Global Journal of Mathematical Sciences: Theory and Practical.
ISSN 0974-3200 Volume 6, Number 1 (2014), pp. 9-18

http://www.irphouse.com

To Study of Magneto hydrodynamics Shocks in Astrophysical Medium

Krishna Kumari and Prof R.K.Budhaulia

Research Scholar in Applied Mathematics, Guide/Supervisor-
Bhagwant university,Ajmer,Rajsthan

Abstract

The aim of the study is to discuss the effect of Magneto hydrodynamics shocks on the dynamics of Astrophysical plasma. The solution of Riemann problems to compute numerical solutions of Ideal magneto hydrodynamics for an arbitrary initial condition is also obtained which is partly based on the algorithm proposed by Torrilhon (2002) .

Key words-Ideal Magneto hydrodynamics-MHD, turbulence

1.Introduction

Shocks and discontinuities are transition layers where the plasma properties change from one equilibrium state to another. The plasma is a fluid composed by charged particles in addition to this, a moving charged particle creates a magnetic field which also interacts with the other charged particles. The relation between the plasma properties on both sides of a shock or a discontinuity can be obtained from the conservatives form of the magneto hydrodynamics equations, assuming conservative of mass,momentum energy and of in the study of compressible fluid through nozzle the continuity consideration led . In some case to the formation of a discontinuity surface across which these are jumps in pressure,density, temperature etc. such surface are called shock wave these may also result from their causes, for example detonation of explosives supersonic flights of projectile and so on.

Magneto hydrodynamic (MHD) flow is governed by classical fluid dynamics and electromagnetic.

Examples of such fluids include liquid metals, plasmas, and salt water or electrolytes. Application to MHD can be derive and control flows in astronomy geo-physics, network, electromagnetic casting of metal and MHD power generation Many Astrophysical phenomena is based on MHD flows of plasma.

The system of ideal MHD is not strictly hyperbolic. Gogosov (1962) investigated the wave-pattern of the solution in MHD Riemann problems on the existence and uniqueness of solutions of ideal MHD Riemann, considering only the evolutionary waves and switch-off waves.Torrilhon (2002) investigated the uniqueness of the solution considering the intermediate shocks.

The Riemann problem is a kind of initial value problems for hyperbolic systems like system of MHD equations with discontinuous initial condition in the form

$$u(x)|_{t=0} = u^1, x < 0 \qquad u(x)|_{t=0} = u^0, x > 0$$

,Solving Riemann problems is one of main tasks in numerical schemes for MHD flow because there is no convincing criterion for the physically relevant solution. The entropy condition admits only the shocks across .

The objective of present work is to explain Ideal MHD equations and exact solution of Riemann solutions are used to obtain numerical fluxes and problems to compute numerical solutions of Ideal MHD equations and also discussed the internal rotation motion of stars and turbulence which is one of basic problems in astrophysics.

The paper is organized as follows. Sec. 2 deals Ideal MHD equations and simple waves in Ideal MHD and discontinuity. Sec .3 deal a brief review of shock waves which are constituents of the solution of the Riemann problems. Sec. 4 discuss the importance of magnetic field in stars and turbulence. Finally conclusion that have been drawn from this work are presented in Sec.5.

2.Ideal MHD equation

The ideal MHD equations consist of the continuity equation, the Cauchy momentum equation, Ampere's Law neglecting displacement current, and a temperature evolution equation. Ideal MHD is only strictly applicable when the plasma is strongly collision, so that the time scale of collisions is shorter than the other characteristic times in the system, and the particle distributions are therefore close to Maxwell's equation.

In plane symmetry the MHD equation are given by

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho u) = 0 \tag{2.1}$$

$$\frac{\partial \rho u}{\partial t} + \nabla.\left[\rho u u + P + \frac{B^2}{2}\right] = 0 \tag{2.2}$$

$$\frac{\partial e}{\partial t} + \nabla.\left[\left(e + P + \frac{B^2}{2}\right)u - B(B.u)\right] = 0 \tag{2.3}$$

$$\frac{\partial B}{\partial t} - \nabla \times (u \times B) = 0 \tag{2.4}$$

Where ρ, u, P and B are density, flow velocity, pressure and the magnetic field respectively. Total energy density

$$e = \frac{P}{\gamma - 1} + \frac{1}{2}\rho u^2 + \frac{B^2}{2} \tag{2.5}$$

For 1D plane symmetric flow $\nabla . B = 0$

Equations (2.1)-(2.5) can be written in conservative form as

$$\frac{\partial U}{\partial t} + \frac{\partial F}{\partial x} = 0$$

The vector of conserved variables $U = \left(\rho, \rho u_x, \rho u_y, \rho u_z, e, B_y, B_z\right)^T$

