Global Journal of Mathematical Sciences: Theory and Practical.
ISSN 0974-3200 Volume 9, Number 2 (2017), pp. 169-188

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Effect of Radiation on MHD Three-Dimensional Free-Convection Flow through A Porous Medium with Chemical Reaction

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Abstract

The present article deals with the effect of thermal radiation on free-convection flow of a viscous incompressible fluid through a porous medium bounded by an infinite vertical porous plate with chemical reaction. A magnetic field is applied normal to the flow. Roseland approximation is used to describe the radiative heat flux in the energy equation. The problem is solved using finite difference and perturbation methods, for which numerical simulation is carried out by coding in C-Program**.** The results obtained for velocity, and temperature are discussed and analyzed. It is observed that the velocity, thermal boundary layer thickness and Nusselt number decrease for increasing values of radiation parameter R. It is also observed that for increasing values of the chemical reaction parameter Ch there is a fall in the concentration of the fluid.

Key words: Thermal radiation**;** Chemical reaction; volumetric rate of heat absorption; Magnetic field; Porous medium; Finite deference method

1. INTRODUCTION

In the recent years, the flows through porous medium are of principal interest because these are quite prevalent in nature. Such flows have many scientific and engineering applications, viz., in the fields of agricultural engineering to study the underground water resources, seepage of water in river beds; in chemical engineering for filtration and purification processes In view of these applications, a series of investigations have been made by Raptis *et al.* [1-3] in to the steady flow past a vertical wall. Raptis [4] studied the unsteady flow through porous medium bounded by an infinite porous plate subjected to a constant suction and variable temperature. Raptis and Perdikis [5] further studied the problem of free convective flow through a porous medium bounded by a vertical porous plate with constant suction where the free stream velocity oscillates in time about a constant mean value.

In all the studies mentioned above the permeability of the porous medium has been assumed as constant. In fact, a porous material containing the fluid is a non-homogeneous medium and there can be numerous in homogeneities present in a porous medium. Therefore, the permeability of the porous medium may not necessarily be constant. Sing and Suresh Kumar [6] have analyzed a free convective two dimensional unsteady flow through a highly porous medium bounded by an infinite vertical porous plate when the permeability of the medium fluctuates in time about a constant mean. Most of the investigators have restricted themselves to two-dimensional flows only by assuming either constant or time dependent permeability of the porous medium. However, there may arise situations where the flow field may be essentially three dimensional, for example, when variation of the permeability distribution is transverse to the potential flow. The effect of such a transverse permeability distribution of the porous medium bounded by horizontal flat plate has been studied by Sing and Verma [7] and Singh *et al* [8]. The effect of magnetic field on three dimensional flow of a viscous, incompressible and electrically conducting fluid past an infinite porous plate with transverse sinusoidal suction was discussed by Singh [9]. Singh, [10] studied hydro magnetic effects on the three dimensional oscillatory flow of an electrically conducting viscous incompressible fluid past an infinite porous plate subjected to a transverse sinusoidal sections. Singh and Sharma [11] studied the effect of transverse periodic variation of the permeability on the heat transfer and the free-convective of a viscous incompressible fluid through a highly porous medium bounded by a vertical porous plate. Later this same study with mass transfer was extended by Varshney and Singh [12]. Jain et al. [14] analysed the effects of periodic temperature and periodic permeability on three-dimensional free convective flow through porous medium in slip flow regime. Srihari and Anand Rao [15] analysed the effect of magnetic field on three-dimensional free-convective heat and mass transfer flow through a porous medium with periodic permeability. Ahmed [16] obtained an analytical solution for three-dimensional mixed convective flow with mass transfer along an infinite vertical porous plate in the presence of a magnetic field effect. Ravinder Reddy et al [17]analyzed the effect of heat sink in the presence of magnetic field on three-dimensional free convective heat and mass transfer flow through a porous medium with periodic permeability.Hayat et al. [18] investigated the three-dimensional flow of viscous fluid with convective boundary conditions and heat generation/absorption. Baag and Mishra [19] have made an analysis of heat and mass transfer on three-dimensional hydromagnetic nano-fluid flow.

Moreover, when the radiative heat transfer occurred in the fluid is considered as electrically conducting. In such case one cannot ignore the effect of magnetic field on the flow field. But in the recent years the subject of Magneto hydrodynamics has fascinated the attention of many researchers due to the growing number of various applications to problems of geophysical and astrophysical importance.In the previous three-dimensional studies, the effect of radiation has not been considered. But in science & technology many systems occurred at high temperature and the knowledge of radiation heat transfer plays an significant role for the design of the pertinent equipment, Nuclear power plants, gas turbines and the different propulsion devices for air craft, satellites and missiles. In such cases one has to consider the effects of radiation. So the objective of the present article is to analyze the effect of thermal radiation on free-convection flow of a viscous incompressible fluid through a porous medium bounded by an infinite vertical porous plate with chemical reaction. Usually obtaining closed form solution for three-dimensional flow problems is practically not feasible due to its high degree of non-linearity. Moreover, in all the previous three-dimensional attempts much emphasis was given to obtain an analytical solution and also numerical studies pertaining to such type of 3-D flow problems are very limited. So the problem is solved using finite difference and perturbation methods.

2. MATHEMATICAL ANALYSIS:

The flow of a viscous incompressible fluid through a highly porous medium bounded by an infinite vertical porous plate with constant suction is considered. The plate is lying vertically on the x^*-z^* plane with x^*-axis taken along the plate in the upward direction. The y^*-axis is taken normal to the plane of plate and directed into the fluid flowing laminarly with a uniform free stream velocity U. Since the plate is considered infinite in x^* -direction, so all physical quantities will be independent of x^*. A magnetic field of uniform strength is applied normal to the flow, i.e. along y^* -axis.

The permeability of the porous medium is assumed to be of the form.

$$K^*(z^*) = \frac{K_0^*}{(1+\varepsilon\cos \pi z^*/L)} \tag{1}$$

Where ε (<<1) is the amplitude of the permeability variation. The problem becomes three-dimensional due to such a permeability variation. All fluid properties are assumed constant except that the influence of the density variation with temperature and concentration is considered only in the body force term.

