Global Journal of Mathematical Sciences: Theory and Practical.
ISSN 0974-3200 Volume 9, Number 2 (2017), pp. 103-116

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δωα-Closed Sets in Topological Spaces

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Abstract

In this paper we introduced new type of closed sets is called δωα-closed sets in the topological spaces. Moreover we discuss the relations between δωα-closed sets and already available various closed sets. Also we studied some properties and applications of δωα-closed, δωα- interior and δωα-closure.

Key Words: δωα-closed sets, δωα-open sets , δωα- interior and δωα- closure.

AMS subject classification: 54C55,54A05

1. INTRODUCTION

In 1968 Velicko introduced δ-closed set in Topological spaces [19]. Using δ-closed set several results introduced by many researcher. ωα-closed set[1] introduced by S. S. Benchalli,etal., in the year 2009. Since the advent of these types of notions, several author have been introduced interesting results. δg-closed set introduced by Dontchev[3](1999). In 1965 Njastad[10] introduced α-open sets. The aim of the present paper is study the concept of δωα-closed sets and its various characterizations are investigated. Also, we further studied about δωα-interior δωα-closure in topological spaces.

2. PRELIMINARIES

Throughout this research paper (X, τ) (or simply X) represent topological spaces ,For a subset A of X, cl(A), int(A) and A^c denote the closure of A, the interior of A and the complement of A respectively.

Let us recall the following definition, which are useful in the sequel.

Definition: 2.1

A subset A of (X,τ) is called if

1) Regular closed set [18] if cl(int(A))=A.
2) Semi closed set [6] if int(cl(A)))⊆ A
3) α- closed set [10] if cl(int(cl(A)))⊆ A
4) δ-closed [19] if A = cl_δ(A), where cl_δ(A) = {x ∈X: int(cl(G)) ∩A ≠ϕ, G∈τ and x∈G}.
5) g-closed set [5] if cl(A) ⊆ U whenever A⊆ U , U is open in (X, τ) .
6) αg-closed set[7] if αcl(A)⊆ U whenever A⊆ U , U is open in (X, τ).
7) gα-closed set[8] if αcl(A)⊆ U whenever A⊆ U , U is α open in (X, τ).
8) δg-closed set [3] if δcl(A)⊆ U whenever A⊆ U , U is open in (X, τ).
9) gδ-closed set [4] if cl(A)⊆ U whenever A⊆ U , U is δ-open in (X, τ).
10) $\delta g^{\#}$-closed set[20] if δcl(A)⊆ U whenever A⊆ U , U is δ-open in (X, τ).
11) ω-closed set [16] if cl(A)⊆ U whenever A⊆ U , U is semi open in (X, τ).
12) ωα-closed set [1] if αcl(A) ⊆ U whenever A⊆ U , U is ω- open in (X, τ).
13) αω-closed set [12] if ω-cl(A) ⊆ U whenever A⊆ U , U is α-open in (X, τ).
14) δg*-closed set[17] if δcl(A) ⊆ U whenever A⊆U , U is g-open in (X, τ).
15) gωα-closed set [2] if αcl(A) ⊆ U whenever A⊆ U , U is ωα -open in (X, τ).
16) g*ωα -closed set[11] if cl(A) ⊆ U whenever A⊆ U , U is ωα -open in (X, τ).
17) δ (δg)*-closed set[9] if δcl(A) ⊆ U whenever A⊆ U ,U is δg-open in (X, τ).

The complements of the above listed closed sets are their concern open sets.

3. δωα-CLOSED SETS

In this section we introduce δωα-closed set in topological space (X ,τ).

Definition: 3.1

A subset A of a topological space (X, τ) is called δωα-closed set if $cl_{\delta}(A)\subseteq U$ whenever A⊆ U , U is ωα- open set in (X, τ) .the complement of δωα-closed set is called δωα-open set.

Proposition 3.2

Every δ-closed set is δωα-closed.