And the flux vector function

$$F = \begin{pmatrix} \rho u_x, \rho u_x^2 + P + \frac{B^2}{2} - B_x^2, \rho u_x \rho u_y - B_x B_y, \rho u_x u_z - B_x B_z, \left(e + P + \frac{B^2}{2}\right) u_x - B_x\left(B_x u_x + B_y u_y + B_z u_z\right), \\ B_y u_x - B_x u_y, B_z u_x - B_x u_z \end{pmatrix}^T$$

2.1 Simple waves in Ideal MHD

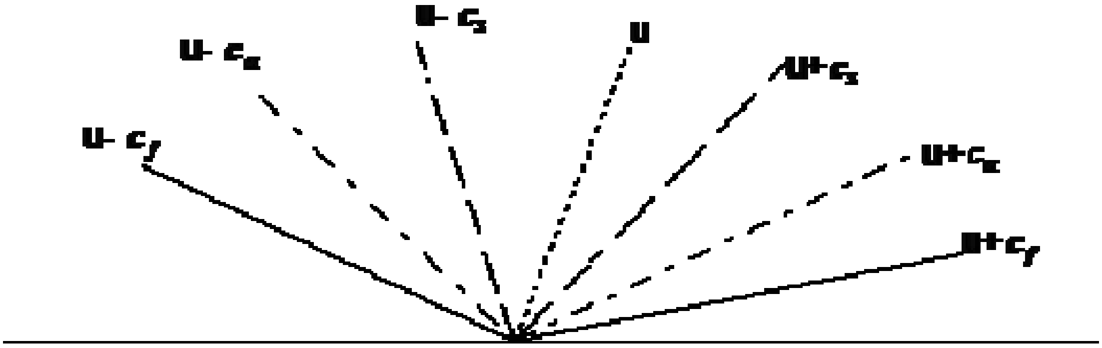
The jacobian matrix $J = \frac{\partial F}{\partial U}$ has 7 real eigenvalues. Ideal MHD equations are hyperbolic one for each wave.The dynamics of magnetized plasmas can be interpreted using solution of the properties of linear and nonlinear wave . The properties of linear waves can be study using the dispersion relation $e^{i(\omega t + \lambda . x)}$, where ω is the frequency and λ is the wave vector. The MHD wave families are fast, Alfven,slow and entropy.

1. Fastmagnetosonic waves $u \mp c_f$ are longitudinal waves with variations in pressure and density.Fast magnetosonic waves are correlated with magnetic field.
2. Alfven waves $u \mp c_a$ are transverse waves with no variation in pressure and density.Alfen waves can be polarized.The sum of linear polarizations can lead to circularly polarized alfven,slow and entropy waves.
3. Slowmagnetosonic waves $u \mp c_s$ are longitudinal waves with variations in pressure and density but slow magnetosonic waves are anticorrelated with magnetic field.
4. Entropy wave u is a contact discontinuity with no variation in pressure and velocity.In these waves,the minus (plus) sign is applied to left-going (right-going) waves. MHD waves involve transversrs motion.

The characteristic speeds are expressed as

$$c_{f,s} = \left[\frac{1}{2}\left(\frac{B_x^2 + B_y^2 + B_z^2}{\rho} + a^2\right) \pm \sqrt{\frac{1}{4}\left(\frac{B_x^2 + B_y^2 + B_z^2}{\rho} + a^2\right)^2 - \frac{a^2 B_x^2}{\rho}}\right]^{1/2}, c_a = \sqrt{\frac{B_x^2}{\rho}}$$

Where a is the sound speed given by $a = \sqrt{\frac{\gamma P}{\rho}}$.The slow or fast signal could be shocks or rarefaction.



Since $c_s \le c_a \le c_f$ .This reflect that eigen values may coinside at a special point. Hence,the system of Ideal MHD equations is not strictly hyperbolic.There are certain cases where two or more eigen value collaps to the same value.

