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Note on a Family of Integers of the form $N_p(6) = p^6+6^p$

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Abstract

In the current research paper we discuss the nature of a families of integers of the form $N_p(6) = p^6+6^p$. Here we observe that the unit digits of $N_p(6)$ will be 0, 1, 5 & 7 for all prime numbers $p \geq 2$. Also we observe that $N_p(6)$ is a family of composite integersfor all prime numbers $p \geq 2$.

Keywords: Composite Number, Family of Integers, Fermat Number, Mersenne Prime, and Prime Number**.**

INTRODUCTION

Prime numbers as well as composite numbers are the building blocks of the number theory. Elementary discussion of prime numbers and composite numbers is the need of our current research work.

A natural number is called a prime number (or a prime) if it has exactly two positive divisors, 1 and the number itself.[2] Natural numbers greater than 1 that are not primes are called composite. The crucial importance of prime numbers to number theory and mathematics in general stems from the fundamental theorem of arithmetic, which states that every integer larger than 1 can be written as a product of one or more primes in a way that is unique except for the order of the prime factors.**[3]**

In addition to concept of conjectures related families of the integers of the form revolving about primes has been posed. If we look a family of the integers of the form $2^{2^n} + 1$ are prime (they are called Fermat numbers) and he verified this up to $n = 4\ (or\ 2^{16} + 1)$. However, the very next Fermat number $2^{32} + 1$ is composite (one of its prime factors is 641), as Euler discovered later, and in fact no further Fermat numbers are known to be prime. Also we look another family of the integers of the form $2^p - 1$, with p a prime. They are called Mersenne primes in his honor. In 1747 he showed that the even perfect numbers are precisely the integers of the form $2^{p-1}(2^p - 1)$, where the second factor is a Mersenne prime.**[1]**

Continuing the discussion of conjectures related families of the integers of the various forms of integers in number theory we are discussing here some important results concerned with the nature of the family of integers of the form $N_p(6) = p^6$+6^p.

Theorem 1: Let p be an even prime number**,** then the number $N_p(6) = p^6$+6^p has 0 as the unit digit and also it is divisible by 5.

Proof- Since we know that the even prime number is only 2. Therefore for $p = 2$, $N_p(6) = p^6$+6^p=$2^6 + 6^2 \equiv 0(mod\ 10)$ this implies that $N_p(6)$ is divisible by 10. Which implies that, the number $N_p(6)$ has 0 as the unit digit and also it is divisible by 5 for $p = 2$.

Theorem 2: Let p =3**,** then the number $N_p(6) = p^6$+6^p has 5 as the unit digit and also it is divisible by 5.

Proof- Since we have $p =$ 3. Therefore for $p = 3, N_p(6) = p^6$+6^p=$3^6 + 6^3 \equiv 5(mod\ 10)$this implies that $N_p(6)$ has 5 as the unit digit. Since we know that a number with unit digit 5 is always divisible by 5.

Therefore we find out that, the number $N_p(6)$ has 5 as the unit digit and also it is divisible by 5 for p = 3.

Theorem 3: Let p =5**,** then the number $N_p(6) = p^6$+6^p has 1 as the unit digit and also it is divisible by 7.

Proof- Since we have $p =$ 5. Therefore for $p = 5$, $N_p(6) = p^6$+6^p=$5^6 + 6^5 \equiv 1(mod\ 10)$ this implies that $N_p(6)$ has 1 as the unit digit.

Now we check that $5^6 + 6^5$ is divisible by 7- by using the congruence we find out that $5^6 + 6^5 \equiv 0\ (mod\ 7) \Rightarrow N_p(6)$ divisible by 7. Therefore we find out that, the number $N_p(6)$ has 1 as the unit digit and also it is divisible by *7 for p* = 5.

Theorem 4: Let p =7**,** then the number $N_p(6) = p^6+6^p$ has 5 as the unit digit and also it is divisible by 5.

Proof- Since we have p = 7. Therefore for $p = 7$, $N_p(6) = p^6+6^p=7^6 + 6^7 \equiv 5(mod\ 10)$ this implies that $N_p(6)$ has 5 as the unit digit.

Now we check that $7^6 + 6^7$ is divisible by 5- by using the congruence we find out that $7^6 + 6^7 \equiv 0\ (mod\ 5) \Rightarrow N_p(6)$ divisible by 5 .Therefore we find out that, the number $N_p(6)$ has 5 as the unit digit and also it is divisible by 5 for p = 7.

Theorem 5: The unit digits of $N_p(6)$ will be 5, and 7 for prime for all prime numbers $p \geq 11$ respectively.

Proof: Since we know that the prime numbers ≥ 11 has the unit digits of 1, 3, 7, and 9 respectively.