Proof:

Assume that A is a δ- closed and Let U be any ωα-open set such that A ⊆ U.Here A is δ-closed. A=$cl_{\delta}(A)$ for any subset A in (X, τ) .This implies that $cl_{\delta}(A)\subseteq U$. Here A is δωα-closed Set.

Remark: 3.3

Converse of the above theorem need not be true from the following example.

Let (X, τ) be a topological space with X= {a, b, c}and $\tau = \{\phi, X, \{a\}, \{a, c\}, \{c\}, \{b, c\}\}$. Here we take a subset A={c}, A is δωα closed but not δ-closed.

Proposition 3.4

Every regular closed set is δωα-closed.

Proof:

Assume that A is a regular closed set and Let U be any ωα-open set such that A ⊆ U.Here A is regular closed set. This implies A is a δ- closed and A=$cl_{\delta}(A)$ for any subset A in (X, τ) .This implies that $cl_{\delta}(A)\subseteq U$. Here A is δωα-closed Set.

Remark: 3.5

Converse of the above theorem need not be true from the following example.

Let (X, τ) be a topological space with X= {a, b, c, d, e}and

$\tau = \{\phi, X, \{a\}, \{b\}, \{d\}, \{a, b\}, \{a, d\}, \{b, d\}, \{a, b, d\}\}$Here we take a subset A={a,b,c,e}.A is δωα closed but not r-closed.

Proposition 3.6

Every δωα-closed set is δg-closed.

Proof:

Consider, A is a δωα closed set and Let U be an open such that A⊆ U.We know that, every open set is ωα-open set. Therefore U is an ωα open set such that A⊆ U.Here A is δωα-closed set, it imply that $cl_{\delta}(A) \subseteq U$. Hence A is δg-closed in (X, τ).

Remark: 3.7

Converse of the above the theorem need not be from thefollowing example.

Let (X, τ) be a topological space with X= {a,b,c} and $\tau = \{\phi, X, \{a\}, \{a, c\}, \{c\}, \{b, c\}\}$.

Here we take a subset A={a,c}.A is δg closed not δωα-closed set in (X, τ).

Proposition:3.8

Every δωα-closed set is αg-closed.

Proof:

Let A be any δωα-closed set in (X, τ) and V be an open set such that A ⊆ V. Therefore V is an ωα- open set such that A⊆ V .Since A is δωα-closed set, it implies that $cl_\delta(A) \subseteq V$. But αcl(A) ⊆ $cl_\delta(A)$ for any A. Therefore . αcl(A)⊆V. V is an open set in (X, τ) .Hence A is αg-closed.

Remark: 3.9

Converse of the above theorem need not be true from the following example,

Let (X, τ) be a topological space with X= $\{a, b, c, d\}$and $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}$.Here wetake a subset A= $\{d\}$.A is αg-closed but not δωα- set in (X, τ) .

Proposition 3.10

Every δωα-closed set is αω -closed.

Proof:

Let A be a δωα-closed and U be α-open set in (X, τ) such that A⊆ U. Since every α-open set is ωα- open set and A is δωα-closed, then ωcl(A) ⊆ cl(A) ⊆ $cl_\delta(A)$)⊆U Where U is ωα- open set. It implies that ωcl(A) ⊆U. Hence A is αω-closed in (X, τ).

Remark: 3.11

Converse of the above the theorem need not be from the following example.

Let (X, τ) be a topological space with X= {a,b,c,d} and $\tau = \{\phi, X, \{a\}, \{b\} \{a, b\}, \{a, c\}, \{a, b, c\}\}$.

Here we take a subset A={a,b,c}.A is αω closed not δωα-closedset in (X, τ).

Proposition 3.12

Every δωα-closed set is gωα-closed but converse need not be true.

Proof:

Let A be a δωα-closed and U be an ωα-open set in (X, τ) .such that A⊆ U. Since A is δωα-closed, then $cl_{\delta}(A)) \subseteq U$. But $cl(A) \subseteq cl_{\delta}(A)) \subseteq U$ [4], it implies that cl(A) ⊆ U. Hence A is gωα-closed set.