1. If $B_y = 0 = B_z$ and $a = \sqrt{\frac{\gamma P}{\rho}} \neq \sqrt{\frac{B_x^2}{\rho}} = c_a$,In this case either both fast or slow eigen values collapse with Alfven speed.
2. If $B_y = 0 = B_z$ and $a = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{B_x^2}{\rho}} = c_a$.In this case Slow and Fast eigen values collapse with Alfven eigen values.
3. If $B_x = 0$.Both Slow and Alfven waves collapse to entropy wave.

When Eigen values collapse, there exists possibility of over compressive and under compressible Shocks. This fact makes finding solutions to the equations to ideal MHD more complicated. Today, the most important method for solving the MHD equations are numerical methods. But the development of numerical technique to solve MHD has been slower due to complexity of the MHD flow. Grid based methods to solve MHD equations is based on explicity finite difference scheme called the total variation dimising (TVD) scheme.In this method the conserved variables are discretized on a grid, with volume averaged values stored at cell centers. One of grid code for MHD is Athena . Athena implements a higher-order Godunov scheme .In this method,the difference in cell average values at each grid interface define set of Riemann problems. Solution of Riemann problems averaged over cell give time-evolution of cell average value. Riemann solvers available to compute the fluxes.

The electric currents transmitted in an electrolyte solution interact with the magnetic field to form Lorentz body forces that in turn, drive fluid motion.Lorentz force is the flow in the direction perpendicular to both magnetic and electric fields in conductive, solutions in the MHD system with initial velocity and initial magnetic field by keeping electric field as negligible.The current flow in a direction perpendicular to the direction of magnetic field causes the fluid to experience a force.

The force is in a direction perpendicular to both the magnetic field and the current flow.However,in many situation the changes in pressure and temperature are sufficient small that the changes in density are negligible .That flow can be modelled as an incompressible flow .

For gases, to determine whether fow to use compressible or incompressible fluid dynamics, the Mach number of the flow is tobe evaluated. Compressible effect can be ignored at Mach numbers below approximately 0.3.

For liquids, the incompressible flow depends on fluid properties and the flow condition how close to critical pressure the actual flow pressure becomes .The phenomenon of strong and weak discontinuities in a compressible fluid has been of great interest amongst the scientist and mathematician .Rankine Hugonoit developed the jump conditions for sudden charges in physical parameters such as temp pressure and velocity etc. with discontinuity. This sudden change may occur on account of sudden explosions in compressible gases, collision of clouds, movements of super sonic jets with high mach numbers.

Since most of the problems occurring are non-linear are in nature, the similarity condition play an important role in solving such problems. Analytical similarity solutions as well as numerical solution exist for electrically conducting compressible flows across such discontinuities. It is therefore proposed that study of such non-linear problems nil come out in presence of conducting and non conducting medium where conditions may be adiabatic or isothermal or both.

2.2. Discontinuities in ideal MHD

In one-dimensional u_n and B are scalar normal component of vector variables u and B in the direction of the space variable and the two-dimensional transversal parts u_t and B_t. If x is the space direction. We have $B=(B_x, B_y, B_z)=(B_n, B_t)$ and following Torrilhon (2002) MHD discontinuity satisfy the Rankine-Hugoniot relations, which in ideal MHD are expressed as

$$\hat{P}-1+\gamma M_0^2(\hat{u}-1)+\tfrac{1}{2}(\hat{B}_t^2-A^2)=0\,, \tag{2.21}$$

$$\gamma M_o^2(\hat{u}\hat{B}_t-A)-B^2(\hat{B}_t^2-A^2)=0\,, \tag{2.22}$$

$$M_0\left[\frac{1}{\gamma-1}\left(\hat{P}\hat{u}-1\right)+1/2\,(\hat{u}-1)(\hat{P}+1)+1/4\,(\hat{u}-1)(\hat{B}_t-A)^2\right]=0 \tag{2.23}$$

where $u=\tfrac{1}{\rho}$ is the specific volume .Fixing the upstream normalize quanttiies

$$A:=\frac{B_{t0}}{\sqrt{P_0}}, B:=\frac{B_n}{\sqrt{P_0}}, M_0=\frac{u_{n0}}{a_0} \tag{2.24}$$

Solving the equation (2.21)-(2.23),Using the quantities

$$\hat{P}:=\frac{P_1}{P_0}, \hat{B}_t:=\frac{B_{t1}}{\sqrt{P_0}}, \hat{u}:=\frac{u}{u_0} \tag{2.25}$$

The other downstream quantities can be calculated as

$$u_{x1} = \hat{u} u_{x0}$$

$$u_{y1} = u_{y0} \pm \frac{a_0 B}{\gamma M_0} \left[\hat{B}_{y0} \right] \quad (2.26)$$

The plus and minus signs correspond to the left-and right-going discontinuities respectively.

