Global Journal of Mathematical Sciences: Theory and Practical
ISSN 0974-3200 Volume 6, Number 1 (2014), pp. 59-62

http://www.irphouse.com

Fuzzy Semi Continuity and Fuzzy Weak-Continuity

B.Chellappa[1] and R.Saral Manjula[2]

*[1]Associate Professor, Alagappa Govt. Arts College,
Karaikudi-630 003, Tamilnadu, India.
[2]Research Scholar (Part Time-Mathematics),
Manonmaniam Sundaranar University,
Abishekapapti, Tirunelveli-627 012, Tamilnadu, India.*

Abstract

A function f of a fuzzy topological space X into a fuzzy topological space Y to be fuzzy weakly-continuous if for each x ∈ X and each fuzzy open neighborhood V of f(x) there exists a fuzzy open neighborhood U of x such that f(U) ⊂ Fcl (V). where Fcl/(V) denotes the fuzzy closure of V.

Definition: (a)
A fuzzy subset S of a fuzzy topological space X is said to be fuzzy semi-open if there exists a fuzzy open set U of X such that U ⊂S⊂ Fcl(U). The family of all fuzzy semi-open sets in X is denoted by FSO (X).

Definition: (b)
A function f: X →Y to be fuzzy semi-continuous if f^{-1} (V) ∈ FSO (X) for every fuzzy open set V of Y. It has been known that the fuzzy semi-continuity is equivalent to the fuzzy quasi-continuity.

Definition: (c)
A function f: X →Y to be fuzzy semi-open if f(U) ∈ FSO (Y) for every fuzzy open set of U of X.

Definition: (d)
A function f: X →Y to be fuzzy irresolute (resp. fuzzy pre-semi-open) if for each V ∈ FSO (Y) (resp. U∈ FSO (X)), f^{-1} (V) ∈ FSO (X) (resp f (U)) ∈FSO (Y)).

The purpose of the present paper is to investigate the interrelation among the fuzzy weak-continuity, the fuzzy semi-continuity and some fuzzy weak forms of fuzzy open functions.

A fuzzy semi-continuous function is fuzzy irresolute if it is either fuzzy weakly-open injective or fuzzy almost-open.

A fuzzy semi-open function is fuzzy pre-semi-open if it is either fuzzy weakly-continuous or fuzzy almost-continuous.

A fuzzy semi-continuous function is fuzzy weakly-continuous if the domain is extremely disconnected.

1.FUZZY IRRESOLUTE FUNCTIONS

Definition 1:1

A function f: X →Y is said to be fuzzy weakly-open if

f (∪) ⊂ Int (f(Fcl(U))) for every fuzzy open set ∪ of X.

Definition 1:2

A function f: X →Y is said to be fuzzy almost-open for every fuzzy regular open set ∪ of X, f(U) is fuzzy open in V.

Definition 1:3

A function f: X →Y is said to be fuzzy almost-open if

f^{-1} (Fcl(V)) ⊂ Fcl (f^{-1} (V)) for every fuzzy open set V of Y.

Lemma 1:4

If f: X →Y is a fuzzy almost open function then it is fuzzy weakly-open

Proof:

Let U be a fuzzy open set of X. Since f is fuzzy almost open,

f (Int (Fcl(U))) is fuzzy open in Y and hence

f(U) ⊂ f(Int (Fcl(U))) ⊂ Int (f(Fcl (U))).

The converse to Lemma 1:4 is not necessarily true.

Example:

Let X = { a, b, c, d} & σ = { X, {a, b, d}, {a,b} {d}, 0}.

Let Y = {x,y,z} & τ = {Y, {x,y}, {y,z}, {y}, {z}, 0}.

Let f: (X, σ) →(Y, τ) he a function defined as follows

f(a) = x f(b)=z, f(c) = f(d)=y. Then f is fuzzy weakly-open but it is nut fuzzy almost open.

Definition 1:5

A function f: X →Y is said to be fuzzy somewhat continuous if for each fuzzy open V of Y with f^{-1} (V) ≠ 0 there exists a fuzzy open set U of X such that 0 ≠ U ⊂ f^{-1} (V).

Theorem: 1:6

If f: X →Y is a fuzzy weakly-open somewhat continuous injection then it is fuzzy irresolute.

Proof:

Let V ∈ FSO (Y) and x ∈ f^{-1} (V) Put y = f(x) and let U be any fuzzy open neighborhood of x, since f is fuzzy weakly-open, we have

y ∈ f(U) ∩ V ⊂ Int (f (Fcl (U))) ∩ V ∈ FSO (Y).

There exists a fuzzy open set G such that

0 ≠ G ⊂ Int (f (Fcl (U))) ∩V. Since f is Fuzzy some what continuous and f^{-1} (G) ≠ 0, there exists an fuzzy open set W of X such that 0 ≠ W ⊂ f^{-1} (G).

Therefore, we obtain W ⊂ Fcl (U) ∩ f^{-1} (V) and hence

W ⊂ Fcl (U)∩ Int (f^{-1} (V)) because f is injective. Thus, we have

0 ≠Fcl (U)∩ Int (f^{-1} (V)) and hence

0 ≠U ∩ Int (f^{-1} (V)). This shows that

x ∈ Fcl (Int (f^{-1} (V))) and f^{-1} (V) ∈ FSO (X).

Theorem 1.7
If a function f: X →Y is a fuzzy almost open and fuzzy semi-continuous then it is fuzzy irresolute.

Proof:
Let V ∈ FSO (Y). Then there exists a fuzzy open set G of Y such that G ⊂ V ⊂ Fcl (G), hence f^{-1} (G) ⊂ f^{-1} (V) ⊂ f^{-1}(Fcl (G)). Since f is fuzzy semi-continuous, f^{-1} (G) ∈ FSO (X) and hence f^{-1} (G) ⊂ Fcl (Int (f^{-1} (G))).