Remark 3.13

Converse of the above theorem need not be true from the following example,

Let (X, τ) be a topological space with X= $\{a, b, c\}$ and $\tau = \{\phi, X, \{a\}, \{a, b\}\}$. Here we take a subset A={c}.A is gωα-closed but not δωα-closed.

Proposition 3.14

Every δωα-closed set is g*ωα-closed but converse need not be true.

Proof:

Let A be a δωα-closed and U be an ωα-open set in (X, τ) such that A⊆ U. SinceA is δωα-closed, then $cl_{\delta}(A))\subseteq U$.But $\alpha cl(A)\subseteq cl_{\delta}(A))\subseteq U$,it implies that αcl(A) ⊆U.Hence A is g*ωα-closed set.

Remark 3.15

Converse of the above theorem need not be true from the following example, Let (X, τ) be a topological space with X =$\{a, b, c\}$ and $\tau = \{\phi, X, \{a\}, \{a, b\}\}$. Here we take a subset A= {c}.A is g*ωα- closed but not δωα closed.

Theorem 3.16

Every δωα-closed set is gα-closedset set but converse need not be true.

Proof:

Let A be a δωα-closed and U be an α-open set in (X, τ) .such that A⊆ U. Since A is δωα-closed, then $cl_{\delta}(A)) \subseteq U$. because every α-open set is ωα-open set in (X, τ) .But $\alpha cl(A) \subseteq cl_{\delta}(A)) \subseteq U$, it implies that αcl(A)⊆U. Hence A is gα-closed set.

Remark 3.17

Converse of the above theorem need not be true from the following example,

Let (X, τ) be a topological space with X = $\{a, b, c, \}$and$\tau = \{\phi, X, \{a\}\}$.

Here we take a subset A={c}.A is gα-closed but not δωα closed.

Theorem 3.18

Every δωα-closed set is $\delta g^{\#}$-closed but converse need not be true.

Proof:

Let A be a δωα-closed and U be an δ-open set in (X, τ) such that A⊆ U. Since A is δωα-closed, then $cl_\delta(A))$ ⊆Uwhere U is ωα-open set .it implies that $cl_\delta(A))$ ⊆ U. Hence A is $\delta g^{\#}$-closed set.

Remark 3.19

Converse of the above theorem need not be true from the following example,

Let (X, τ) be a topological space with $X = \{a, b, c, \}$and $\tau = \{\phi, X, \{a\}\}$. Here we take a subset A={c}.Ais $\delta g^{\#}$-closed but not δωα closed.

Theorem 3.20

Every δωα-closed set is gδ-closed but converse is notneed be true.

Proof:

Let A be a δωα-closed and U be anδ-open set in (X, τ) such that A⊆ U. Since A is δωα-closed, then $cl_\delta(A))$ ⊆ U. But cl(A) ⊆$cl_\delta(A))$ ⊆ U , it implies that cl(A) ⊆ U. Hence A is gδ-closed set.

Remark 3.21

Converse of the above theorem need not be true from the following example,

Let (X, τ) be a topological space with $X = \{a, b, c, \}$and $\tau = \{\phi, X, \{a\}\}$.

Here we take a subset A={c}.A is gδ -closed but not δωα closed.

Remark 3.22

δωα-closed set is independent with following closed sets are

closed set ,α-closed set, ωα-closed set , δg* -closed set ,and δ(δg)* closed set

above result we can prove by using followingexamples.

Example 3.23

Let (X, τ) be a topological space with X={a,b,c} with τ = { ϕ, { a }, {a ,b},X}.

Here we take a subset A={ c }.A is closed, α-closed and ωα-closed set but not δωα-closed set.

Example 3.24

Let (X, τ) be a topological space with X={a,b,c} with τ = { ϕ,{ a },{ b,c },X}.

The subset A={ b } is δωα-closed set but not closed, α-closed and ωα-closed in (X, τ).