3.SHOCKS WAVE

When macroscopic motion with supersonic speed occurs in an interplanetary atmosphere occurs, strong and weak discontinuities popularly known as shock wave come into picture . In other word,A shocks wave is a special kind of wave as a steep finite pressure wave. In some situations shocks are undesirable because they interfere with the normal flow behaviour as turbomachines Parker (1963) studied the flow produced in solar wind using similarity method. Lee and Chen (1969) attempted the only self consistent similarly variable model of flow generated in a conducting plasma verma etal (1986) studied the effect of magnetic field on shock in a rotating interplanetary gases. . The normal shock wave is perpendicular to one dimensional flow . Shock may occur on account of supersonic flow developed on account of local accelerations. These shocks may be normal or inclined to the direction of local flow ;They may cause boundary layer separation and deviation of flow from its designed direction. To study the complex gas motion behind the shock waves,One has to solve two points boundary value problems for a set of non linear partial differential equation . Other undesirable forms of shock waves are the sonic boom created by supersonic aircrafts and the blast waves generated by an explosion. One of the boundaries e.g. the point of explosion is fixed in space and has boundary conditions arising out of special symmetry about it. The other boundary is the shock waves across which the unknown flow variable satisfies certain condition laws . On account of high temperature that prevail in many phenomenon with shocks.

Since the conservation laws apply across the phenomenon of non-linear discontinuities in a conducting plasma the analytical solutions may be obtained in some cases, which restricts the propagation of these discontinuities along some characteristics. Riemann give the method of characteristics for analytic expressions for the velocity of shocks wave.

Some useful applications of shock waves are in the shock tubes and supersonic compressors. A strong moving shock wave is utilised o accelerate the flow to a high Mach number in shock tube where flow behaviour at high Mach numbers can be studied.On account of abrupt changes of pressure, density, etc. Across shock waves, they are profitably used in supersonic compressors to obtain considerably high pressure ratios in one stage; in such compressors the pressure ratio developed per stage may be as high as 10.0. The thickness of such waves is of the order of 10^{-3} mm which is comparable with mean free path of the gas molecules.

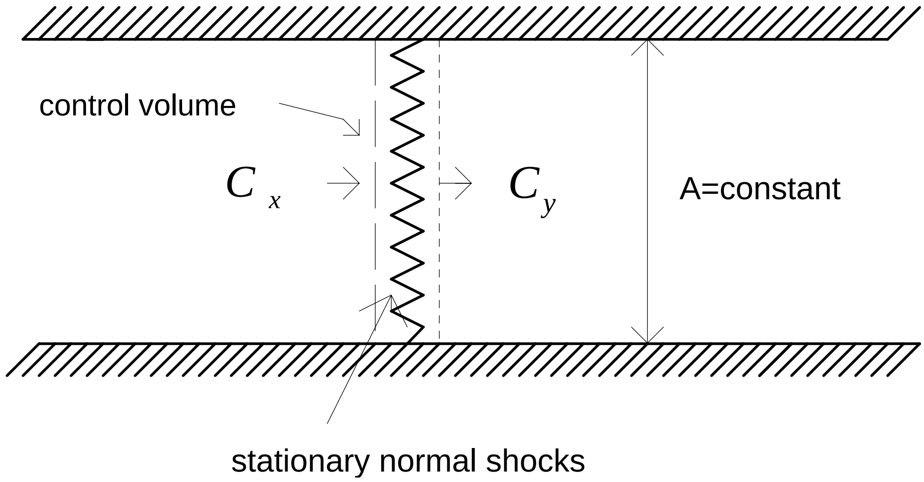


Figure shows a normal shock wave in a constant area frictionless duct, the shock wave is considered to be contained in a control volume.

As the mathematical expressions which derive the non linear problems of propagation of shocks in conducting radiative medium an analytical solution may be obtained as far as possible but in most cases it become very difficult to achieve a complete solutions due to its non-linearly .

In such case, solutions shall be derived numerically on a computer by many well known methods such as Runga-Kutta approximetors or Newton’s Rephoson and other iterative methods. These approximate solutions however still project a near-complete picture of patterns of pressure, temperature density and velocity distributions across these discontinuities.

