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Mathematical Modelling Involving $\overline{H}$-function on Effect of Environmental Pollution

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Abstract

The object of the present paper is to discuss an application of Mathematical Modeling involving the $\overline{H}$-Function on effect of environmental pollution on the Growth and existence of Biological Populations which was introduced by Inayat Hussain in 1987 [1, 2]. The results established in this paper are general nature & hence encompass several cases of interest.

I. INTRODUCTION

In 1987, Inayat-Hussain [1, 2] was introduced generalization form of Fox's H-Function, Which is popularly known as $\overline{H}$-Function. Now $\overline{H}$-Function stands on fairly firm footing through the research & contribution of various authors [1-3, 4-5]. $\overline{H}$-Function is defined and represented in the following manner [6].

$$\overline{H}^{m,n}_{p,q}[z] = \overline{H}^{m,n}_{p,q}\left[z \left| \begin{matrix} (a_j,\alpha_j;A_j)_{1,n}, & (a_j,\alpha_j)_{n+1,p} \\ (b_j,\beta_j;B_j)_{1,m}, & (b_j,\beta_j)_{m+1,q} \end{matrix} \right. \right] = \frac{1}{2\pi i}\int_L z^{\xi}\overline{\phi}(\xi)\,d\xi$$

$$(z \neq 0) \qquad (1.1)$$

Where,

$$\overline{\phi}(\xi) = \frac{\prod_{j=1}^{m}\Gamma(b_j-\beta_j\xi)\prod_{j=1}^{n}\left\{\Gamma(1-a_j+\alpha_j\xi)\right\}^{A_j}}{\prod_{j=m+1}^{q}\left\{\Gamma(1-b_j+\beta_j\xi)\right\}^{B_j}\prod_{j=n+1}^{p}\Gamma(a_j-\alpha_j\xi)} \qquad (1.2)$$

It may be noted that the $\overline{\phi}(\xi)$ contains fractional powers of some of the gamma functions and m, n, p, q are integers such that 1≤m≤q, 1≤n≤p $(\alpha_j)_{1,p}$, $(\beta_j)_{1,q}$ are positive real number and $(A_j)_{1,n}$, $(B_j)_{m+1,q}$ may take non-integer values, which we assume to be positive for standardization purpose. $(\alpha_j)_{1,p}$ and $(\beta_j)_{1,q}$ are complex numbers.

The nature of contour L, sufficient conditions of convergence of defining integral (1.1) and other details about the $\overline{H}$ -Function can be seen in paper [4, 5].

The behavior of the $\overline{H}$ -Function for the small values of $|z|$ follows easily from a result given by Rathie [3]:

$$\overline{H}_{p,q}^{m,n}[z] = O\left(|z|^{\alpha}\right)$$

$$\alpha = \min_{1\le j\le m} \operatorname{Re}\left(\frac{b_j}{\alpha_j}\right), |z| \to 0 \tag{1.3}$$

$$\mu_1 = \sum_{j=1}^{m} |B_j| + \sum_{j=1}^{q} |b_j B_j| - \sum_{j=1}^{n} |a_j A_j| - \sum_{j=n+1}^{q} |A_j| > 0, 0 < |z| < \infty \tag{1.4}$$

Pollution is a worldwide problem and its potential to influence the physiology of human population is great one of the greatest problems that the world is facing today is that of environmental pollution increasing with every passing year and causing grave and irreparable damage to the earth. Environmental Pollution is the contamination of the physical and biological components of the earth/atmosphere system to such an extent that normal environmental processes are adversely affected. Since long both various kinds of industrial discharges and wastes, causing damage to our ecosystems, are polluting our atmosphere and the aquatic environment.

The biological and ecological consequences of pollution in our environment may be considered in several ways depending upon the toxic level of pollutants (acute or chronic) and the ecotoxicological situations. One such situation is where the pollutants can adversely affect the natural resources, thereby influencing the growth of other biological populations, which may be depending upon these resources. The other such situation is where the pollutants can affect directly the species accompanied by rapid injury to the principal physiological and biochemical systems of the organism, and results in lethal toxication, elimination of individual species and populations or causes profound pathological alterations on the level of individual organisms, individual populations, and occasionally, on entire ecosystems which might change the carrying capacity of the environment [6, 7, 8, 9, 10, 11]. Various investigations have been carried out in this direction, both experimentally and mathematically [8, 9, 10, 11, 12, 13, 14, 15].

The deleterious effect of environmental pollution on interacting biological populations depends upon the toxicity and the level of pollutant, the sort of damage it causes to the physiological and biochemical systems of the populations and their environment. To study this situation, in this paper, a mathematical model is presented by considering that the growth rate of species and the carrying capacity of its environment are directly affected by pollution and decrease as the concentration of the pollutant increases.

In view of the above, in this paper, we have studied the effect of environmental pollution on the growth and existence of two interacting biological populations in the situation where the pollutant causes injury to the principal physiological and biochemical systems of the populations and their environment.

To study this situation, a mathematical model is presented here be considering that the growth rate of species and carrying capacity of its environment are directly affected by pollution and decrease as the concentration of the pollutant increases.

II. Mathematical Model:

Consider the growth of interacting and dispersing biological species of density N_i (x, t), (i = 1, 2) in a one dimensional linear habitat 0 5 x ≤ L, whose growth rate and the carrying capacity of the environment decreasing due to the environmental pollution present in the habitat.