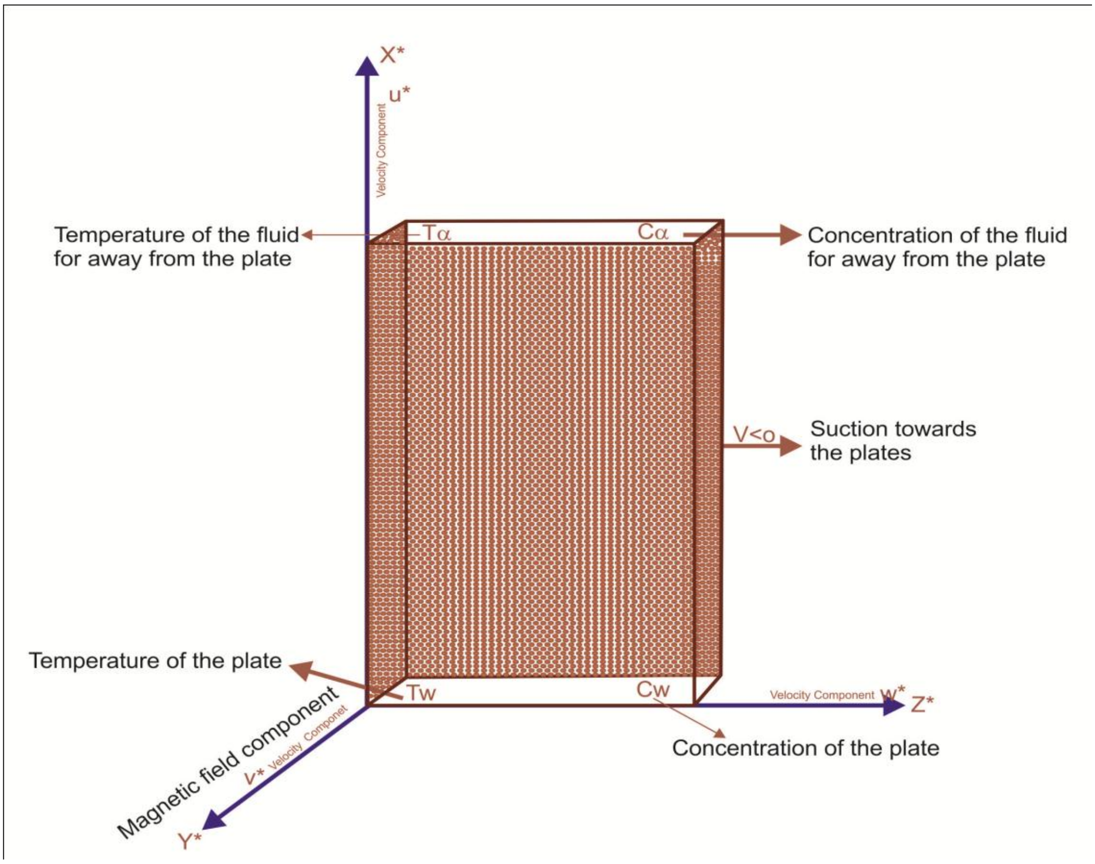

Figure 2.1 Schematic of the flow configuration

Thus, denoting velocity components by u^*, v^*, w^* in the directions of x^*, y^*, z^* respectively and the temperature by the T^* and concentration by C^*, the flow through a highly porous medium is governed by following non-dimensional equations:

$$\frac{\partial v^*}{\partial y^*} + \frac{\partial w^*}{\partial z^*} = 0 \tag{2}$$

$$v^* \frac{\partial v^*}{\partial y^*} + w^* \frac{\partial v^*}{\partial z^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial y^*} + \upsilon\left(\frac{\partial^2 v^*}{\partial y^{*2}} + \frac{\partial^2 v^*}{\partial z^{*2}}\right) - \frac{\upsilon}{K^*} v^* \tag{3}$$

$$v^* \frac{\partial w^*}{\partial y^*} + w^* \frac{\partial w^*}{\partial z^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial z^*} + \upsilon\left(\frac{\partial^2 w^*}{\partial y^{*2}} + \frac{\partial^2 w^*}{\partial z^{*2}}\right) - \frac{\upsilon}{K^*} w^* - \frac{\sigma B_0^2}{\rho} w^* \tag{4}$$

$$v^* \frac{\partial T^*}{\partial y^*} + w^* \frac{\partial T^*}{\partial z^*} = \frac{k}{\rho C_p}\left(\frac{\partial^2 T^*}{\partial y^{*2}} + \frac{\partial^2 T^*}{\partial z^{*2}}\right) - \frac{Q(T^* - T_\infty^*)}{\rho C_p} - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial y^*} \tag{5}$$

$$v^* \frac{\partial C^*}{\partial y^*} + w^* \frac{\partial C^*}{\partial z^*} = D\left(\frac{\partial^2 C^*}{\partial y^{*2}} + \frac{\partial^2 C^*}{\partial z^{*2}}\right) - Ch^*(C^* - C_\infty) \tag{6}$$

The boundary conditions of the problem are:

$$\begin{aligned} &y^* = 0;\ u^* = 0, v^* = -V, w^* = 0, T^* = T_w^*, C^* = C_w^* \\ &y^* \to \infty;\ u^* \to U, w^* \to 0, p^* \to p_\infty^*, T^* \to T_\infty^*, C^* \to C_\infty^* \end{aligned} \tag{7}$$

Where V>0 is a constant and the negative sign indicates that suction towards the plate.

Using the Rosseland diffusion, the approximation of radiative flux q_r as a Fourier type gradient function is given by [20].

$$q_r = -\frac{4\sigma^*}{3a_R}\frac{\partial T^4}{\partial y^*}, \text{ it implies, } \quad \frac{\partial q_r}{\partial y*} = -\frac{4\sigma^*}{3a_R}\frac{\partial^2 T^4}{\partial y^{*2}} \tag{8}$$

T^4 may be expressed as a linear function T under the assumption that the temperature differences within the flow are adequately small. Expanding T^4 in a Taylor series about T_∞, the following is obtained

$$T^4 = T_\infty^4 + 4(T - T_\infty)T_\infty^3 + 12\frac{(T - T_\infty)^2}{2!}T_\infty^2 + \ldots \quad , \quad \text{where, } f(T) = T^4, \tag{9}$$

Neglecting the higher order terms in (9), we have

$$T^4 \approx 4T_0^3 T - 3T_0^4 \tag{10}$$

Substituting (10) in (8) and then using (8) in (5), we get

$$\left(v^* \frac{\partial T^*}{\partial y^*} + w^* \frac{\partial T^*}{\partial z^*}\right) = \frac{k}{\rho C_p}\left(\frac{\partial^2 T^*}{\partial y^{*2}} + \frac{\partial^2 T^*}{\partial z^{*2}}\right) - \frac{Q(T^* - T_\infty^*)}{\rho C_p} + \frac{16\sigma^* T_0^3}{3\rho c_p a_R}\frac{\partial^2 T^*}{\partial y'^2} \tag{11}$$

