Global Journal of Mathematical Sciences: Theory and Practical.
ISSN 0974-3200 Volume 5, Number 3 (2013), pp. 177-184

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Note on new Classes of Separation axioms

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Abstract

In this paper we introduce new class of spaces such as $\hat{\Omega}$-regular and $\hat{\Omega}$-normal spaces and investigate the basic properties of these spaces via $\hat{\Omega}$-closed sets.

AMS subject classification: 57D05.
Keywords: Semi open sets, δ-open sets, δ-closure, $sker$, $\delta\omega$-closed sets.

1. Introduction

Levine [6] initiated the study of generalized closed sets (briefly g-closed) in general topology. The concept of weak separation axioms such as g-regular and g-normal spaces are introduced and investigated by Munshi [7] by utilizing g-closed sets. The objective is to introduce and study new separation axioms namely $\hat{\Omega}$-regular and $\hat{\Omega}$-normal spaces and their properties by applying $\hat{\Omega}$-closed sets.

2. Preliminaries

Throughout this paper (X, τ) (or briefly X) represent a topological space with no separation axioms assumed unless otherwise explicitly stated. For a subset A of (X, τ), we denote the closure of A, the interior of A and the complement of A as $cl(A), int(A)$ and A^c respectively. The family of all open (resp. δ-open, $\hat{\Omega}$-open) sets on X are denoted by $O(X)$ (resp. $\delta O(X), \hat{\Omega}O(X)$). The family of all $\hat{\Omega}$-closed sets on X are denoted by $\hat{\Omega}C(X)$.

- $O(X,x) = \{U \in X : x \in U \in O(X)\}$
- $\delta O(X,x) = \{U \in X : x \in U \in \delta O(X)\}$
- $\hat{\Omega} O(X,x) = \{U \in X : x \in U \in \hat{\Omega} O(X)\}$

Let us recall the following definitions, which are useful in the sequel.

Definition 2.1. [9] A subset A of X is called δ-closed in a topological space (X,τ) if $A = \delta cl(A)$, where $\delta cl(A) = \{x \in X : int(cl(U)) \cap A \neq \emptyset, U \in O(X,x)\}$. The complement of δ-closed set in (X,τ) is called δ-open set in (X,τ). From [1], lemma 3, $\delta cl(A) = \cap\{F \in \delta C(X) : A \subseteq F\}$ and from corollary 4, $\delta cl(A)$ is a δ-closed for a subset A in a topological space (X,τ).

Definition 2.2. A subset A of a topological space (X,τ) is called

(i) *semiopen* set in [5] (X,τ) if $A \subseteq cl(int(A))$.

(ii) $\hat{\Omega}$-closed set [2] if $\delta cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi open in (X,τ).

The complement of $\hat{\Omega}$-closed (resp. semi open) is said to be $\hat{\Omega}$-open (resp. semi closed).

Definition 2.3. A function $f : (X,\tau) \to (Y,\sigma)$ is said to be $\hat{\Omega}$-irresolute if $f^{-1}(V)$ is $\hat{\Omega}$-open set in (X,τ) for every $\hat{\Omega}$-open set V in (Y,σ).

Definition 2.4. A space X is said to be [4]

(i) $\hat{\Omega}$-T_0 if for any distinct pair of points x and y of X, there exists a $\hat{\Omega}$-open set U of X containing x but not y (or) containing y but not x.

(ii) $\hat{\Omega}$-T_1 if for any distinct pair of points x and y of X, there exists a $\hat{\Omega}$-open set U of X containing x but not y and a $\hat{\Omega}$-open set V of X containing y but not x.

(iii) $\hat{\Omega}$-T_2 if for any distinct pair of points x and y of X, there exists disjoint $\hat{\Omega}$-open sets U and V of X containing x and y respectively.

Definition 2.5. A space X is said to be R_0 [8] if for every open set U such that $x \in U$, then $cl(\{x\}) \subseteq U$.

3. $\hat{\Omega}$-regular and $\hat{\Omega}$-normal spaces

Definition 3.1. A space is said to be $\hat{\Omega}$-regular if for every closed set F and each point $x \notin F$, there exists two disjoint $\hat{\Omega}$-open sets U and V in X such that $F \subseteq U, x \in V$.

