Global Journal of Mathematical Sciences: Theory and Practical.
ISSN 0974-3200 Volume 7, Number 1 (2015), pp. 53-57

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Fuzzy Almost – Open Functions

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Abstract

Let S be a fuzzy subset of a fuzzy topological space X. The fuzzy closure of S and fuzzy interior of S in X are denoted byFcl_x(S) and $Fint_x$(S) (briefly Fcl(s) &Fint(S))Fcl(S) and Fint (S) respectively. A fuzzy subset S of X is said to be fuzzy regular open (fuzzy regular closed) if Fint (Fcl(S)) = S (resp. Fcl (Fint (S) = S). A function f:X→ Y to be fuzzy almost open (briefly f.a.o.S) If for each fuzzy regular open set U of X, f (U) is fuzzy open in Y. Where X and Y fuzzy topological spaces.
A function f: X→ Y to be fuzzy almost – open(briefly f.a.o.R) if for each fuzzy open set U of X, f (U) ⊂ Fint (Fcl (f (U)).

2.FUZZY SEMI-OPEN

A fuzzy subset S of a fuzzy topological space X is said to be fuzzy semi – open if there exists an fuzzy open set U of X such that U ⊂ S⊂ Fcl (U). The complment of a fuzzy semi – open set is called Fuzzy semi – closed.

Theorem 2.1:
A function f: X→ Y is f.a.o.S if and only if for each fuzzy semi – closed set F of X, f (Fint (F)) ⊂ Fint (f(F)).

Proof:
Suppose that f is f.a.o.S and let F be a fuzzy semi – closed set of X. Then Fint (F) = Fint(Fcl (F))⊂ F and f (Fint (Fcl (F)) is fuzzy open in Y. Therefore, we havef (Fint (F)) ⊂ Fint (f (F)) conversely, let U be a fuzzy regular open set of X. Then U is fuzzy semi – closed. By hypothesis, we have f (U) = f (Fint (U)) ⊂ Fint (f(U)). Thus f (U) is open in Y and hence f is f.a.o.S.

Theorem 2.2:
A function f: X → Y is f.a.o.S if and only if for any fuzzy subset S of Y and any fuzzy regular closed set F of X containing $f^{-1}(S)$, thereexists a fuzzy closed set G of Y containing S such that $f^{-1}(G) \subset F$.

Proof:
Suppose that f is f.a.o.S. Let S⊂ Y and F be a fuzzy regular closed set of X containing $f^{-1}(S)$ put G = Y – f (X-F). Since $f^{-1}(S) \subset F$, we have S⊂G.since f is f.a.o.S and F is fuzzy regular closed in X, G is fuzzy closed in Y. It follows from a straightforward calculation that $f^{-1}(G) \subset F$.
Conversely, let U be a fuzzy regular open set of X and put S = Y – f (U). Then X – U is a fuzzy regular closed set containing $f^{-1}(S)$. By hypothesis, there exists a fuzzy closed set G of Y containing S such that $f^{-1}(G) \subset X – U$. Thus, we have f (U) ⊂ Y – G. On the other hand we have,f (U) = Y – S ⊃ Y – G and hence f (U) = Y – G. Consequently, f (U) is open in Y and f is f.a.o.S.

Theorem 2.3:
If a function f: X→ Y is f.a.o.S and A is fuzzy regular open set of X, then the restriction f/A: A → Y is f.a.o.S.

Proof:
Let U be a fuzzy regular open set in the fuzzy subspace A. Since A is fuzzy regular open in X, so is U and hence f (U) is fuzzy open in Y. Therefore f/A is f.a.o.S.

Theorem 2.4:
Let f: X → Y be anf.a.o.S function. If A is an fuzzy open set of X such that $A= f^{-1}(B)$ for some fuzzy subset B of Y,then a function f_A:A → B defined by $f_A(x) = f(x)$ all x ϵ A is f.a.o.S.

Proof:
Let U be a fuzzy regular open set in the fuzzy subspace A. Since A is fuzzy open in X, we have $U = Fint_A (Fcl_A (U)) = A \cap Fint_X (Fcl_X(U))$. Since f is f.a.o.S, $f (Fint_X (Fcl_X(U)))$ is fuzzy open in y. Therefore $f_A (U) = B \cap f (Fint_X(Fcl_X (U)))$ is fuzzy open in the fuzzy subspace B and hence $f_A$is f.a.o.S.