Introducing the following non dimensional quantities:

$$y = \frac{y^*}{L},\ z = \frac{z^*}{L}, \quad u = \frac{u^*}{U}, \quad v = \frac{v^*}{V},\ w = \frac{w^*}{V}, \quad p = \frac{p^*}{\rho U^2}, \quad \theta = \frac{T^* - T_\infty^*}{T_w^* - T_\infty^*},$$

$$\phi = \frac{C^* - C_\infty^*}{C_w^* - C_\infty^*}$$

in to the equations (2),(3), (4),(6) and (11), the following are obtained:

$$\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{12}$$

$$v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = Gr\,\mathrm{Re}\,\theta + Gm\,\mathrm{Re}\,\phi + \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) - \frac{(u-1)\,(1+\varepsilon\cos\pi z)}{\mathrm{Re}\,K_0} - \frac{M^2}{\mathrm{Re}}u \tag{13}$$

$$v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{\partial p}{\partial y} + \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) - \frac{(1+\varepsilon\cos\pi z)\,v}{\mathrm{Re}\,K_0} \tag{14}$$

$$v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{\partial p}{\partial z} + \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) - \frac{(1+\varepsilon\cos\pi z)w}{\mathrm{Re}\,K_0} - \frac{M^2}{\mathrm{Re}}w \tag{15}$$

$$v\frac{\partial \theta}{\partial y} + w\frac{\partial \theta}{\partial z} = \frac{1}{\mathrm{Re}\,\mathrm{Pr}}\left(\left(1+\frac{4}{3R}\right)\frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2}\right) - \frac{S}{\mathrm{Re}\,\mathrm{Pr}}\theta \tag{16}$$

$$v\frac{\partial \phi}{\partial y} + w\frac{\partial \phi}{\partial z} = \frac{1}{\mathrm{Re}\,Sc}\left(\frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2}\right) - Ch\phi \tag{17}$$

Where, $Gr = \dfrac{\upsilon g\beta\left(T_w^* - T_\infty^*\right)}{UV^2},\ Gm = \dfrac{\upsilon g\beta^*\left(C_w^* - C_\infty^*\right)}{UV^2},\ \mathrm{Re} = \dfrac{VL}{\upsilon},\ \mathrm{Pr} = \dfrac{\mu C_p}{k},\ Sc = \dfrac{\upsilon}{D},$

$K_0 = \dfrac{K_0^*}{L^2},\ M = B_0 L\sqrt{\dfrac{\sigma}{\mu}}\ ,\ S = \dfrac{QL^2}{k},\ Ch = \dfrac{\overline{Ch}.L}{V},\ R = \dfrac{k\,a_R}{4\sigma^* T_0^3}$

The corresponding boundary conditions reduce to

$$y = 0;\ u = 0,\ v = -1,\ w = 0,\ \theta = 1,\ \phi = 1 \tag{18}$$

$$y \to \infty;\ u \to 1,\ w \to 1,\ p \to p_\infty,\ \theta \to 0,\ \phi \to 0$$

3. METHOD OF SOLUTION

In order to solve the problem, solutions are assumed in the following form because the amplitude $\varepsilon(\langle\langle 1)$ is very small:

$$f(y, z) = f_0(y) + \varepsilon f_1(y,z) + \varepsilon^2 f_2(y,z) + \ldots \tag{19}$$

where f stands for u, v, w, p, θ, ϕ and $\varepsilon <<1$.

Case (i): Zeroth order equations for ε=0

When $\varepsilon = 0$, the problem is governed by the following equations:

$$\frac{dv_0}{dy} = 0 \tag{20}$$

$$\frac{d^2u_0}{dy^2} - v_0 \operatorname{Re}\frac{du_0}{dy} - \left(M^2 + \frac{1}{K_0}\right)u_0 = -Gr\operatorname{Re}^2\theta_0 - Gm\operatorname{Re}^2\phi_0 - \frac{1}{K_0} \tag{21}$$

$$\left(1+\frac{4}{3R}\right)\frac{d^2\theta_0}{dy^2} - v_0 \operatorname{Re}\Pr\frac{d\theta_0}{dy} - S\theta_0 = 0 \tag{22}$$

$$\frac{d^2\phi_0}{dy^2} - v_0 \operatorname{Re} Sc\frac{d\phi_0}{dy} - Ch.\operatorname{Re}.Sc.\phi_0 = 0 \tag{23}$$

The corresponding boundary conditions become

$$y = 0;\ u_0 = 0,\ v_0 = -1,\ \theta_0 = 1,\ \phi_0 = 1 \tag{24}$$

$$y \to \infty;\ u_0 \to 1,\ p_0 \to p_\infty,\ \theta_0 \to 0,\ \phi_0 \to 0$$

The solutions of the equations (20) to (23) under the boundary conditions (24) are given by

$$u_0 = 1 + (L_0 + L_1 - 1)e^{-r_3 y} - L_0 e^{-r_1 y} - L_1 e^{-r_2 y} \tag{25}$$

$$\theta_0 = e^{-r_1 y} \quad (26), \qquad \phi_0 = e^{-r_2 y} \tag{27}$$

$$v_0 = -1,\ w_0 = 0,\ \text{and}\ p_0 = p_\infty \tag{28}$$

where

$$a_1 = \frac{r_1^2\ So\ Sc}{r_1^2 - \operatorname{Re} Sc\ r_1 - Ch\operatorname{Re} Sc},\ L_0 = \frac{Gr\ \operatorname{Re}^2}{r_1^2 - \operatorname{Re} r_1 - \left(M^2 + \frac{1}{K_0}\right)}$$

$$L_1 = \frac{Gm\,\mathrm{Re}^2}{r_2^2 - \mathrm{Re}\,r_2 - \left(M^2 + \frac{1}{K_0}\right)}, \quad r_1 = \frac{\mathrm{Re}\,\mathrm{Pr} + \sqrt{\mathrm{Re}^2\,\mathrm{Pr}^2 + 4(1+4/3R)S}}{2(1+4/3R)}$$

$$r_2 = \frac{\mathrm{Re}\,Sc + \sqrt{\mathrm{Re}^2\,Sc^2 + 4Ch\,\mathrm{Re}\,Sc}}{2}, \qquad r_3 = \frac{\mathrm{Re} + \sqrt{\mathrm{Re}^2 + 4\left(M^2 + \frac{1}{K_0}\right)}}{2}$$