However with the expansion of knowledge computer softwares.It is now a day more prominent to achieve the numerical approximations in case a analytical solution is not obtained various software such as RKGS etc. are available to cope up such difficulties and an approximate numerical solutions may easily be achieved in getting non-linear equations.

it is therefore,proposed that the non-linear equations shall be integrated with the help these software whenever required. The respective physical parameter thus, will exhibit the pattern of flow and field distribution (of course with the help of graph theory) of problem. Jeffrey and Taniuti (1964),the solution of Riemann problems is generally not unique in the sense of the weak solution and other conditions should be imposed to single out the physically relevant one.

Entropy condition, which admits only the shocks across which the entropy increases. The entropy condition discards manifestly unphysical solutions which include expanding shocks, across which the entropy is decreased. Takashi and Yamada (2013),In ideal MHD, some initial conditions have more than one solutions that satisfy the entropy condition . Therefore the so-called evolutionary conditions are introduced, which define physically relevant shocks should be structurally stable. Structurally stable shocks just remain close to the initial discontinuity.

If $M_0 \neq 0$ and $\hat{u} > 1$ The solutions of (2.21)-(2.23) is compressed as it passes through the discontinuities, are called shock waves. In this case the magnetic fields are either planar or coplanar. If $\hat{u} > 1$and $u_{n1} = \hat{u}u_{n0}$,then we get $\hat{B}_t = \frac{u_{n0}^2 - c_{A0}^2}{u_{n1}^2 - c_{A1}^2} A$,Transverse magnetic fields are coplanar if and only if the upstream flow velocity is super Alfvenic whereas the downstream speed is sub-Alfvenic. The shocks with planar transverse magnetic fields are either fast or slow shocks.The shocks that change the direction of transverse magnetic fields are referred to as intermediate shocks.

4. MHD in Astrophysical

The solar magnetic fields that wave in some may connected with sun's rotation imply large scale magnetic fields may appear in a rapidly rotating planet. The rotation of planet in the presence of a magnetic field is controlled by laws of isorotation i.e. field is symmetric about the axes of rotation and each line of force lies wholly on a surface .The matter in the Sun is in a plasma state, that is, an ionised gas with enough abundance of free charges. One way to have a reasonable description of the plasma under solar conditions, among other applications is magneto hydrodynamics.The effect of magnetic fields to the dynamics and evolution of astrophysical plasmas comes from observations of the outer layers of the Sun.The dynamo activity involves patterns of magnetohydrodynamics flows, the interaction of differential rotation and convection needed to magnetic field in large scale.Both the presence of sunspots in the photosphere, and structures such as filaments, prominences, and flares in the solar corona, demonstrate the key role that magnetic fields play in shaping the dynamics. In fact, the very existence of the hot corona is now interpreted as due to heating by MHD effects..

It is thought that most of the magnetic activity of the Sun is driven by the combination of rotation and turbulent flows in the convection zone. In fact, the properties of MHD turbulence driven by convection was one of the problems that first interested Chandra in plasma physics.

Understanding the origin and evolution of the Sun's magnetic field via a dynamo process has been a challenging problem for many decades. In addition to generation of the dipole field due to differential rotation,aprocess first proposed by Parker (1955),there are also small-scale mult ipole fields thought to be generated by the convective turbulence that play a role in shaping sunspots and coronal activity. Both the processes that produce sunspots, and the large-scale magnetic field of the Sun, are very active areas of research.

In the case of sunspots, direct numerical simulations of magneto convection in the outer layers, including realistic radiative transfer to capture the outer radiative zone, can now reproduce details of observed sunspots, including the penumbral filaments; a beautiful example is given in Rempel et al. (2009) .In the case of the solar dynamo, the dipole field is now thought to originate in the tachocline, a region of strong shear between the radiative core (which is in solid body rotation, according to results from helioseismology) and the outer convective zone (which is in differential rotation) . However, although the sophistication of modern global MHD simulations of magnetoconvection in spherical and rotating stars is impressive, they still fail to explain both the origin of the differential rotation in the convective zone, and the origin of the cyclic dipole field. Solving the solar dynamo problem is important, as we are unlikely to understand magnetic fields in other stars if we cannot first understand the Sun. These phenomenon usually occur in stellar atmosphere where turbulence in medium is more often .many more scientists discover that in stanleous energy release along a line cylindrical shocks or along point, spherical shocks may propagate with increasing strength without limit .