Theorem 2.5:
Let f: X → Y be a function and $\{ V_\alpha / \alpha \in \nabla\}$ an fuzzy open cover of X. If the restriction $f/V_\alpha:V_\alpha \to Y$ is f.a.o.S for each α ϵ ∇ then f is f.a.o.S.

Proof:
Let U be a fuzzy regular open set of X. Since V_α is fuzzy open in X, $U \cap V_\alpha$is fuzzy regular open in the fuzzy subspace V_α for each α ϵ ∇. Since f/V_α is f.a.o.S. $(f/V_\alpha)(U \cap V_\alpha)$ is fuzzy open in Y and hence $f(U) = \cup\{ (f/V_\alpha) U \cap V_\alpha) / \alpha \in \nabla\}$ is fuzzy open in Y. This shows that f is f.a.o.S.

Corollary 2.6:

Let { V_α / $\alpha \in \nabla$ } be an fuzzy open cover of Y. A fuzzy continuous function f: X → Y is f.a.o.S if and only if $f_\alpha = f / f^{-1}(V_\alpha)$: $f^{-1}(V_\alpha) \rightarrow V_\alpha$ is f.a.o.S for each $\alpha \in \nabla$.

Proof:

This follows from theorem 2.4 and 2.5.

FUZZY PRE – OPEN:

A fuzzy subset S of a fuzzy topological space X is said to be fuzzy pre open if S⊂Fint(Fcl (S)). The complement of a fuzzy preopen set is called fuzzy pre closed

Theorem 3.1:

For a function f: X→ Y, then the following are equivalent:

(a) f is f.a.o.R.

(b) For any fuzzy subset S of Y and any fuzzy closed set F of X, containing $f^{-1}(S)$, there exists a fuzzy pre closed set G of Y containing S such that $f^{-1}(G) \subset F$.

(c) For any fuzzy set B of Y, f^{-1} (Fcl (Fint (B))) ⊂Fcl (f^{-1} (B)).

(d) For any fuzzy set A of X, f (Fint (A)) ⊂Fint (Fcl (f(A))).

Lemma 3.3:

A function f: X→ Y is f.a.o.R if and only if each fuzzy open set V of Y, f^{-1} (Fcl (U))⊂Fcl (f^{-1}(V)).

Theorem 3.4:

If a function f: X → Y is f.a.o.R and B is fuzzy open in Y then f_A: A → B is f.a.o.R, where A = f^{-1} (B).

Proof

Let V be an fuzzy open set of the fuzzy subspace B. Since B is fuzzy open in Y, so is V and hence f^{-1} (Fcl_Y (V)) ⊂Fcl_X(f^{-1}(V)) by lemma 3.3. Thus we obtain, f_A^{-1} (Fcl_B (V)) = f^{-1}(Fcl_Y (V)) ∩ A ⊂Fcl_A(f_A^{-1}(V)). This shows that f_A is f.a.o.R.

Theorem 3.5:

Let f: X → Y be an function and { V_α / $\alpha \in \nabla$ } an fuzzy open cover of Y.If $f_\alpha = f / f^{-1}(V_\alpha)$: $f^{-1}(V_\alpha) \rightarrow V_\alpha$ is f.a.o.R for each $\alpha \in \nabla$ then f is f.a.o.R.

Proof

Let V be an fuzzy open set of Y and put $U_\alpha = f^{-1}(V_\alpha)$ for each $\alpha \in \nabla$. Since V∩ V_α is fuzzy open in the fuzzy subspace V_α, by lemma 3.3 we have f_α^{-1}(Fcl_α (V∩V_α)) ⊂Fcl_α (f_α^{-1} (V∩V_α)) for each $\alpha \in \nabla$. Since V_α is fuzzy open in Y for each $\alpha \in \nabla$, we obtain, f^{-1}(Fcl_Y (V)) = ∪ f_α^{-1} (Fcl_Y (V∩ V_α) ⊂∪(Fcl_α (V∩V_α)) ⊂∪Fcl_α(f_α^{-1}(V∩ V_α))⊂ Fcl_X (f^{-1}(V)). This shows that f is f.a.o.R.

Corollary 3.6:
Let { V_α /α ϵ ∇} be an fuzzy open cover of Y.A function f: X → Y is f.a.o.R if and only if f_α: f^{-1} (V_α) → V_α is f.a.o.R for each α ϵ ∇

Proof
This follows from Theorem 3.4 and 3.5.