Now, Put F= Y-f(X-Fcl (Int (f^{-1} (G)))). Then F is fuzzy closed in Y because f is fuzzy almost open and Fcl (Int (f^{-1} (G))) is fuzzy regular closed in X. By a straight forward calculation we obtain G⊂F and f^{-1} (F) ⊂ Fcl (Int (f^{-1} (G))).

Therefore, we have f^{-1} (Fcl (G)) ⊂ Fcl (f^{-1} (G)).

Since f^{-1} (G) ∈ FSO (X). we obtain f^{-1} (V) ∈ FSO (X).

Lemma 1.8:
If a fuzzy topological space X is extremely disconnected then
Fcl (U) =U for every U∈ FSO (X).

Proof:
In general, we have S ⊂ Fcl(S) for every fuzzy subset S of X. Thus we shall Show that U ⊃ Fcl (U) for each U ∈ FSO (X).

Let 0 ≠ U∈ FSO (X) and x ∉U, then there exists a V ∈ FSO (X) such that x ∈V, & V∩U=0; hence Int (V) ∩Int (U) = 0. Since X is extremely disconnected, we have Fcl (Int (V)) ∩Fcl (Int (U))=0Therefore, we have x ∉ Fcl (Int(U))=Fcl (U).

Theorem: 1:9
If a fuzzy topological space Y is extremally disconnected and a function f: X →Y is fuzzy semi-open fuzzy semi-continuous then f is fuzzy irresolute.

Proof:
Let V ∈ FSO (Y). There exists a fuzzy open set G of Y such that G ⊂V⊂ Fcl (G),hence f^{-1}(G) ⊂ f^{-1}(V) ⊂ f^{-1}(Fcl(G)). Since Y is extremally disconnected, we have G = Fcl (G) by lemma 1.8, since f is fuzzy semi-open then f^{-1}(G) ⊂ Fcl(f^{-1}(G)). Therefore we obtain f^{-1}(Fcl (G)) ⊂Fcl (f^{-1}(G)). Since f is fuzzy semi-continuous, f^{-1}(G) ∈ FSO (X) and hence f^{-1}(V) ∈ FSO (X).

2. FUZZY PRE-SEMI-OPEN FUNCTIONS

Definition: 2:1
A function f: X →Y is said to be fuzzy almost-continuous if for each x ∈ X and each fuzzy neighborhood V of f(x), Fcl (f^{-1}(V)) is a fuzzy neighborhood of x.

Definition: 2:2
A function f: X →Y is said to be some what fuzzy open if for each non empty fuzzy open set U of X, there exists a fuzzy open set V of Y such that 0 ≠ V ⊂ f(U).

Theorem 2.3
If a function f: X →Y is fuzzy weakly-continuous somewhat fuzzy open, then it is fuzzy pre semi-open.

Proof:
Let A ∈ FSO (X) and y ∈ f(A). Let Vbe any fuzzy open neighborhood of y. There exists x ∈ A such that y = f(x). Since f is fuzzy weakly-continuous, there exists a fuzzy open neighborhood U of x such that f(U) ⊂ Fcl (V).

Since x∈ U ∩A∈ FSO (X) there exists a fuzzy open set W of X such that 0 ≠ W ⊂ U ∩A. Moreover, Since f is fuzzy some what fuzzy open, there exists a fuzzy open set G of Y such that 0 ≠ G⊂ f(W), hence G ⊂ Fcl (V) ∩ (f(A)). Therefore, we have G ⊂ Fcl (V) ∩ Int (f(A)) and hence V ∩ Int (f(A))≠0.

This shows that y ∈ Fcl (Int (f(A))) and hence f(A)⊂Fcl (Int (f(A))). Consequently we obtain f(A) ∈ FSO (Y).

Corollary: 2.4
Every Fuzzy Weakly-continuous fuzzy semi-open function is fuzzy pre-semi open.

Proof:
Since every fuzzy semi-open function is somewhat fuzzy open, this is an immediate consequence of Theorem 2:3.

Theorem 2:5
If a function f: X →Y is fuzzy almost-continuous fuzzy semi open then it is fuzzy pre-semi-open.

Proof:
Let U ∈ FSO (X). There exists a fuzzy open set G of X such that G⊂U ⊂ Fcl (G). Since f is fuzzy almost-continuous, we have f(Fcl (G) ⊂ Fcl (f(G)) and hence f(G) ⊂ f(∪) ⊂ Fcl (f(G)). Since f is fuzzy semi-open, we obtain f(G) ∈ FSO (Y) and f(U) ∈ FSO (Y).

Theorem 2.6
If a fuzzy topological space X is extremally disconnected and a function f: X →Y is fuzzy semi-continuous fuzzy semi-open, then f is fuzzy pre-semi-open.

Proof:
Let U ∈ FSO (X). There exists a fuzzy open set G of X such that G ⊂ U ⊂ Fcl (G) Since X is extremally disconnected. We have Fcl (G) =G by lemma 1.8 since f is fuzzy semi-continuous, we obtain f(G) ⊂Fcl (f(G)) and hence f(G) ⊂f (U) ⊂ Fcl (f(G)). Since f is fuzzy semi-open, we have f(G) ∈ FSO (Y) and F(U) ∈ FSO (Y).

Example: 2:7
Let X=Y = {a, b,c,d} σ = { X, {a,b}, {a}, {b}, o} and τ = {Y, {b,c,d}, {a,b} {a}, {b}, 0}

Let f: (X, σ) → (Y, τ) be the identify function. Then f is fuzzy open and fuzzy semi-continuous but it is not fuzzy pre-semi-open.