Case (ii): First order equations for $\boldsymbol{\varepsilon \neq 0}$

When $\varepsilon \neq 0$, substituting (19) in equations (12) to (17), and comparing the coefficients of identical power of ε, the following equations are obtained with the help of equation (28)

$$\frac{\partial v_1}{\partial y} + \frac{\partial w_1}{\partial z} = 0 \tag{29}$$

$$v_1 \frac{\partial u_0}{\partial y} - \frac{\partial u_1}{\partial y} = Gr\,\mathrm{Re}\,\theta_1 + Gm\,\mathrm{Re}\,\phi_1 + \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_1}{\partial z^2}\right) - \frac{(u_0 - 1)\varepsilon\cos\pi z + u_1}{\mathrm{Re}\,K_0} - \frac{M^2}{\mathrm{Re}} u_1 \tag{30}$$

$$-\frac{\partial v_1}{\partial y} = -\frac{\partial p_1}{\partial y} + \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 v_1}{\partial y^2} + \frac{\partial^2 v_1}{\partial z^2}\right) - \frac{(v_1 - \cos\pi z)}{\mathrm{Re}\,K_0} \tag{31}$$

$$-\frac{\partial w_1}{\partial y} = -\frac{\partial p_1}{\partial z} + \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 w_1}{\partial y^2} + \frac{\partial^2 w_1}{\partial z^2}\right) - \frac{w_1}{\mathrm{Re}\,K_0} - \frac{M^2}{\mathrm{Re}} w_1 \tag{32}$$

$$v_1 \frac{\partial \theta_0}{\partial y} - \frac{\partial \theta_1}{\partial y} = \frac{1}{\mathrm{Re}\,\mathrm{Pr}}\left[\left(1 + \frac{4}{3R}\right)\frac{\partial^2 \theta_1}{\partial y^2} + \frac{\partial^2 \theta_1}{\partial z^2}\right] - S\theta_0 \tag{33}$$

$$v_1 \frac{\partial \phi_0}{\partial y} - \frac{\partial \phi_1}{\partial y} = \frac{1}{\mathrm{Re}\,Sc}\left(\frac{\partial^2 \phi_1}{\partial y^2} + \frac{\partial^2 \phi_1}{\partial z^2}\right) - Ch\,\phi_1 \tag{34}$$

The corresponding boundary conditions are:

$$y = 0;\ u_1 = 0,\ v_1 = 0,\ w_1 = 0,\ \theta_1 = 0,\ \phi_1 = 0 \tag{35}$$

$$y \to \infty;\ u_1 \to 0,\ w_1 \to 0,\ p_1 \to 0,\ \theta_1 \to 0,\ \phi_1 \to 0$$

Equations (29) to (34) are the partial differential equations, which describe three-dimensional flow. In order to solve these equations, we separate the variables y and z in the following manner.

$$v_1(y,\,z) = -v_{11}(y)\cos\pi z \tag{36}$$

$$w_1(y,\,z) = \frac{1}{\pi}v'_{11}(y)\sin\pi z \tag{37}$$

$$p_1(y,\,z) = p_{11}(y)\cos\pi z \tag{38}$$

Where expressions $v_1(y,\,z)$ and $w_1(y,\,z)$ have been chosen so that the equation of continuity (29) is satisfied. Substituting the expressions (36), (37) & (38) **in** (31) and (32), the following differential equations are obtained:

$$\frac{dv_{11}}{dy} = -\frac{dp_{11}}{dy} + \frac{1}{\mathrm{Re}}\left(\pi^2 v_{11} - \frac{d^2 v_{11}}{dy^2}\right) + \frac{1}{\mathrm{Re}\,K_0}(v_{11}+1) \tag{39}$$

$$\frac{-1}{\pi}\frac{d^2 v_{11}}{dy^2} = p_{11}\pi + \frac{1}{\mathrm{Re}}\left(\frac{1}{\pi}\frac{d^3 v_{11}}{dy^3} - \pi\frac{dv_{11}}{dy}\right) - \frac{1}{\pi\,\mathrm{Re}\,K_0}\frac{dv_{11}}{dy} - \frac{M^2}{\mathrm{Re}\,\pi}\frac{dv_{11}}{dy} \tag{40}$$

Elimination of the pressure gradient terms $\frac{dp_{11}}{dy}, p_{11}$ in (39) and (40), implies

$$\frac{d^4 v_{11}}{dy^4} + \mathrm{Re}\frac{d^3 v_{11}}{dy^3} - \left(M^2 + \frac{1}{K_0} + 2\pi^2\right)\frac{d^2 v_{11}}{dy^2} - \mathrm{Re}\,\pi^2\frac{dv_{11}}{dy} + \left(\pi^4 + \frac{\pi^2}{K_0}\right)v_{11} = -\frac{\pi^2}{K_0} \tag{41}$$

The corresponding boundary conditions transformed to

$$\begin{aligned} &y = 0 : v_{11} = 0,\; v'_{11} = 0 \\ &y \to \infty : v_{11} = 0 \end{aligned} \tag{42}$$

In order to solve equations (30), (33) and (34), the following are supposed

$$u_1(y,z) = u_{11}(y)\cos\pi z \tag{43}$$

$$\theta_1(y,z) = \theta_{11}(y)\cos\pi z \tag{44}$$

$$\phi_1(y,z) = \phi_{11}(y)\cos\pi z\ . \tag{45}$$

Substitution of (43), (44) and (45) in (30), (33) and (34), gives the following :

$$\frac{d^2 u_{11}}{dy^2} + \mathrm{Re}\frac{du_{11}}{dy} - \left(M^2 + \frac{1}{K_0} + \pi^2\right)u_{11} = -\mathrm{Re}\,v_{11}\frac{du_0}{dy} - Gr\,\mathrm{Re}^2\,\theta_{11} - Gm\,\mathrm{Re}^2\,\phi_{11} + \frac{u_0 - 1}{K_0} \tag{46}$$

$$\left(1 + \frac{4}{3R}\right)\frac{d^2\theta_{11}}{dy^2} + \mathrm{Re}\,\mathrm{Pr}\frac{d\theta_{11}}{dy} - (\pi^2 + S)\theta_{11} = -\mathrm{Re}\,\mathrm{Pr}\,v_{11}\frac{d\theta_0}{dy} \tag{47}$$

$$\frac{d^2\phi_{11}}{dy^2}+\operatorname{Re}Sc\frac{d\phi_{11}}{dy}-(Ch\operatorname{Re}Sc+\pi^2)\phi_{11}=-\operatorname{Re}Sc\,v_{11}\frac{d\phi_0}{dy} \tag{48}$$