Example 3.25

Let (X, τ) be a topological space with X={a,b,c,d} with τ ={ ϕ, {c}, {a,b},{c ,d},{a, b, c},X}.

Here we take a subset A={b,c,d}.A is not δg*and ,δ(δg)* closed set but δωα-closed set.

Example 3.26

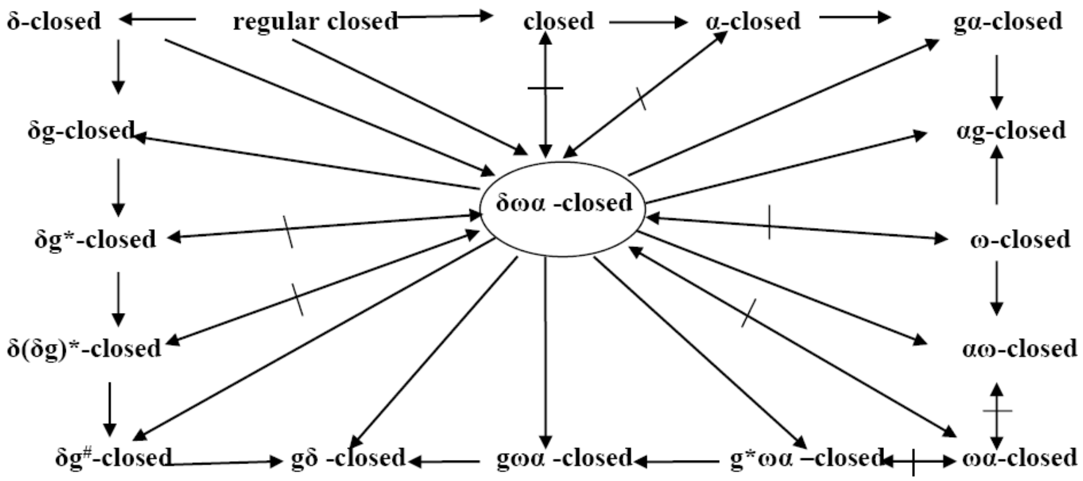
Let (X, τ) be a topological space with X={a,b,c} with τ = { ϕ, { a },{ a,b },X}.

Here we take a subset A ={ c}.A is δg* and δ(δg)* closed set but not δωα-closed set.

Diagram-I

Where A ⟶ B represents A implies B but B does not implies A

Where A ⟷̸ B represents A independent B each other.



4. PROPERTIES AND CHARACTERIZATION OF δωα-CLOSED

Theorem 4.1

The finite union of δωα-closed sets is δωα-closed.

Proof:

Let $\{A_i / i = 1,2 \ldots n\}$ be finite class of δωα-closed subsets of a space (X,τ). Then for each ωα-open Set U_i in (X, τ) containing A_i, $cl_\delta(A_i) \subseteq U_i$ $i \in \{1,2,...n\}$.Hence $\bigcup_{i=1}^{n} A_i \subseteq \bigcup_{i=1}^{n} U_i = V$. Since arbitrary union of ωα-open sets in (X,τ) is also ωα -open Set in (X,τ), V is ωα -open in (X,τ). Also ,$\bigcup_i cl_\delta(A_i) = cl_\delta(U_i A_i) \subseteq V$. Therefore $U_{i,}$ A_i is δωα-closed in (X,τ).

Remark 4.2:

Intersection of any two δωα-closed sets in (X,τ) need not be δωα-closed.

Let (X, τ) be a topological space with X= {a, b, c}with $\tau = \{\phi, X, \{a\}, \{c\}, \{a, c\}, \{b, c\}\}$.

Here we take two subsets {a,b} and{b,c} are δωα-closed sets but their intersection of two sets {b} is not δωα-closed.

Theorem 4.3

Let A be δωα-closed Set of (X,τ). Then cl_δ(A)-A does not contain a non empty ωα-Closed set.