Example 3.2. If $X = \{a,b,c\}$ and $\tau = \{\emptyset, \{a\}, \{b,c\}, X\}$, then $\hat{\Omega} O(X) = \mathcal{P}(X)$. It is $\hat{\Omega}$-regular.

Theorem 3.3. Every $\hat{\Omega}$-regular and T_0 space is $\hat{\Omega}$-T_2 space.

Proof. Suppose that x and y are any two disjoint points of X. Since X is T_0, there exists an open set U in (X, τ) containing any one of these points.Without loss of generality, assume that $x \in U$ and $y \notin U$. Therefore, $X \setminus U$ is a closed set and $x \notin X \setminus U$. By the $\hat{\Omega}$-regularity of X, there exists disjoint $\hat{\Omega}$-open sets V and W such that $X \setminus U \subseteq V$ and $x \in W$. Now we have two disjoint $\hat{\Omega}$-open sets U and V such that $x \in W$ and $y \in V$. Thus (X, τ) is a $\hat{\Omega}$-T_2 space. ∎

Theorem 3.4. If (X, τ) is $\hat{\Omega}$-regular and $semi$-$T_{\frac{1}{2}}$, then X is regular.

Proof. Suppose that $x \in X$ is arbitrary and F is any closed set in X such that $x \notin F$. By hypothesis, there exists two disjoint $\hat{\Omega}$-open sets U and V in X such that $F \subseteq U, x \in V$. By [3] theorem 3.17, U and V are open sets in X. Therefore, X is regular. ∎

Let us characterize $\hat{\Omega}$-regular space as follows.

Theorem 3.5. In a topological space (X, τ) the following statements hold.

(i) X is $\hat{\Omega}$-regular.

(ii) For each $x \in X$ and each open set U containing x, there is a $\hat{\Omega}$-open set V containing x such that $\hat{\Omega}cl(V) \subseteq U$.

(iii) For every non-empty set A,disjoint from an open set U, there exists a $\hat{\Omega}$-open set V such that $A \cap V \neq \emptyset$ and $\hat{\Omega}cl(V) \subseteq U$.

(iv) For every non-empty set A,disjoint from a closed set F, there exists two disjoints $\hat{\Omega}$-open subsets U and V of X such that $A \cap V \neq \emptyset$ and $F \subseteq U$.

Proof. i) $\Rightarrow$ ii) Suppose that U is any open set in (X, τ) and $x \in X$ such that $x \in U$. Then $X \setminus U$ is a closed set does not containing x. Since X is $\hat{\Omega}$-regular, there exists two disjoint $\hat{\Omega}$-open sets V and W such that $X \setminus U \subseteq W, x \in V \subseteq \hat{\Omega}cl(V)$. Suppose that $z \in X$ such that $z \notin U$, then $z \in X \setminus U \subseteq W$. Now W is a $\hat{\Omega}$-open set containing z such that $V \cap W = \emptyset$. By [2] theorem 5.11, $z \notin \hat{\Omega}cl(V)$ and hence $\hat{\Omega}cl(V) \subseteq U$. Now we have a $\hat{\Omega}$-open set V containing x such that $\hat{\Omega}cl(V) \subseteq U$.
ii) $\Rightarrow$ iii) Suppose that A is a non-empty set which is disjoint from a open subset U of X. Choose $x \in A \cap U$. Then, U is a open set containing x. By hypothesis, there exits a $\hat{\Omega}$-open subset $V$of X such that $x \in V \subseteq \hat{\Omega}cl(V) \subseteq U$. Therefore, $x \in A \cap V$ which implies that $A \cap V \neq \emptyset$. Thus, there exists a $\hat{\Omega}$-open subset $V$of X such that $A \cap V \neq \emptyset$ and $\hat{\Omega}cl(V) \subseteq U$.
iii) $\Rightarrow$ iv) Suppose that A is a non-empty set which is disjoint from a closed subset F of X. Then, $X \setminus F$ is a open set such that $A \cap X \setminus F \neq \emptyset$. By hypothesis, there exists a $\hat{\Omega}$-open subset $V$of X such that $A \cap V \neq \emptyset$ and $\hat{\Omega}cl(V) \subseteq X \setminus F$. Then $F \subseteq X \setminus \hat{\Omega}cl(V)$, where $X \setminus \hat{\Omega}cl(V)$ is a $\hat{\Omega}$-open subset of X. By letting $U = X \setminus \hat{\Omega}cl(V)$, it is shown that there exists two disjoint $\hat{\Omega}$-open sets U and V in X such that $A \cap V \neq \emptyset$ and $F \subseteq U$.