The corresponding boundary conditions reduced to

$$y=0:\quad u_{11}=0,\ \theta_{11}=0,\ \phi_{11}=0 \tag{49}$$

$$y\to\infty: u_{11}\to 0,\ \theta_{11}\to 0,\ \phi_{11}\to 0\,.$$

Substitution of the following finite difference formulae

$$\frac{d^2v_{11}}{dy^2}=\frac{v_{11}(i+1)-v_{11}(i-1)}{2h},\qquad \frac{d^2v_{11}}{dy^2}=\frac{v_{11}(i+1)-2v_{11}(i)+v_{11}(i-1)}{h^2}$$

$$\frac{d^3v_{11}}{dy^3}=\frac{v_{11}(i+2)-2v_{11}(i+1)+2v_{11}(i-1)-v_{11}(i-2)}{2h^3}$$

$$\frac{d^4v_{11}}{dy^4}=\frac{v_{11}(i+2)-4v_{11}(i+1)+6v_{11}(i)-4v_{11}(i-1)+v_{11}(i-2)}{h^4}$$

in equation (41) provides the following finite difference form

$$A_1v_{11}(i+2)-A_2v_{11}(i+1)+A_3v_{11}(i)-A_4v_{11}(i-1)+A_5v_{11}(i-2)=-2\frac{\pi^2h^4}{K_0} \tag{50}$$

The corresponding boundary conditions in finite difference form are given by:

$$v_{11}[i]=0,\quad for\quad i=0$$
$$v_{11}[i]=0,\quad for\quad i=10$$

Where $A_1=(2+\operatorname{Re}h)$, $A_2=\left(8+2\operatorname{Re}h+2h^2\left(M^2+\frac{1}{K_0}+2\pi^2\right)+\operatorname{Re}h^3\pi^2\right)$

$$A_3=\left[12+4h^2\left(M^2+\frac{1}{K_0}+2\pi^2\right)+2h^4\left(\pi^4+\frac{\pi^2}{K_0}\right)\right],\quad A_5=(2-\operatorname{Re}h)$$

$$A_4=\left[8-2\operatorname{Re}h+2h^2\left(M^2+\frac{1}{K_0}+2\pi^2\right)-\operatorname{Re}h^3\pi^2\right],\ .$$

Similarly substituting of the following finite difference formulae

$$\frac{du_{11}}{dy}=\frac{u_{11}(i+1)-u_{11}(i-1)}{2h},\qquad \frac{d^2u_{11}}{dy^2}=\frac{u_{11}(i+1)-2u_{11}(i)+u_{11}(i-1)}{h^2}$$

$$\frac{d\theta_{11}}{dy}=\frac{\theta_{11}(i+1)-\theta_{11}(i-1)}{2h} \qquad \frac{d^2\theta_{11}}{dy^2}=\frac{\theta_{11}(i+1)-2\theta_{11}(i)+\theta_{11}(i-1)}{h^2}$$

$\frac{d^2\phi_{11}}{dy^2}=\frac{\phi_{11}(i+1)-2\phi_{11}(i)+\phi_{11}(i-1)}{h^2}$, in equations (46) to (48), we obtain the following

$$A_1u_{11}(i+1)-A_6u_{11}(i)+A_5u_{11}(i-1)=A(i) \tag{51}$$

$$B_1\theta_{11}(i+1)-B_2\theta_{11}(i)+B_3\theta_{11}(i-1)=B(i) \tag{52}$$

$$C_1\phi_{11}(i+1)-C_2\phi_{11}(i)+C_3\phi_{11}(i-1)=C(i) \tag{53}$$

The boundary conditions in finite difference form are given by:

$$u_{11}[i]=0,\ \theta_{11}[i]=0, \Phi_{11}[i]=0\ \ for\ \ i=0$$

$$u_{11}[i]=0,\ \theta_{11}[i]=0, \Phi_{11}[i]=0\ for\ i=10$$

Where i stands plate divisions with step length h=0.1 and $y=i\,h$, and A_1, A_5, L_0, L_1 have already been defined and

$B_1=\left[2(1+4/3R)+\mathrm{Re}\,\mathrm{Pr}\,h\right]$, $B_2=\left\{4(1+4/3R)+2h^2(\pi^2+S)\right\}$, $B_3=\left[2(1+4/3R)-\mathrm{Re}\,\mathrm{Pr}\,h\right]$

$B(i)=2h^2r_1\,\mathrm{Re}\,\mathrm{Pr}\,v_{11}(i)e^{-r_1ih}$, $C_1=2+\mathrm{Re}\,Sch$, $C_3=2-\mathrm{Re}\,Sch$,

$C_2=4+2h^2(\pi^2+Ch\,\mathrm{Re}\,Sc)$,

$$A_6=4+2h^2\left(M^2+\frac{1}{K_0}+\pi^2\right),\ C(i)=2h^2\,\mathrm{Re}\,Scv_{11}(i)\left((r_2e^{-r_2y}\right)$$

$$A(i)=-2h^2\,\mathrm{Re}\,v_{11}(i)A_7(i)-2(h\,\mathrm{Re})^2\left(Gr\theta_{11}(i)-Gm\phi_{11}(i)\right)+\frac{2h^2}{K_0}A_8(i)$$

$$A_7(i)=-r_3(L_0+L_1-1)e^{-r_3ih}+r_1L_0e^{-r_1ih}+r_2L_1e^{-r_2ih},$$

$$A_8(i)=(L_0+L_1-1)e^{-r_3ih}-L_0e^{-r_1ih}-L_1e^{-r_2ih}$$

Equations (50), (51), (52) and (53) have been solved by Gauss-seidel iteration method, for which numerical simulation is carried out by coding in C-Program. In order to obtain the numerical solution with least total error and to prove the convergence of present numerical scheme, a grid independent test is applied by experimenting with different grid sizes i.e. the computation is carried out by a slight changed values of *h.* This process is repeated until we get the results up to the desired degree of accuracy 10^{-8}. No considerable change is observed in the values of temperature profile (θ), main and cross flow velocities $w \& \theta$ respectively.

4. SKIN-FRICTION COEFFICIENT:

Skin friction components in the x^* -direction in the non-dimensional form is given by:

$$\tau = \frac{\tau^*}{\rho U V} = \frac{\upsilon}{VL}\left(\frac{\partial u}{\partial y}\right)_{y=0} = \frac{1}{\mathrm{Re}}\left(\frac{du_0}{dy} + \varepsilon \frac{du_{11}}{dy}\cos \pi z\right)_{y=0} \tag{54}$$

5. NUSSELT -NUMBER:

From the temperature field the rate of heat transfer coefficient in terms of Nusselt number Nu is given by

$$Nu = \frac{-q^*}{\rho V C_p \left(T_w^* - T_\infty^*\right)} = \frac{k}{\rho V C_p L}\left(\frac{\partial \theta}{\partial y}\right)_{y=0} = \frac{1}{\mathrm{Re}\,\mathrm{Pr}}\left(\frac{d\Theta_0}{dy} + \varepsilon \frac{d\Theta_{11}}{dy}\cos \pi z\right)_{y=0} \tag{55}$$

6. RESULTS AND DISCUSSION

In order to describe the physics of the problem, the problem of three-dimensional free-convection flow of a viscous incompressible fluid through a porous medium in the presence of thermal radiation is solved using finite difference and perturbation methods. The effects of main parameters, which are appeared in the governing equations, are discussed in the presence of thermal radiation.