Proof:

Suppose that A is δωα-closed. Let G be ωα-closed set contained in cl_δ(A)-A. Now $G^C$is ωα-open set of (X,τ) Such that A⊆G^C. Here A isδωα-closed Set of (X,τ). Thus cl_δ(A) ⊆G^C.

Therefore G⊆$(cl_\delta(A))^C$.Also G⊆cl_δ(A)-A. There G⊆$(cl_\delta(A))^C \cap cl_\delta(A)$=ϕ. Hence G=ϕ. Hence cl_δ(A)-A does not contain a non emptyωα-Closed set.

Theorem 4.4

If A is ωα-open and δωα-closed subset of (X,τ) then A is δ-closed subset of (X,τ).

Proof:

We know that A⊆cl_δ(A).Here A is ωα-open and δωα-closed,its implies that cl_δ(A) ⊆A. Hence A is δ-closed.

Theorem 4.5

The intersection of a δωα-closed set and a δ-closed set is δωα-closed.

Proof:

Let A be δωα-closed and Let G be δ-closed. Take U is an ωα-open set with A∩G⊆U. This imply A⊆U∪G^C.Here U∪G^C is ωα-open set .Therefore cl_δ (A)⊆U∪G^C.Now cl_δ(A∩G) ⊆cl_δ(A)∩cl_δ G= cl_δ(A)∩ G⊆ $(U\cup G^C) \cap G = U$. Here A ∩F is δωα-closed.

Theorem: 4.6

If A is a δωα-clsoed set is space (X,τ) and A⊆B⊆cl_δ(A) then B is also δωα-clsoed set.

Proof:

Let U be a ωα-open set of (X,τ) such that B⊆U. Then A⊆U. Since A is δωα-closed set, cl_δ(A) ⊆U. Since B⊆cl_δ(A), cl_δ(B) ⊆cl_δ(cl_δ(A))= cl_δ(A). Hence cl_δ(B) ⊆U. Therefore B is also a δωα-closed set.

Theorem 4.7.

Let A be δωα-clsoed of (X,τ). Then A is δ-closed set if cl_δ(A)-A is ωα-closed.

Proof:

Let A be a δ-closed Subset of X. Then cl_δ(A)=A and so cl_g(A)-A=ϕ which is ωα -closed.

Conversely ,Here A is δωα- closed by theorem 4.3 cl_δ(A)-A does not contain a non-empty ωα-closed set. This imply cl_δ(A)-A=ϕ. Therefore cl_δ(A)=A. Hence A is δ-closed.

5. δωα- CLOSURE AND δωα- INTERIOR IN TOPOLOGICAL SPACES

In this section, we introduce the notion of δωα- Closure and δωα-Interior of topological Spaces.

Definition 5.1.

The δωα- closure of a Subset A of X is denoted by δωα-cl(A) and is defined as the intersection of all δωα-closed sets containing A.Therefore

δωα-cl(A) = ∩ {F ⊆ X: A ⊆ F and F is δωα-closed set }.

Definition 5.2

The δωα-interior of subset A of X is denoted by δωα-(int A) and is defined as the union of all δωα-open sets contained in A . Therefore

δωα-int(A)= ∪ {G ⊆X: G ⊆ A and G is δωα- open set}.

Remark 5.3

If A⊆ X then,

(i) A⊆δωα-cl(A) ⊆cl_δ(A).

(ii) int_δ(A) ⊆int (A) ⊆ A.

Theorem: 5.4

Let A and B be any two subsets of a space X then the following properties are true.

(i) A is δωα-closed set if and only of δωα-cl(A) = A.

(ii) δωα-cl(A) is the smallest δωα- closed subset of X containing A.

(iii) δωα- cl(ϕ) = ϕ and ϕδωα-cl(X) = X.

(iv) δωα-cl (A) is a δωα- closed set in (X, τ) ..