iv) $\Rightarrow$ i) Suppose that F is any closed subset of X and $x \notin F$. Then $A \cap F = \emptyset$, where $A = \{x\}$. By hypothesis, there exists two disjoint $\hat{\Omega}$-open sets U and V such that $A \cap V \neq \emptyset$ and $F \subseteq U$. It is shown that there exists two disjoint $\hat{\Omega}$-open sets U and V such that, $x \in V$ and $F \subseteq U$. Thus, X is $\hat{\Omega}$-regular. ■

Theorem 3.6. A space X is $\hat{\Omega}$-regular if and only if for each $x \in X$ and every closed set F such that $x \notin F$, there exists $V \in \hat{\Omega}O(X, x)$ such that $\hat{\Omega}cl(V) \cap F = \emptyset$.

Proof. **Necessity-**Suppose that $x \in X$ and F is any closed set in X such that $x \notin F$. Then $X \setminus F$ is an open set in X containing x. By hypothesis, there exists a $\hat{\Omega}$-open set V containing x such that $\hat{\Omega}cl(V) \subseteq X \setminus F$. Thus, $\hat{\Omega}cl(V) \cap F = \emptyset$.
Sufficiency-Suppose that $x \in X$ is arbitrary and F is any closed set in X such that $x \notin F$. By hypothesis, there exists $V \in \hat{\Omega}O(X, x)$ such that $\hat{\Omega}cl(V) \cap F = \emptyset$. Then $F \subseteq X \setminus \hat{\Omega}cl(V)$. If $U = X \setminus \hat{\Omega}cl(V)$, then U is a $\hat{\Omega}$-open set such that $F \subseteq U, x \in V$ and $U \cap V = \emptyset$. Thus, X is is $\hat{\Omega}$-regular. ■

Theorem 3.7. A space X is $\hat{\Omega}$-regular if and only if for every closed subset F and for each $x \notin F$, there exists $\hat{\Omega}$-open subsets U and V of X such that $x \in U$ and $F \subseteq V$ and $cl(U) \cap cl(V) = \emptyset$.

Proof. **Necessity-**Suppose that F is any closed set not containing the point x of X. By hypothesis, there exists there exists two disjoint $\hat{\Omega}$-open sets U_x and V in X such that $F \subseteq V, x \in U$. Then,$U_x \cap cl(V) = \emptyset$. Since $cl(V)$ is a closed subset of X not containing x, again by hypothesis, there exists two disjoint $\hat{\Omega}$-open sets G and W in X such that $cl(V) \subseteq W$ and $x \in G$. Then, $cl(G) \cap W = \emptyset$. Put $U = U_x \cap G$, then U is a $\hat{\Omega}$-open set disjoint from V and $x \in U, F \subseteq V$. Moreover, $cl(U) \cap cl(V) \subseteq cl(G) \cap W = \emptyset$.
Sufficiency-Suppose that for every closed subset F not containing a point x of X, there exists $\hat{\Omega}$-open subsets U and V of X such that $x \in U$ and $F \subseteq V$ and $cl(U) \cap cl(V) = \emptyset$. Then $U \cap V \subseteq cl(U) \cap cl(V) = \emptyset$. Thus, X is a $\hat{\Omega}$-regular space. ■

Theorem 3.8. Let Y be both open and pre-closed set in a $\hat{\Omega}$-regular space (X, τ). Then the subspace $(Y, \tau|_Y)$ is $\hat{\Omega}$-regular.

Proof. Suppose that F_1 is closed in $(Y, \tau|_Y)$ and x be any point in X such that $x \in Y \setminus F_1$. Then $F_1 = F \cap Y$ for some closed set F in X. Since $x \in Y \setminus F_1, x \notin F$. By hypothesis, there exists disjoint $\hat{\Omega}$-open sets U and V in X such that $F \subseteq U, x \in V$. By [2] theorem 6.10, $U \cap Y$ and $V \cap Y$ are disjoint $\hat{\Omega}$-open sets in $(Y, \tau|_Y)$ such that $F_1 \subseteq U \cap Y, x \in V \cap Y$. Therefore, $(Y, \tau|_Y)$ is $\hat{\Omega}$-regular. ■

Theorem 3.9. Let X be a $\hat{\Omega}$-regular space. If A is $\hat{\Omega}$-closed relative to X, then A is gp-closed subset of X.

