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S-Inverse in a Super Fuzzy Matrix and Its Application

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Abstract

Several new models are created in the interesting field of super fuzzy matrices. For the first time an inverse which we call as S-inverse of a Fuzzy matrix is introduced. How S-inverse is applied in super fuzzy models is explained in this paper. Some properties of this new S-Inverse are also included in this paper.

Keywords: Super Fuzzy, S-Inverse

I. INTRODUCTION

In modern times Fuzzy matrices have become an integral part of mathematics one of the more interesting branch of Fuzzy mathematics is super Fuzzy matrix. Several new models in super Fuzzy matrices are already existing. In this paper a new concept called S-inverse is introduced. This paper consists of the following sections. Basic concepts of super Fuzzy matrices, Definition of S-inverse and its properties, Application of S-inverse to super Fuzzy models.

II. BASIC CONCEPTS OF SUPER FUZZY MATRICES

Let
$$A = \begin{bmatrix} A_{11} & A_{12} & . & . & . & A_{1n} \\ A_{21} & A_{22} & . & . & . & A_{2n} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ A_{m1} & A_{m2} & . & . & . & A_{mn} \end{bmatrix}$$

Where $A_{ij}^{'}$ s are fuzzy sub matrices, $i = 1, 2...m$

and $j = 1, 2...n$. If the number of columns in $A_{11}, A_{21}, \ldots A_{m1}$ are equal, $A_{12}, A_{22}, \ldots A_{m2}$ are equal and so on. Similarly the number of rows in $A_{11}, A_{12}, \ldots A_{1n}$ are equal, $A_{21}, A_{22}, \ldots A_{2n}$ are equal and so on, then A is defined to be a super fuzzy matrix.

Example of a super Fuzzy matrix

$$\left[\begin{array}{cc|cccc|ccc}
0.2 & 0.3 & 1 & 0 & 0.5 & 0.6 & 1 & 0.7 & 0.2 \\
1 & 0 & 0.2 & 1 & 0.3 & 0.9 & 0 & 0.2 & 0 \\
\hline
0 & 1 & 0.3 & 0 & 0.4 & 1 & 0 & 0.2 & 0 \\
0.9 & 0.3 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\
0.6 & 1 & 0 & 0.4 & 1 & 0.3 & 1 & 0 & 0 \\
\hline
0.2 & 0 & 1 & 0 & 1 & 0.4 & 0 & 1 & 0.3 \\
0.3 & 1 & 0.3 & 1 & 0 & 0.5 & 1 & 0.1 & 0 \\
1 & 0.4 & 0.2 & 1 & 0 & 1 & 0.3 & 0 & 0.3 \\
\hline
0 & 1 & 0.1 & 0 & 0 & 1 & 0.2 & 0.5 & 0.3 \\
0.4 & 0.3 & 1 & 0.2 & 0.5 & 0.7 & 1 & 1 & 0
\end{array}\right]$$

Let

$$A_{11} = \begin{bmatrix} 0.2 & 0.3 \\ 1 & 0 \end{bmatrix}$$

$$A_{12} = \begin{bmatrix} 1 & 0 & 0.5 & 0.6 \\ 0.2 & 1 & 0.3 & 0.9 \end{bmatrix} A_{12} = \begin{bmatrix} 1 & 0 & 0.5 & 0.6 \\ 0.2 & 1 & 0.3 & 0.9 \end{bmatrix}$$

$$A_{21} = \begin{bmatrix} 0 & 1 \\ 0.9 & 0.3 \\ 0.6 & 1 \end{bmatrix}$$

$$A_{22} = \begin{bmatrix} 0.3 & 0 & 0.4 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0.4 & 1 & 0.3 \end{bmatrix}$$

$$A_{23} = \begin{bmatrix} 0 & 0.2 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$A_{31} = \begin{bmatrix} 0.2 & 0 \\ 0.3 & 1 \\ 1 & 0.4 \end{bmatrix}$$

$$A_{32} = \begin{bmatrix} 1 & 0 & 1 & 0.4 \\ 0.3 & 0.1 & 0 & 0.5 \\ 0.2 & 1 & 0 & 1 \end{bmatrix}$$

$$A_{33} = \begin{bmatrix} 0 & 1 & 0.3 \\ 1 & 0.1 & 0 \\ 0.3 & 0 & 0.3 \end{bmatrix}$$

$$A_{41} = \begin{bmatrix} 0 & 1 \\ 0.4 & 0.3 \end{bmatrix}$$

$$A_{42} = \begin{bmatrix} 0.1 & 0 & 0 & 1 \\ 1 & 0.2 & 0.5 & 0.7 \end{bmatrix} A_{43} = \begin{bmatrix} 0.2 & 0.5 & 0.3 \\ 1 & 1 & 0 \end{bmatrix}$$

Thus we see that

A_{11}, A_{12} and A_{13} have 2 rows each.