The dynamical equations governing the growth of the species are assumed to be given by the following system of non-linear partial differential equations

$$\partial N_i/\partial t = N_i F_i\,(N_1, N_2, r_i\,(C), K_i\,(C)) + D_i\,(\partial^2 N_i/\partial t^2),\ i = 1, 2 \tag{2.1}$$

Where, F_i (N_1, N_2, r_i (C), K_i (C)) determines the interaction function of the species. r_i (C) and K_i (C) are the intrinsic growth rate and the carrying capacity of the environment respectively which are affected by the concentration C (x, t) of pollutant. The positive constant D_i (i = 1, 2) is the dispersion coefficient of the species.

The dynamics of the concentration C (x, t) of the pollutant is considered to be given by the following equation

$$\partial C/\partial t = Q_0 - \alpha C + D_c\,(\partial^2 C/\partial x^2) \tag{2.2}$$

Where, $Q_0 > 0$ is the constant determining the exogenous rate of input of pollutant into the habitat, $\alpha > 0$ represents the first order decay constant as a result of biological (including consumption by the species), chemical or geological processes. $D_c > 0$ is the diffusion coefficient of the pollutant. In the formulation of the model it has been assumed that the organismal uptake of the pollutant is proportional to the concentration of the pollutant present in the environment of the population.

On the basis of the mathematical model (2.2) the solution of this mathematical equation will be obtained with the help of $\overline{H}$ -function in the subsequent part of this section.

III. Result In Terms Of $\overline{H}$ -Function

Choose concentration $C(x,t)$ in terms of $\overline{H}$ -function as

$$C(x,t) = \overline{H}_{p,q}^{m,n}\left[zx^{\sigma}t^{\mu} \left| \begin{matrix} (a_j,\alpha_j;A_j)_{1,n}, & (a_j,\alpha_j)_{n+1,p} \\ (b_j,\beta_j;B_j)_{1,m}, & (b_j,\beta_j)_{m+1,q} \end{matrix} \right. \right] \tag{3.1}$$

Where $\sigma > 0, \mu > 0, |\arg z| < 1/2\pi A,$ where A is given in

Now differentiate it with respect to x and t partially, we get

$$\frac{\partial C}{\partial t} = (1/t)\overline{H}^{m,n+1}_{p+1,q+1}\left[zx^{\sigma}t^{\mu} \left| \begin{matrix} (0,\mu),\left(a_j,\alpha_j;A_j\right)_{1,n}, & \left(a_j,\alpha_j\right)_{n+1,p} \\ \left(b_j,\beta_j;B_j\right)_{1,m}, & \left(b_j,\beta_j\right)_{m+1,q},(1,\mu) \end{matrix} \right. \right] \tag{3.2}$$

and

$$\frac{\partial^2 C}{\partial x^2} = (1/x^2)\overline{H}^{m,n+1}_{p+1,q+1}\left[zx^{\sigma}t^{\mu} \left| \begin{matrix} (0,\sigma),\left(a_j,\alpha_j;A_j\right)_{1,n}, & \left(a_j,\alpha_j\right)_{n+1,p} \\ \left(b_j,\beta_j;B_j\right)_{1,m}, & \left(b_j,\beta_j\right)_{m+1,q},(1,\sigma) \end{matrix} \right. \right] \tag{3.3}$$

Now after using (3.1), (3.2), and (3.3) in (2.2), we get following result

$$(1/t)\overline{H}^{m,n+1}_{p+1,q+1}\left[zx^{\sigma}t^{\mu} \left| \begin{matrix} (0,\mu),\left(a_j,\alpha_j;A_j\right)_{1,n}, & \left(a_j,\alpha_j\right)_{n+1,p} \\ \left(b_j,\beta_j;B_j\right)_{1,m}, & \left(b_j,\beta_j\right)_{m+1,q},(1,\mu) \end{matrix} \right. \right]$$

$$= Q_0 - \alpha\overline{H}^{m,n}_{p,q}\left[zx^{\sigma}t^{\mu} \left| \begin{matrix} \left(a_j,\alpha_j;A_j\right)_{1,n}, & \left(a_j,\alpha_j\right)_{n+1,p} \\ \left(b_j,\beta_j;B_j\right)_{1,m}, & \left(b_j,\beta_j\right)_{m+1,q} \end{matrix} \right. \right]$$

$$+D_c(1/x^2)\overline{H}^{m,n+1}_{p+1,q+1}\left[zx^{\sigma}t^{\mu} \left| \begin{matrix} (0,\sigma),\left(a_j,\alpha_j;A_j\right)_{1,n}, & \left(a_j,\alpha_j\right)_{n+1,p} \\ \left(b_j,\beta_j;B_j\right)_{1,m}, & \left(b_j,\beta_j\right)_{m+1,q},(1,\sigma) \end{matrix} \right. \right] \tag{3.4}$$

Where $\sigma > 0, \mu > 0, |\arg z| < 1/2\pi\mu_1,$ where μ_1 is given in (1.4).

CONCLUSION

In this paper, we have studied the effect of environmental pollution on the growth and existence of two interacting biological populations. Here we have presented a Mathematical Model by considering that the growth rate of species and carrying capacity of its environment are directly affected by pollution and decrease as the concentration of the pollutant increases. It is shown that the habitat still remain asymptotically stable but at much reduced levels implying that if the concentration of pollutant continues to increase in the environment unabatedly, the species may not exist for long. A number of other functions that are special cases of $\overline{H}$-function can also we obtained from (3.4). Therefore the result (3.4) is useful in literature in Mathematics and other branches.

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