Proof. Suppose that X is a $\hat{\Omega}$-regular space and A is any $\hat{\Omega}$-closed relative to X. Let U be any open subset of X such that $A \subseteq U$. Let $x \in A$ be arbitrary. Then U is an open subset of X such that $x \in U$. By theorem 3.5 (ii), there is a $\hat{\Omega}$-open set V_x containing x such that $\hat{\Omega}cl(V_x) \subseteq U$. Then the family $\{V_x : x \in A, x \in V \text{ and } \hat{\Omega}cl(V_x) \subseteq U\}$ is a $\hat{\Omega}$-open cover of A. Since A is $\hat{\Omega}$-closed relative to X, there exists a finite number of points $x_1, x_2, \ldots, x_n$ in A such that $A \subseteq \bigcup_{i=1}^{i=n} \hat{\Omega}cl(V_{x_i}) \subseteq U$. By [2] remark 5.2, each $\hat{\Omega}cl(V_{x_i})$ is $\hat{\Omega}$-closed and by [2] theorem 4.12, finite union of $\hat{\Omega}$-closed sets is $\hat{\Omega}$-closed set in X. Then, $A \subseteq \hat{\Omega}cl(A) \subseteq \bigcup_{i=1}^{i=n} \hat{\Omega}cl(V_{x_i}) \subseteq U$. Since $pcl(A) \subseteq \hat{\Omega}cl(A)$, it follows that A is gp-closed subset of X. ■

Theorem 3.10. If $f : (X, \tau) \to (Y, \sigma)$ is continuous bijective such that every $\hat{\Omega}$-open set is $\hat{\Omega}$-open,then the image of a $\hat{\Omega}$-regular space is $\hat{\Omega}$-regular.

Proof. Suppose that (X, τ) is a $\hat{\Omega}$-regular space and $y \in Y$ be arbitrary and F is any closed set in Y such that $y \notin F$. Since f is surjective, $y = f(x)$ for some $x \in X$. Since f is continuous, $f^{-1}(F)$ is a closed set in X such that $x \notin f^{-1}(F)$. Since X is $\hat{\Omega}$-regular, there exists disjoint $\hat{\Omega}$-open sets U and V in X such that $x \in U$ and $f^{-1}(F) \subseteq V$. By hypothesis, $f(U)$ and $f(V)$ are disjoint $\hat{\Omega}$-open sets in Y such that $y \in f(U)$ and $F \subseteq f(V)$. Therefore, Y is $\hat{\Omega}$-regular space. ■

Theorem 3.11. If $f : (X, \tau) \to (Y, \sigma)$ is closed injective $\hat{\Omega}$-irresolute and Y is $\hat{\Omega}$-regular, then X is $\hat{\Omega}$-regular space.

Proof. Suppose that (Y, σ) is a $\hat{\Omega}$-regular space and $x \in X$ be arbitrary and F is any closed set in X such that $x \notin F$. Since f is closed, $f(F)$ is a closed set in Y such that $f(x) \notin f(F)$. Since Y is $\hat{\Omega}$-regular, there exists disjoint $\hat{\Omega}$-open sets U and V in Y such that $f(x) \in U$ and $f(F) \subseteq V$. Since f is $\hat{\Omega}$-irresolute, $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint $\hat{\Omega}$-open sets in X such that $x \in f^{-1}(U)$ and $F \subseteq f^{-1}(V)$. Therefore, Y is $\hat{\Omega}$-regular space. ■

Definition 3.12. A space (X, τ) is said to be $\hat{\Omega}$-normal if for every pair of disjoint closed sets A and B of X, there exists a pair of disjoint $\hat{\Omega}$-open sets $U$and V such that $A \subseteq U$ and $B \subseteq V$.

Example 3.13. If $X = \{a, b, c\}$ and $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, X\}$, then $\hat{\Omega}O(X) = \tau$. Here X is $\hat{\Omega}$-normal.

Theorem 3.14. Every $\hat{\Omega}$-regular space is $\hat{\Omega}$-normal.