RELATIONS WITH SOME WEAK FORMS OF FUZZY CONTINUITY

A function f: X → Y is said to be fuzzy almost – continuous (f.a.c.S) if for each xϵX and each fuzzy open neighborhood V of f(x), there exists an fuzzy open neighborhood U of X such that f (U) ⊂Fint (Fcl (V)) (resp. f(Fcl (U) ⊂Fcl (V), f(U)⊂Fcl (V). It has been fuzzy continuous implies f.a.c.S implies θ-fuzzy continuous implies fuzzy weakly continous. A function f: X→ Y is said to be fuzzy almost continuous (f.a.c.H) if for each x ϵ X and each fuzzy open neighborhood V of f (x), Fcl (f^{-1}(V)) is a neighborhood of x.
A function f: X → Y is said to be fuzzy semi – continuous (f.s.c) if for each fuzzy open set V of Y, f^{-1}(V) is fuzzy semi – open in X.

Example 4.1:
Let X be the fuzzy set of real numbers and σ the fuzzy topology for X. Let Y = {a, b} and τ = { φ, {a}, Y}. Define a function f: (X, σ) → (Y, τ) as follows f (x) = a if x is rational and f (x) = b if x is irrational. Then f is f.a.c.S but not f.s.c.

Theorem 4.2:
If a function f: X → Y is f.a.o.S and f.a.c.H then f is f.a.o.R.

Proof
A function f: X → Y is f.a.c.Hif and only if f (Fcl (U)) ⊂Fcl (f(U)) for all fuzzy open set U of X. Let U be an fuzzy open set of X. Then we have f (Fcl (U))⊂Fcl (f (U)). Since f is f.a.o.S,f (Fint (Fcl (U))) is fuzzy open in Y and hence f (U) ⊂ f (Fint (Fcl (U))) ⊂ Fint (Fcl (f (U))). This shows that f is f.a.o.R.

Theorem 4.3:
If a functionf: X → Y is f.a.o.S and f.s.c.Sthen f is f.a.o.R.

Proof
By using lemma 3.3, we shall show that f is f.a.o.R. Let V be an fuzzy open set of Y. Then f^{-1}(V) is fuzzy semi – open in X and hence, f^{-1}(V) ⊂Fcl (Fint (f^{-1}(V))). Since f is f.a.o.S and Fcl(Fint (f^{-1}(V))) is fuzzy regular closed in X. By theorem 3.2 there exists a fuzzy closed set F of Y containing V such that f^{-1}(F) ⊂Fcl(Fint (f^{-1}(v))). Therefore we obtain f^{-1}(Fcl (V)) ⊂Fcl(f^{-1}(V)).

Example 4.4:
Shows that an f.a.o.S and f.a.c function is not necessarily f.a.o.R on the other hand, a fuzzy continuous f.a.o.R function is not necessarily f.a.o.S, as the following examples shows.

Example: Let X = {a,b,c } and τ = { φ, {a},{b,c}, X}. Let σ be the discrete fuzzy topology for X and f: (X, σ) $\rightarrow$ (X, τ) be the identity function. Then f is fuzzy continuous and f.a.o.R, but is not f.a.o.S. A function f: X $\rightarrow$ Y is fuzzy weakly continuous then, Fcl $(f^{-1}(V)) \subset f^{-1}$(Fcl (V)) for each fuzzy open set V of Y and that the converse is true if f is f.a.c.H.

Theorem 4.5:
A function f:X$\rightarrow$ Y is θ – fuzzy continuous (f.a.c.S) if and only if for each open set V of y $Fcl_\theta(f^{-1}(V)) \subset f^{-1}$(Fcl (V)).

Proof
Let x ϵ X and V be an fuzzy open set containing f (x). Since Y-Fcl (v) is fuzzy open in Y. By hypothesis we have $Fcl_\theta(X-f^{-1}$(Fcl (V))) $\subset f^{-1}$(Y-Fint (Fcl (V))) and hence x ϵ $f^{-1}(V) \subset X-Fcl_\theta$ $(X-f^{-1}$(Fcl (V))). Therefore, there exists an fuzzy open set U containing x such that Fcl (U) $\subset f^{-1}$(Fcl (v)). Hence f (Fcl(U) $\subset$Fcl (V). This completes the proof.

Corollary 4.6:
Let f:X $\rightarrow$Y be an f.a.o.R function. Then f is θ – fuzzy continuous if and only if$Fcl_\theta(f^{-1}(V)) \subset f^{-1}$(Fcl (V)) for each fuzzy open set V of Y

Proof
This is a consequence of lemma 3.3 and theorem 4.5.

References

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