If prime number has unit digit as 1 then $p^6 \equiv 1\ (mod\ 10)$ and $6^p \equiv 6\ (mod\ 10)$ this implies that $N_p(6)$ has 7 as the unit digit.

If prime number has unit digit as 3 then $p^6 \equiv 9\ (mod\ 10)$ and $6^p \equiv 6\ (mod\ 10)$ this implies that $N_p(6)$ has 5 as the unit digit.

If prime number has unit digit as 7 then $p^6 \equiv 9\ (mod\ 10)$ and $6^p \equiv 6\ (mod\ 10)$ this implies that $N_p(6)$ has 5 as the unit digit.

If prime number has unit digit as 9 then $p^6 \equiv 1\ (mod\ 10)$ and $6^p \equiv 6\ (mod\ 10)$ this implies that $N_p(6)$ has 7 as the unit digit.

Theorem 6: With the unit digits 0 and 5 $N_p(6)$ will be divisible by 5 for all prime numbers $p \geq 2$.

Proof- Since we know that the multiples of 5 has the unit digits of 0, and 5 respectively. Therefore $N_p(6)$ with the unit digits 0 and 5 is divisible by 5.

Theorem 7: If p is a prime number with the unit digits 1, 3, 7 and 9 then $N_p(6)$ will be divisible by 5 for all prime numbers $p \geq 11$ those have the unit digits 3 and 7 respectively.

Proof- It is easily proved by using congruence -

If prime number has unit digit as 1 then $p^6 \equiv 1\ (mod\ 5)$ and $6^p \equiv 1\ (mod\ 5)$ this implies that $N_p(6) \equiv 2\ (mod\ 5) \Longrightarrow$ 5 is not a divisor of $N_p(6)$.

If prime number has unit digit as 3 then $p^6 \equiv 4\ (mod\ 5)$ and $6^p \equiv 1\ (mod\ 5)$ this implies that $N_p(6) \equiv 0\ (mod\ 5) \Longrightarrow$ 5 is a divisor of $N_p(6)$.

If prime number has unit digit as 7 then $p^6 \equiv 4\ (mod\ 5)$ and $6^p \equiv 1\ (mod\ 5)$ this implies that $N_p(6) \equiv 0\ (mod\ 5) \Longrightarrow$ 5 is a divisor of $N_p(6)$.

If prime number has unit digit as 9 then $p^6 \equiv 1\ (mod\ 5)$ and $6^p \equiv 1\ (mod\ 5)$ this implies that $N_p(6) \equiv 2\ (mod\ 5) \Longrightarrow$ 5 is not a divisor of $N_p(6)$.

Hence we proved that $N_p(6)$ will be divisible by 5 for all prime numbers $p \geq 11$ those have the unit digits 3 and 7 respectively.

Theorem:8 If p is a prime number with the unit digits 1, 3, 7 and 9 then $N_p(6)$ will be divisible by 7 for all prime numbers $p \geq 11$.

Proof- It is easily proved by using congruence -

If prime number has unit digit as 1 then $p^6 \equiv 1\ (mod\ 7)$ and $6^p \equiv 6\ (mod\ 7)$ this implies that $N_p(6) \equiv 0\ (mod\ 7) \Longrightarrow 7$ is a divisor of $N_p(6)$.

If prime number has unit digit as 3 then $p^6 \equiv 1\ (mod\ 7)$ and $6^p \equiv 6\ (mod\ 7)$ this implies that $N_p(6) \equiv 0(mod\ 7) \Longrightarrow 7$ is a divisor of $N_p(6)$.

If prime number has unit digit as 7 then $p^6 \equiv 1\ (mod\ 7)$ and $6^p \equiv 6\ (mod\ 7)$ this implies that $N_p(6) \equiv 0\ (mod\ 7) \Longrightarrow 7$ is a divisor of $N_p(6)$.

If prime number has unit digit as 9 then $p^6 \equiv 1\ (mod\ 7)$ and $6^p \equiv 6\ (mod\ 7)$ this implies that $N_p(6) \equiv 0\ (mod\ 7) \Longrightarrow 7$ is a divisor of $N_p(6)$.

Hence we proved that $N_p(6)$ will be divisible by 7 for all prime numbers $p \geq 11$.

CONCLUSIONS

We conclude that the unit digits of $N_p(6)$ will be 0, 1, 5 & 7 for all prime numbers $p \geq 2$. Secondly we conclude that $N_p(6)$ with unit digits $0,$ & 5 is divisible by 5 for all prime numbers $p \geq 2$, and $N_p(6)$ with unit digits $1, 3, 5$ & 7 is divisible by 7 for all prime numbers $p > 2$ excluding 7 i.e. we observe that $N_p(6)$ is a family of composite integers for all prime numbers $p \geq 2$.

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