A_{21}, A_{22} and A_{23} have 3 rows each.

A_{31}, A_{32} and A_{33} have 3rows each

A_{41}, A_{42} and A_{43} have 2 rows each

A_{11}, A_{21} ,$A_{31}$and A_{41} have 2 columns each.

A_{12}, A_{22} ,$A_{32}$and A_{42} have 4 columns each.

A_{13}, A_{23} ,$A_{33}$and A_{43} have 3 columns each.

Therefore A is a super Fuzzy matrix

III. DEFINITION OF S-INVERSE AND ITS PROPERTIES

Let $A = \begin{bmatrix} a_{11} & a_{12} & . & . & . & a_{1n} \\ a_{21} & a_{22} & . & . & . & a_{2n} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ a_{m1} & a_{m2} & . & . & . & a_{mn} \end{bmatrix}$ be a Fuzzy matrix. that is each a_{ij} satisfies

$0 \le a_{ij} \le 1$, then the S-inverse which we denote as A^S is defined as

$$A^S = \begin{bmatrix} 1-a_{11} & 1-a_{12} & . & . & . & 1-a_{1n} \\ 1-a_{21} & 1-a_{22} & . & . & . & 1-a_{2n} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ 1-a_{m1} & 1-a_{m2} & . & . & . & 1-a_{mn} \end{bmatrix}$$

Here the strength of each element is exactly fuzzily reversed and hence we call the inverse as S-inverse.

Main properties

$i)$ $\left(A^S\right)^S = A$

$ii)$ A+$A^S = U$, where U is a unitary matrix where each element of the matrix is 1.

IV. APPLICATION OF S-INVERSE TO SUPER FUZZY MODEL

Suppose 4 experts are studying about school dropouts. Suppose D_1, D_2,… are the attributes associated with school dropouts. D_1 may denote poverty; D_2 may denote lack of proper treatment by teachers and so on.

Let S_1,S_2,… be different types of schools. Suppose the four experts have different set of domain attributes (D_1, D_2, … are different) and different set of range attributes (S_1, S_2,… are different) then in this case they make use of diagonal super fuzzy model.

The model S_D given by experts is as follows.

$$\left[\begin{array}{cccc|ccc|ccccc|cccc}
0.6 & 0.1 & 0.2 & 0.3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.8 & 0.9 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0.8 & 0.9 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.9 & 0.2 & 0.4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.3 & 0.2 & 0.1 & 0.7 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0.8 & 0.8 & 0.4 & 0.8 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.7 & 0.9 & 0.6 & 0.1 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.3 & 0.4 & 0.1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.8 & 0.9 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.1 & 0 & 1 & 0.8 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.4 & 0.1 & 0.6 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.2 & 0.3 & 0.3
\end{array}\right]$$

Suppose X is any row vector of the domain space that is

X= [1 0 1 / 0 1 1 1 / 0 0 1 / 0 0 0 1 0]

Take S-inverse

$$S_D^S = \left[\begin{array}{cccc|ccc|ccccc|cccc}
0.4 & 0.9 & 0.8 & 0.7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.2 & 0.1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0.2 & 0.1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.1 & 0.8 & 0.6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0.7 & 0.8 & 0.9 & 0.3 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0.2 & 0.2 & 0.6 & 0.2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0.3 & 0.1 & 0.4 & 0.9 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.7 & 0.6 & 0.9 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0.2 & 0.1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.9 & 1 & 0 & 0.2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.6 & 0.9 & 0.4 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0.8 & 0.7 & 0.7
\end{array}\right]$$

X^S= [0 1 0 / 1 0 0 0 / 1 1 0 / 1 1 1 0 1]

$X^S \circ S_D{}^S \mapsto y$ ($\circ$ denotes usual multiplication)

The product of 2 fuzzy matrices need not be fuzzy. Therefore a concept called thresholding is introduced. Thresholding is replacing every entry by 1, if that entry is a positive integer and by 0 if it is a negative integer or zero. $\mapsto$ denotes thresholding.

Find $Y \circ \left(S_D{}^S\right)^T$ and so on till we reach stability. It can be observed that $S_D{}^S$ has more 0's compared to S_D and hence enabling simple calculations. Therefore S-inverse is useful in a diagonal super fuzzy model.

V. CONCLUSION

In this paper newly introduced S-inverse was applied to a diagonal super fuzzy matrix. The fact that double negative is positive is fully exploited. How S-inverse can be used in other areas of fuzzy is open for research. Any other discovery will strengthen the introduction of S-inverse.

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