Figures (1), (3),(6) and (7) show that the effect of thermal radiation on velocity, temperature and Nusslet number respectively. It is observed that the velocity, Nusselt number and temperature of the fluid an increase with the increasing values of radiation parameter R. This is owing to the fact that for decreasing values of radiation parameter R, the rate of radiative heat $(\partial q_r / \partial y^*)$, transferred to the fluid increases. This result easily can be established from the non-dimensional radiation parameter $R = k\, a_R / 4\sigma^* T_\infty^3$ that the Roseland radiation absorbtivity a_R decreases as the value of radiation parameter R decreases forgiven k and T_∞. Also from the relation (8) it is observed that there is an indirectly proportionality between a_R and $\partial q_r / \partial y^*$. Therefore the reduction of radiation absorbtivity a_R causes the enhancement in the divergence of the radiative heat flux ($\partial q_r / \partial y^*$). Consequently the temperature and velocity of the particles enhance for the decreasing values of radiation parameter R.

Figure (5) shows temperature field for various values of Pr. It is observed that the temperature of the fluid decreases with the increasing values of Prandtl number Pr. This is a good conformity with physical fact that the thermal boundary layer thickness diminish for the increasing values of Pr. The cause underlying such performance is that the a fluid with higher Pandtl number has comparatively lower thermal conductivity. Figure (2) and (9) demonstrate the velocity and temperature profiles for various values of heat absorption parameter (S). It is clear from the figures that an increase in the heat absorption parameter (S) leads to decrease in the temperature and velocity of the fluid as the effect of heat sink is to decrease the rate of heat transport to the fluid there by falling the temperature of the fluid. Consequently, velocity of the fluid particles also reduces. From figure (8), it is observed that the velocity of the flow reduces for the increasing values of Hartman number. This is owing to the physical fact that the introduction of magnetic field perpendicular to the fluid flow has a tendency to gives rise to a resistive-type force called the Lorentz force, which acts against the fluid flow and hence results in falling the velocity of the flow due to this type of magnetic pull of Lorentz force. The effect of Chemical reaction parameter (Ch) on the concentration field is shown in figure (10). From this it is seen that declining the concentration of the fluid for the increasing values of the Ch. From figure (4), it is noted that temperature of the fluid decreases as the value of Re increases.

7. CONCLUSIONS

The following **conclusions** have been drawn from the above results:

1. Temperature of the fluid reduces, due to the heat sink parameter and therefore velocity of its particles also decreases.
2. A growing radiation parameter reduces the temperature, velocity and Nusselt number.
3. An increase in chemical reaction parameter Ch leads decrease in the concentration of the fluid.

8. NOMENCLATURE

g-Acceleration due to gravity, β-Coefficient of volumetric thermal expansion, β^*-Coefficient of mass expansion, p^*-Pressure, ρ-Density, υ-Kinematics viscosity, μ-Viscosity, k-Thermal conductivity, C_p-Specific heat at constant pressure, D-Concentration diffusivity, C_w^*-Concentration of the plate, T_w^* - Temperature of the plate, T_∞^*-Temperature of the fluid far away from the plate, C_∞^*-Concentration of the fluid far away from the plate, *Gr*- Grashof number, Gm-Modified Grashof number, *Re*- Reynolds number, B_0 -Magnetic field component, Sc- Schmidt number, K_0-Permeability, *Q*-Volumetric rate of Heat absorption, M-Hartmann number, L-Wave length of the permeability, K_0^*-Mean permeability of the medium, S-Heat absorption parameter, Ch-Chemical reaction parameter, R-Radiation parameter

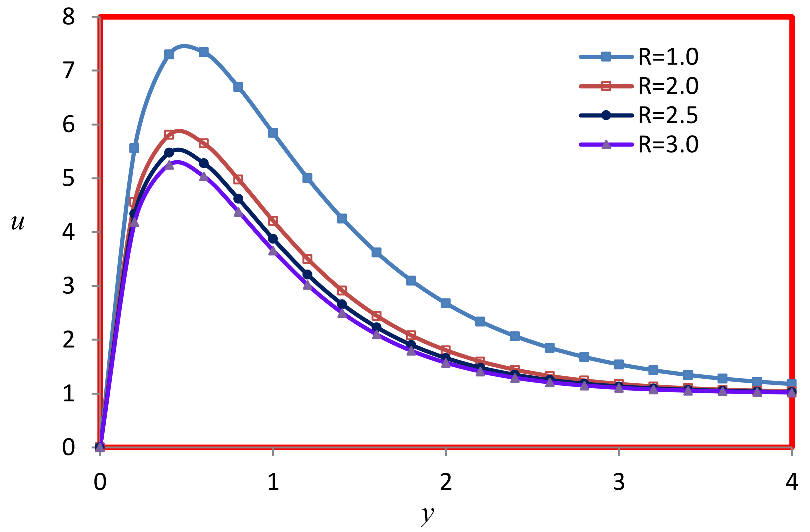

Fig.1-Effect of Radiation on velocity field u
(Gr=5.0, Gm=1.0, Re=3.0, Pr=0.71, S=0.5, Sc=0.66, Ko=1.0, ε=0.1and Z=0.0)

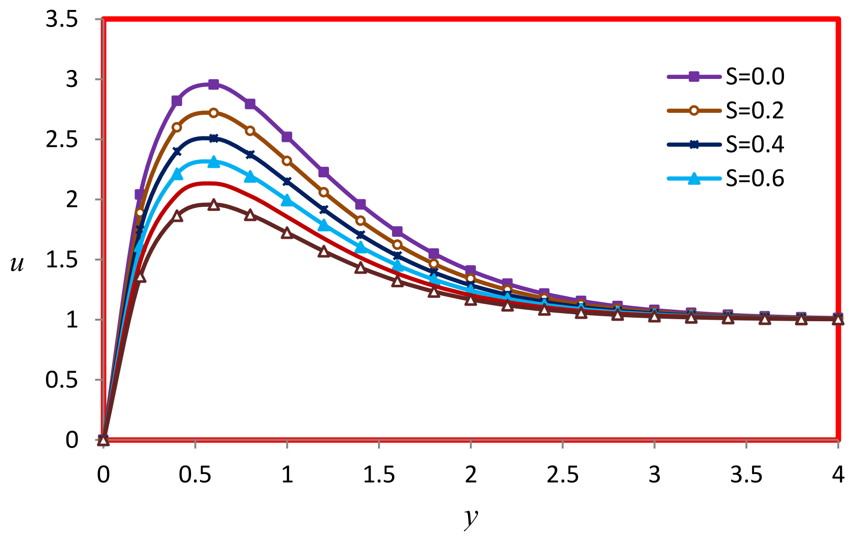

Fig.2-Effect of heat sink on velocity field u
(Gr=5.0, Gm=1.0, Ko=1.0, R=1.0, M=1.0, Re=3.0, Ch=0.5,Pr=0.71, Sc=0.66, ε=0.1 and Z=0.0)

