1 \leq i \leq \frac{n-2}{2}$, $f(y''_i) = 6n + 6i - 6, 1 \leq i \leq \frac{n-2}{2}$. Since $e_f(1) = \frac{13n-18}{2}$ and $e_f(0) = \frac{13n-20}{2}$, f is a difference cordial labeling of $DA(Q_n) \odot K_2$.

Case (iii): The two squares starts from u_2 and ends with u_n.
Label the vertices $v_i, v'_i, v''_i, w_i, w'_i, w''_i$ $\left(1 \leq i \leq \frac{n-1}{2}\right)$ as in case (ii) and define $f(u_i) = 3n + 3i - 5, 1 \leq i \leq n$, $f(u'_i) = 3n + 3i - 4, 1 \leq i \leq n$, $f(u''_i) = 3n + 3i - 3, 1 \leq i \leq n$, $f(x_i) = 6n + 6i - 8, 1 \leq i \leq \frac{n-1}{2}$, $f(x'_i) = 6n + 6i - 7, 1 \leq i \leq \frac{n-1}{2}$, $f(x''_i) = 6n + 6i - 6, 1 \leq i \leq \frac{n-1}{2}$, $f(y_i) = 6n + 6i - 5, 1 \leq i \leq \frac{n-1}{2}$, $f(y'_i) = 6n + 6i - 4, 1 \leq i \leq \frac{n-1}{2}$, $f(y''_i) = 6n + 6i - 3, 1 \leq i \leq \frac{n-1}{2}$. Since $e_f(1) = \frac{13n-9}{2}$ and $e_f(0) = \frac{13n-11}{2}$, f is a difference cordial labeling of $DA(Q_n) \odot K_2$. ∎

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