Proof. Suppose that A and B are any pair of disjoint closed sets in X and $x \in A$. Then $x \notin B$. By hypothesis, there exists disjoint $\hat{\Omega}$-open sets U_x and V_x such that $x \in U_x$ and $B \subseteq V_x$. If we take $U = \bigcup_{x \in A} U_x$, then by [2] theorem 4.16, U is $\hat{\Omega}$-open set in X such that $A \subseteq U$. Also $U \cap V_x = \emptyset$. Thus,X is $\hat{\Omega}$-normal. ■

Remark 3.15. It is noted from the example 3.10 that $\hat{\Omega}$-normal space can not, in general, be $\hat{\Omega}$-regular space.

Let us prove the necessary condition under which $\hat{\Omega}$-normal space become $\hat{\Omega}$-regular.

Theorem 3.16. If a space X is $\hat{\Omega}$-normal and R_0,then X is $\hat{\Omega}$-regular space.

Proof. Suppose that F is any closed set in X and $x \in X \setminus F$ is an arbitrary point. Then $X \setminus F$ is open in X containing x. Since X is $R_0, cl(\{x\}) \subseteq X \setminus F$ and hence $cl(\{x\}) \cap F = \emptyset$. Since X is $\hat{\Omega}$-normal,there exists a pair of disjoint $\hat{\Omega}$-open sets U and V in X such that $cl(\{x\}) \subseteq U$ and $F \subseteq V$. Therefore, $x \in U, F \subseteq V$ and $U \cap V = \emptyset$. Thus, X is $\hat{\Omega}$-regular space. ■

Some characterizations of $\hat{\Omega}$-normal spaces.

Theorem 3.17. A space X is $\hat{\Omega}$-normal if and only if for every closed set A and every open set B containing A, there exists a $\hat{\Omega}$-open set U in X such that $A \subseteq U \subseteq cl(U) \subseteq B$.

Proof. **Necessity-**Suppose that X is $\hat{\Omega}$-normal. Suppose that A is any closed set and B is any open set in X such that $A \subseteq B$. Then A and $X \setminus B$ are a pair of disjoint $\hat{\Omega}$-closed sets in X. By hypothesis, there exists a pair of $\hat{\Omega}$-open sets U and V in X such that $A \subseteq U, X \setminus B \subseteq V$Then $A \subseteq U \subseteq X \setminus V \subseteq B$. By [2] remark 5.2, $A \subseteq U \subseteq \hat{\Omega}cl(U) \subseteq X \setminus V \subseteq B$.
Sufficiency-Suppose that A and B are disjoint $\hat{\Omega}$-closed sets in X. Then $A \subseteq X \setminus B$. By hypothesis, there exists a $\hat{\Omega}$-open set U in X such that $A \subseteq U \subseteq cl(U) \subseteq X \setminus B$. If $V = X \setminus \hat{\Omega}cl(U)$, then V is a $\hat{\Omega}$-open set in X such that $A \subseteq U, B \subseteq V$ and $U \cap V = \emptyset$. Thus, X is $\hat{\Omega}$-normal. ■

Theorem 3.18. A space (X, τ) is $\hat{\Omega}$-normal if and only if for every pair of disjoint closed sets A and B of X, there exists a $\hat{\Omega}$-open set U in X containing A and $\hat{\Omega}cl(U) \cap B = \emptyset$.

Proof. **Necessity-**Suppose that X is a $\hat{\Omega}$-normal space and suppose that A and B are any two disjoint closed sets in X. Then $A \subseteq X \setminus B$. By theorem 3.14, there exists a $\hat{\Omega}$-open set U in X such that $A \subseteq U \subseteq \hat{\Omega}cl(U) \subseteq X \setminus B$. Then $\hat{\Omega}cl(U) \cap B = \emptyset$.
Sufficiency-Suppose that A and B are any two disjoint closed sets in X. By hypothesis,there exists a $\hat{\Omega}$-open set U in X containing A and $\hat{\Omega}cl(U) \cap B = \emptyset$. If $V = X \setminus \hat{\Omega}cl(U)$, then V is a $\hat{\Omega}$-open set in X containing B such that $U \cap V = \emptyset$. Thus, X is $\hat{\Omega}$-normal. ■

Theorem 3.19. Let X be $semi\text{-}T_{\frac{1}{2}}$. Then a space X is $\hat{\Omega}$-normal if and only if for every pair of disjoint closed sets A and B of X, there exists a pair of $\hat{\Omega}$-open sets $U$and V such that $A \subseteq U$ and $B \subseteq V$ and $\hat{\Omega}cl(U) \cap \hat{\Omega}cl(V) = \emptyset$.