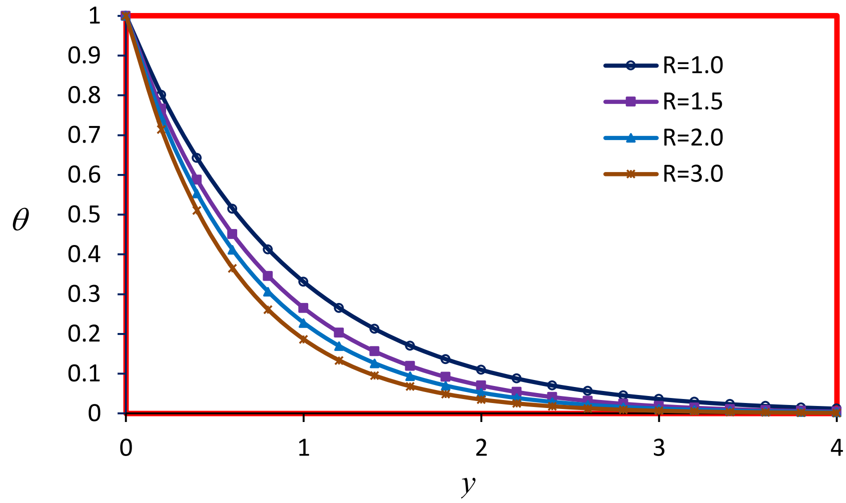

Fig.3-Effect of Radiation 'R' on Temperature field
(Pr=0.71, Re=3.0, Ko =1.0, M=1.0, ε=0.1 and Z=0.0)

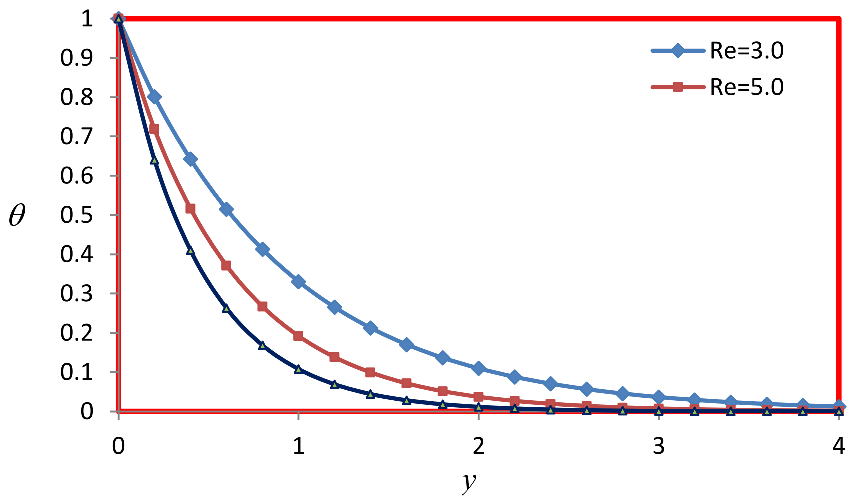

Fig.4-Effect of Reynolds number 'Re' on temperature field
(Pr=0.71, R=1.0, Ko =1.0, M=1.0, ε=0.1 and Z=0.0)

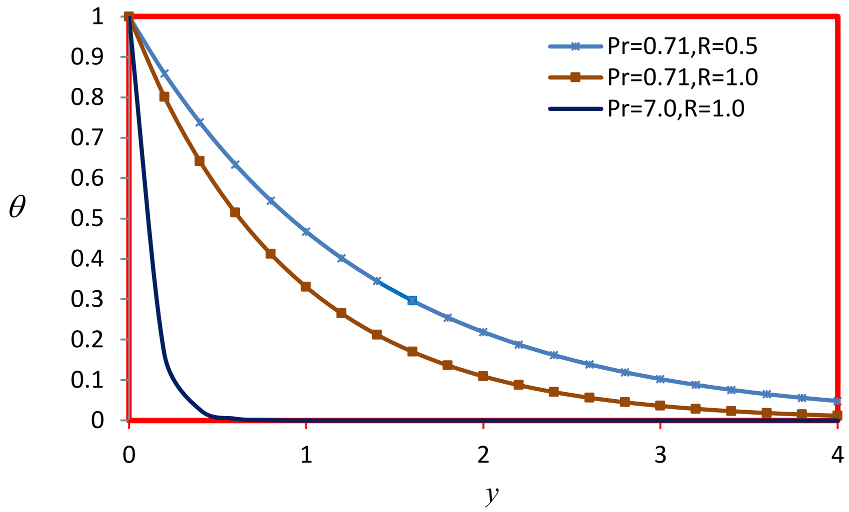

Fig.5-Effect of Prandtl number 'Pr' on temperature field
(Pr=0.71, Re=3.0, Ko =1.0, M=1.0, ε=0.1 and Z=0.0)

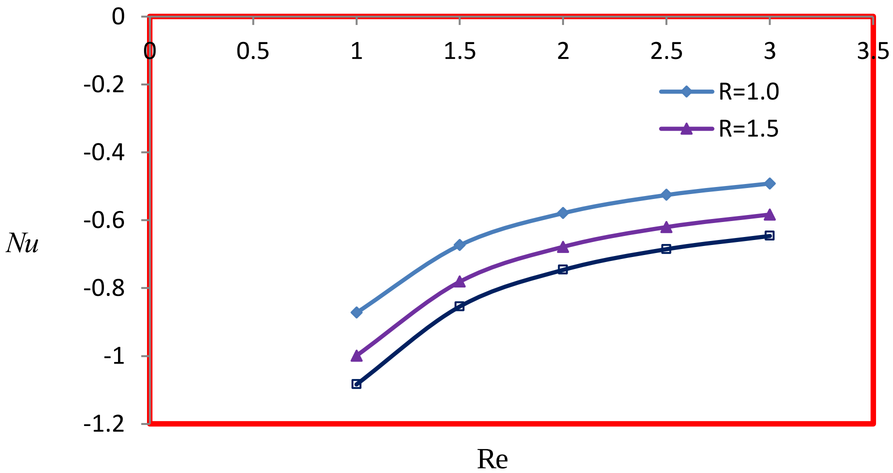

Fig 6-Nusselt number 'Nu' versus 'Re' in the presence of radiation
(Pr=0.71, Ko=1.0, M=1.0, ε=0.1 and Z=0.0)