(v) If A ⊆B, then δωα-cl(A) ⊆δωα- cl(B)

(vi) δωα-cl (A ∪B) = δωα-cl(A) ∪δωα-cl(B)

(vii) δωα-cl(A∩B) ⊆δωα-cl(A) ∩ δωα-cl(B)

(viii) δωα-cl(δωα-cl(A)) = δωα-cl(A)

Proof:

(i) we know that A ⊆δωα-cl(A) for any subset A of X. Let A be a δωα- closed set in (X, τ) .Also A ⊆ A and A ∈{F ⊆X: A ⊆F and F is δωα- closed Set}, it implies that A= ∩{F⊆X: A ⊆F and F is δωα- closed Set} ⊆ A. Then δωα-cl(A) ⊆ A. . Hence A= δωα-cl(A).converse is true from the direct definition.

(ii) By the definition of δωα-cl(A) ,the intersection of any collection sets is closed. There δωα-cl(A) is closed.Also if B is any δωα-closed set containing then δωα-cl(A)⊆B.Therefore δωα-cl(A)is the smallest δωα-closed set in(X, τ) containingA.

(iii) and (iv) –it follows from the Definition .

(v) We know that B ⊆δωα-cl(B)for every B. if A⊆ B then A ⊆δωα-cl(B) .So δωα-cl(B) is the δωα-closed set containing A. But δωα-cl(A) is smallest δωα-closed set containing A . Therefore δωα-cl(A) ⊆δωα-cl(B).

(vi) We know that A⊆ A ∪ B and B ⊆ A ∪ B, applying (v) ,we get δωα-cl(A)⊆δωα-cl(A ∪B) and δωα-cl(B) ⊆δωα-cl(A ∪B). Then δωα-cl(A) ∪δωα-cl (B) ⊆δωα-cl (A∪B). onthe other hand, δωα-cl(A) is δωα- closed set containing A and δωα-cl(B) is δωα-closed set containing B. Therefore δωα-cl(A) ∪δωα-cl (B) is δωα-closed set containing A∪B. But δωα-cl(A∪B) is δωα-closed set containing A∪B.Thereforeδωα-cl(A) ∪δωα-cl (B)⊇δωα-cl(A ∪B).Therefore δωα-cl (A ∪ B) = δωα-cl (A)∪δωα-cl (B).

(vii) We know that A ∩ B ⊆ A and A ∩ B ⊆ B. by using (v) we have, δωα-cl(A ∩B) ⊆δωα- cl(A) and δωα- cl(A∩B) ⊆δωα- cl(A) ∩δωα- cl(B).

(viii) we know that δωα-cl(A) is a δωα- closed set in (X, τ) .Let δωα- cl(A) = G, then G is δωα- closed Set in (X, τ).From (i) we have, δωα-cl(G) = G. It implies that δωα- cl(δωα- cl(A)) = δωα- cl(A).In the example3.5, subsets A= {a} and B= {c}, then δωα- cl(A)= {a,c}, δωα- cl(B)= {b,c} and δωα- cl(A ∩ B) = ϕ.

Therefore {c} = δωα- cl(A) ∩ δωα-cl(B) ⊆δωα- cl(A∩B)= ϕ.

Definition 5.5

Let A be any subset of a space X. Then, ωα-kernel of A is denoted by ωα-kernel (A) and Is defined as intersection of all ωα open sets containing A.

Then ωα-kernel(A) = ∩ {U ⊆X: A ⊆U and U is ωα open set}.

Theorem 5.6

A subset of a space X is δωα-closed set if and only if $cl_\delta(A)$ ⊆ωαker(A).

Proof:

Let U be a ωα-open set containing A then $cl_\delta(A)$⊆ωαker(A) ⊆ U. Therefore A is δωα- closed set.

Conversely suppose A is δωα- closed in (X, τ) .Then $cl_\delta(A)$ ⊆ U where U is ωα-open in (X, τ).

Let x ∈$cl_\delta(A)$.if x does not belong to ωα-kernel(A) then there exists an ωα-kernel(A) set U containing A such that x does not belong to U. Then x does not belong to $cl_\delta(A)$, which is a contradiction to the hypotheses. Hence $cl_\delta(A)$ ⊆ωαker(A).