Proof. **Necessity-**Suppose that A and B are any two disjoint closed sets in X. Then $A \subseteq X \setminus B$. By theorem 3.15, there exists a $\hat{\Omega}$-open set U in X such that $A \subseteq U$and $\hat{\Omega}cl(U) \cap B = \emptyset$. Since $\hat{\Omega}cl(U)$ is a $\hat{\Omega}$-closed set and by [3] theorem 3.17, $\hat{\Omega}cl(U)$ is a closed set in X. Again by hypothesis, there exists a $\hat{\Omega}$-open set V in X such that $B \subseteq V$

and $\hat{\Omega}cl(U) \cap \hat{\Omega}cl(V) = \emptyset$.
Sufficiency-Suppose that A and B are any two disjoint closed sets in X. By hypothesis, there exists a pair of $\hat{\Omega}$-open sets $U$and V such that $A \subseteq U$ and $B \subseteq V$ and $\hat{\Omega}cl(U) \cap \hat{\Omega}cl(V) = \emptyset$. Thus, $U \cap V = \emptyset$ and hence X is $\hat{\Omega}$-normal. ∎

Theorem 3.20. Let Y be both open and closed set in a $\hat{\Omega}$-normal space (X, τ). Then the subspace $(Y, \tau|Y)$ is $\hat{\Omega}$-normal.

Proof. Suppose that A and B are disjoint closed subsets of $(Y, \tau|Y)$. Since Y is closed subset of X. A and B are closed a subset of X. Since X is $\hat{\Omega}$-normal, there exists a disjoint pair of $\hat{\Omega}$-open subsets U_1 and U_2 of X such that $A \subseteq U_1$ and $B \subseteq V_1$. Take $U = U_1 \cap Y$ and $V = V_1 \cap Y$. By [2] theorem 6.10, U and V are $\hat{\Omega}$-open sets in the subspace $(Y, \tau|Y)$. Since U_1 and V_1 are disjoint, U and V are disjoint and $A = A \cap Y \subseteq U_1 \cap Y = U, B = B \cap Y \subseteq V_1 \cap Y = V$. Therefore, $(Y, \tau|Y)$ is $\hat{\Omega}$-normal. ∎

Theorem 3.21. If $f : (X, \tau) \to (Y, \sigma)$ is continuous bijective such that every $\hat{\Omega}$-open set is $\hat{\Omega}$-open, then image of a $\hat{\Omega}$-normal space is $\hat{\Omega}$-normal.

Proof. Suppose that (X, τ) is a $\hat{\Omega}$-normal space and A and B are any two disjoint closed sets in Y. Since f is continuous, $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint closed sets in X. Since X is $\hat{\Omega}$-normal, there exists disjoint $\hat{\Omega}$-open sets U and V in X such that $f^{-1}(A) \subseteq\in U$ and $f^{-1}(B) \subseteq V$. By hypothesis, $f(U)$ and $f(V)$ are disjoint $\hat{\Omega}$-open sets in Y such that $A \in f(U)$ and $B \subseteq f(V)$. Therefore, Y is $\hat{\Omega}$-normal space. ∎

Theorem 3.22. If $f : (X, \tau) \to (Y, \sigma)$ is closed injective $\hat{\Omega}$-irresolute and Y is $\hat{\Omega}$-normal, then X is $\hat{\Omega}$-normal space.

Proof. Suppose that (Y, σ) is a $\hat{\Omega}$-normal space and A and B are any two disjoint closed sets in X. Since f is closed, $f(A)$ and $f(B)$ are closed sets in Y. Since f is injective, $f(A)$ and $f(B)$ are disjoint closed sets in Y. Since Y is $\hat{\Omega}$-normal, there exists disjoint $\hat{\Omega}$-open sets U and V in Y such that $f(A) \subseteq U$ and $f(B) \subseteq V$. Since f is $\hat{\Omega}$-irresolute, $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint $\hat{\Omega}$-open sets in X such that $A \subseteq f^{-1}(U)$ and $B \subseteq f^{-1}(V)$. Therefore, Y is $\hat{\Omega}$-normal space. ∎

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