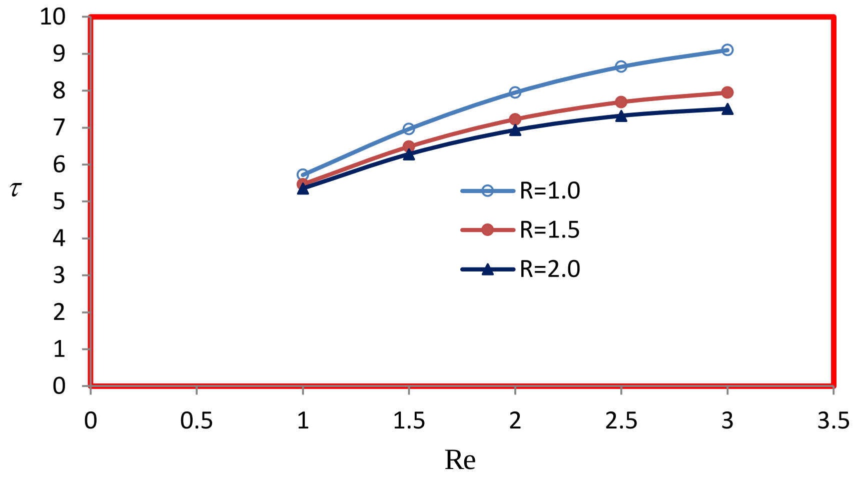

Fig 7- Skin-friction τ versus 'Re**' in the presence of radiation**
(Gr=5.0, Gm=5.0, Re=3.0, Ko=1.0, S=1.0, Pr=0.71, Sc=0.66, Ch=0.5, ε=0.1 and Z=0.0)

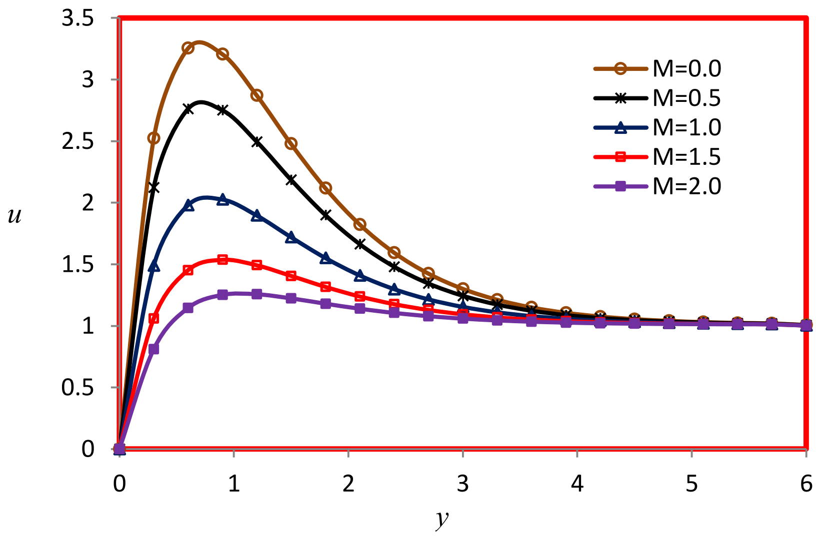

Fig.8-Effect of Hartman number 'M' on velocity field u
(Gr=5.0, Gm=1.0, Re=3.0, Ko=1.0, S=1.0, Pr=0.71, Sc=0.66, ε=0.1 and Z=0.0)

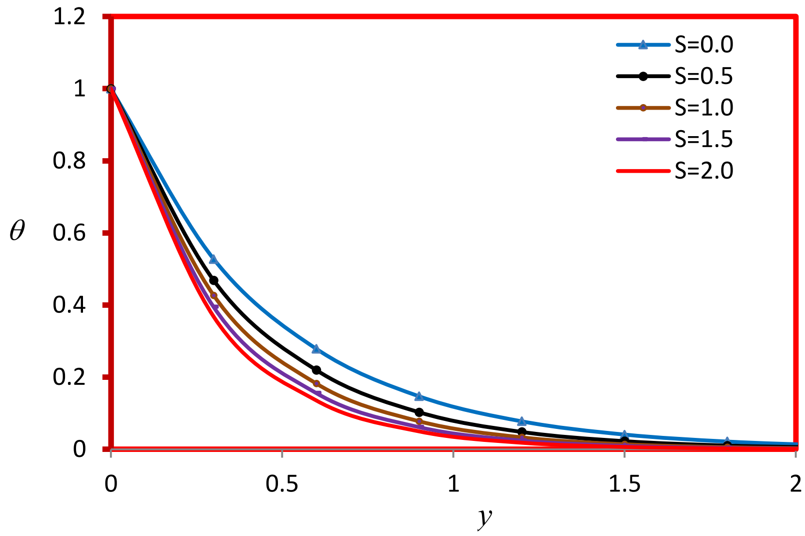

Fig.9-Effect of heat sink on temperature field
(Pr=0.71, Re=3.0, Ko =1.0, M=1.0, ε=0.1 and Z=0.0)

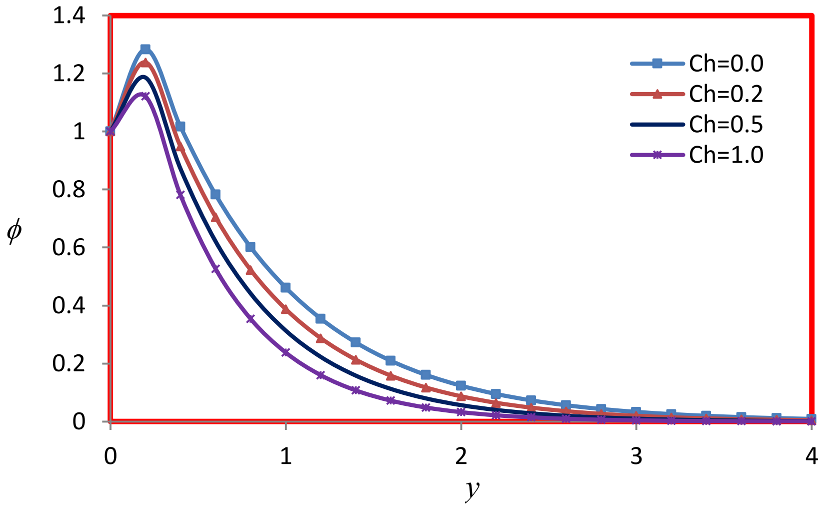

Fig 10-Effect of chemical reaction on Concentration field
(Pr=0.71, Sc=0.66, Re=2.0, Ko=1.0, M=1.0, ε=0.1 and Z=0.0)

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