The non-dimensional similarly conditions define the pattern of propagation of such motions and hence,,it will be a matter of great interest if it is analysed whether the motion of these discontinuities strong or weak really propagate with high energy yields in the problems solar photosphere rocket reentry fission fusion reactions etc. One method to investigate the properties of MHD turbulence is through direct numerical simulation .Lemaster and Stone (2009),Numerical simulations of highly compressible MHD turbulence with both strong and weak magnetic fields. The turbulence is driven with a forcing function whose power spectrum is highly peaked at a wav enumber corresponding to about 1/8 the size of the computational domain. The energy input rate of the driving is held constant, and the turbulence is driven so that the Alfvenic Mach number of the turbulence is about one in the strong field case, and 7 in the weak The spectrum of fluctuations, such simulations can be used to measure properties such as the decay rate of the turbulence, and how it depends on the magnetic field strength. Early predictions suggested the decay rate of strongly magnetized turbulence would be very low, since it would be dominated by incompressible Alfv´en waves. Stone, Ostriker & Gammie (1999),The simulations found the decay rate of supersonic MHD turbulence was very fast, with the decay time about equal to an eddy turn over time on .

5.Conclusion:-

Converging and diverging shocks generated by instanteous energy release over a cylindrical and spherical surface in a conducting medium are of great interest amongst the scientists working in area of solar explosions, detonations, steller turbulence and other astrophysical phenomenon. If the effect of such discontinuities are formulated and these pattern is observed it is a very helpful in designing the space crafts and other missiles entering in such turbulent medium where occurrence these discontinuities is quite regular.

It is therefore, our efforts to provide a complete scenarios of physical distributions which may cause severe hazards to well calculated missiles entering in such medium. Goldreich & Sridhar (1995),The theories of the power spectrum and statistical properties of MHD turbulence can be tested and compared . It is quite clear from the images that in the weak field case, the density fluctuations are isotropic, and the magnetic field is highly tangled. Goldreich & Sridhar (1995), In the strong field case the density fluctuations are elongated along the field lines, and the field is more or less ordered. The suggests that the power spectrum of the turbulence will be anisotropic.

It is impossible to describe studies of Astrophsical MHD shocks without mentioning the important role that numerical method.The types of waves generated and their order are not known a priori in Magneto hydrodynamics Riemann solver..Solution of Riemann are used to obtain numerical fluxes.

Bibliography / References:

[1] Chandrasekhar.S (1949)-APJ,110,329
[2] Dongsu Ryu (2000)-Numerical Magnetohydrodynamics in Astrophysics:Algorithm and tests for one dimensional flow
[3] Elliot: (1961) – Astrophysics and space science vol 155 no. 2,215-224
[4] Gogosov V.V (1962)-On the resolution of an arbitrary discontinuity in Magneto hydrodynamics J .Applied math.Mech.26 (1) 118-129
[5] Goldrich P,Sridhar (1955)-APJ,438,763
[6] Helliwell (1969)-fluid mechanics33, 497-612
[7] James M. Stone (2011) Astrophysical magneto hydrodynamics
[8] Jefrey A and Taniuti T (1964)-Nonlinear wave propagation Academic Press,New yark
[9] K.Takashi and S.Yamada (2013)-Exact Riemann Solver for ideal Magnetohydrodynamics that can handle all types of intermediate shocks and switch on/off waves
[10] Lamster M.N.Stone J.M (2009)-APJ,691,1092
[11] Lee&Chen (1969) A.K.astronautika act 14, 565,582
[12] Parker E.N (1963) interplanetary dynamical Pross Interscience New York
[13] Parker EN (1955)-APJ,122,293
[14] Rampel M.Schiisler M.CameronR.H,Knoker M. (2009)-Science,1325,171
[15] Romain Teyssier and Oscar Agertz (2009)-Computational Astrophysics and computational MHD
[16] Rosenau P (1976) –phy.fluid19, 1989-
[17] Sedov L.I. : (1959)-Similarity and dimensional analysis academicpress, New York
[18] Singh (1979)-App. Space Science 63, 253
[19] Stone J.M,Ostrikar E.C,Gammie C.F (1998)-APJ, 508,99
[20] Summers D (1975)-Astr.A45,151-58
[21] Torrilhon (2002)-Exact solver and uniqueness condition for Riemann problems of Ideal Magnetohydrodynamics
[22] Verma and Singh : (1986) Astrophysical space science 124,53, 62
[23] Wang K.C. : (1964) J.Fluid Mech 20,447