Definition 5.7

Let N be any subset of topological space X them N is said to be δωα- neighborhood (denoted by δωα-nbd) of point x ∈ X if there exist an δωα- open set U such that x ∈ U ⊆ N.

Theorem 5.8

A subset A of topological space X is δωα- closed and x ∈δωα-cl(A) if and only if N ∩ A ≠ ϕ for Any δωα-nbd N of x in (X, τ) .

Proof:

Suppose x∉δωα-cl(A). Then there exists δωα-closed set F of X such that A ⊆ F and x ∉ F. Thus x ∈ (X-F) is δωα- open in X. But A ∩ (X-F) = ϕ which is a contradiction. Hence x∈δωαcl(A).Conversely, suppose that there exists an δωα-nbd N of a point x ∈ X such that N ∩ A = ϕ Then there exists an δωα-open set F of X such that x -open set F of X such that x∈ F ⊆ N. Therefore we have F ∩A = ϕ and x ∈ (X-F). Then δωα-cl(A) ∈ (X-F) and x ∉δωα-cl(A), which is a contradiction to hypothesis that x ∈δωα-cl(A). Therefore N ∩ A ≠ ϕ.

Remark 5.9:

The intersection of any two member of δωα-N(x) is again a member of δωα-N(x).

Definition 5.10

Let a be a subset of topological space X. Then a point $x \in X$ is said to be a δωα-limit point of A if every δωα-oen set of x contains a point of a other than, x that is $G \cap (A-\{x\}) \neq \phi$ for every δωα- open set G of X. In a topological space X, the set of all δωα-limit point of given a subset A of X is called δωα-derived set of A and is denoted by δωα-d(A).

Theorem 5.11

Let A and B be any two subsets of a space X then the following properties are true:

(i)δωαd(ϕ) = ϕ

(ii) If $A \subseteq B$, then δωα-d(A) ⊆δωα-d(B)

(iii) δωα-d(A ∪ B) = δωα-d(A) ∪δωα-cl(B)

(iv) δωα-d(A ∩ B) ⊆δωα-d(A) ∩ δωα-d(B).

Proof:

(i) Let $x \in X$ and x ∈δωα-d(ϕ). Then for every δωα-open set G containing x, we should have $G \cap (A-\{x\})=\phi$, which is impossible. Therfore δωα-d(ϕ) = ϕ.

(ii) Let x ∈δωα-d(A) then x is a limit point of A, $G \cap (A-\{x\}) \neq \phi$ for every δωα-nbd G containing x. if $A \subseteq B$, then $G \cap (B-\{x\}) \neq \phi$. Therefore x ∈δωα-d(B). Hence δωα-d(A) ⊆δωα-d(B)

(iii) We know that $A \subseteq A \cup B$ and $B \subseteq A \cup B$ then from property (ii), δωα-d(A) ∪δωα-d(B) ⊆δωα-d(A ∪ B) .. On the other hand if x ∉δωα-d(A) ∪δωα-d(B), then x ∉δωα-d(A) and x ∉δωα-d(B). Therefore there exist δωα-nbds G_1 and G_2 of x such that $G_1 \cap (A-\{x\}) = \phi$ and $G_2 \cap (B-\{x\}) = \phi$. Since $G_1 \cap G_2$ is δωα-nbd of x, then we get $(G_1 \cap G_2) \cap [A \cup B-\{x\}] = \phi$. Therefore x ∉δωα-d(A ∪ B). Therfore δωα-d(A ∪ B) ⊆δωα-d(A) ∪δωα-d(B) . we get, δωα-d(A ∪ B) = δωα-d(A) ∪δωα-cl(B).

(iv)Since $A \cap B \subseteq A$ and $A \cap B \subseteq B$, then from property (ii) we have δωα-d(A ∪ B) ⊆δωα-d(A) and δωα-d(A ∩ B) ⊆δωα-d(B).Consequently, δωα-d(A ∩B) ⊆δωα-d(A) ∩ δωα-d(B)

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