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Fuzzy semi-regular subset of Fuzzy topological space

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Abstract

The intent of this paper is to study about some subspaces of a fuzzy topological space i.e. fuzzy semi-closed and fuzzy semi-regular sub-space, externally disconnected sub-space. We also obtain some properties of such spaces relative to the fuzzy topological space.

INTRODUCTION:

L.A. Zadeh[1] was the first Mathematician who invented fuzzy set and placed before us very interesting characteristics[1].

A fuzzy set is an universal set X is a mapping . The null fuzzy set O is a mapping from X to [0,1] which admits of the value O and the fuzzy set 1 is mapping from X to [0, 1] which admits of value 1 only.

A fuzzy set λ on X is called a fuzzy singleton if it takes the value (O) for a except one.

The point at which a fuzzy singleton takes the non zero value is called the support of the singleton 2.

A family , where I = [0, 1] is called a fuzzy topology for X if

$$\Im_1 : \tilde{O} \in \Im, \tilde{I} \in \Im$$

$$\Im_2 : \tilde{\lambda} \wedge \mu \in \Im \, \forall \lambda, \mu \in \Im$$

$$\Im_3 : \lambda \tilde{j} \in \Im \, \forall \lambda \tilde{j} \in \Im \, (j = 1, 2, 3, ...)$$

The pair is called a fuzzy topological space[3].

The members of are called -fuzzy open sets. A fuzzy set U is called $\Im$-fuzzy closedet if its complement $U \in \Im$

The closure $\mathrm{Int}\left(\tilde{\lambda}\right)$ and the interior

$\mathrm{Int}\left(\tilde{\lambda}\right)$ of a fuzzy set are defined by

$$\mathrm{Int}\left(\tilde{\lambda}\right) = \wedge\left\{\tilde{U} : \tilde{U}\ \text{i a fuzzy open subset of}\ \tilde{\lambda}\right\}$$

A fuzzy subset $\tilde{\lambda}$ of X is called fuzzy semi open if a fuzzy open set $\tilde{U}$ of X such that

$$\tilde{U} \le \lambda \le \mathrm{cl}\left(\tilde{U}\right)$$

Their $\tilde{\lambda}$ is called fuzzy semi closed [4,5] .

The semi closure $\left\lfloor \mathrm{Int}\left(\tilde{\lambda}\right) \right\rfloor_S$ and the semi interior $\left\lfloor \mathrm{Int}\left(\tilde{\lambda}\right) \right\rfloor_S$ of a fuzzy set $\tilde{\lambda}$ are defined[6] by

$$\left\lfloor \mathrm{cl}\left(\tilde{\lambda}\right) \right\rfloor_S = \wedge\left\{K : K\ \text{is a semi closed superset of}\ \tilde{\lambda}\right\}$$

$$\left\lfloor \mathrm{Int}\left(\tilde{\lambda}\right) \right\rfloor_S = \left\{K : K\ \text{is a semi open subset of}\ \tilde{\lambda}\right\}$$

A sub set of X is called fuzzy semi regular, if it is both fuzzy semi-open and fuzzy semi closed[7].

2. FUZZY SEMI CLOSED AND SEMI REGULAR SUB SPACE :

Definition (2.1):

A topological space $(X, \Im)$ is said to be fuzzy semi closed if corresponding to every cover $C=\{\lambda\alpha\ :\alpha\in\Delta\}$ by fuzzy semi open subsets of X, $\exists$ finite fuzzy subset $\lambda\alpha_0$ a such that

$$X=\{\vee[cl\lambda\alpha_0]_S:\alpha_0\in\Delta\}$$

A fuzzy subset $\tilde{\lambda}$ of X is called fuzzy semi closed relative to $(X,\Im)$ if for every cover $C=\{\lambda\alpha:\alpha\in\Delta\}$ by fuzzy semi open subsets of X, $\exists$ finite fuzzy subset $\lambda\alpha_0$ such that

$$X=\left\{\vee\left[cl\left(\lambda\tilde{\alpha}_0\right)\right]_S:\alpha_0\in\Delta\right\}$$

A fuzzy subset $\tilde{\lambda}$ of X is called fuzzy semi closed relative to $(X,\Im)$ if for every cover $C=\left\{\lambda\tilde{\alpha}:\alpha\in\Delta\right\}$ of by fuzzy semi open sets of X, $\exists$ a finite subset $\lambda\alpha_0$ such that

$$\tilde{\lambda}\le\vee\left\{\left[cl\left(\lambda\tilde{\alpha}_0\right)\right]_S:\alpha_0\in H\right\}$$

Definition (2.2):

A fuzzy topological space $(X,\Im)$ is said to be fuzzy semi-regular if for each fuzzy closed set U and a fuzzy point a pair of disjoint fuzzy semi open sets in such that a pair of disjoint fuzzy semi open sets in X such that .

Theorem (2.3):

A topological space $(X,\Im)$ is fuzzy semi-closed if every proper fuzzy semi-regular subset of X is fuzzy semi-closed relative to $(X,\Im)$.

Proof:

Let p be a proper fuzzy semi-regular subset of X. Let $\{\lambda\alpha:\alpha\in\Delta\}=C$ be a fuzzy cover p such that $\lambda\alpha$ is a member of fuzzy semi open subsets of X for each . Then X-p also fuzzy semi regular $\Rightarrow C\cup\left[1-\tilde{p}\right]$ forms a cover of .

Since X is fuzzy semi closed, $\exists$ a finite sub family such that,

$$X = \vee\left\{\left\lfloor cl\left(\lambda \tilde{\alpha}_0\right)\right\rfloor_S : \alpha_0 \in \Delta \cup \left(1 - \tilde{p}\right)\right\}$$

$$\Rightarrow p \le \vee\left\{\left\lfloor cl\left(\lambda \tilde{\alpha}_0\right)\right\rfloor_S ,\ \alpha_0 \in \Delta\right\}$$

Conversely let $C = \left\{\lambda \tilde{\alpha} : \alpha \in \Delta\right\}$ be a cover of X, where is a member of fuzzy semi open set $\forall \alpha \in \Delta$

$$p = \left\{\left\lfloor cl\left(\lambda \tilde{\alpha}_0\right)\right\rfloor \text{for some} \alpha_0 \in \Delta\right\}$$

Since p is a member of fuzzy semi-regular subsetr of X, so is 1-p and $1 - p \le \vee\{\lambda\alpha : \alpha \in \Delta\}$.

Since 1-p is fuzzy semi closed relative to X, $\exists$ a finite subset such that

$$X - p \le \vee\left\{\left\lfloor cl\left(\lambda \tilde{\alpha}_0\right)\right\rfloor_S , \alpha_0 \in \Delta\right\}$$

$$\Rightarrow X = \vee\left\{[cl(\lambda\alpha)]_S , \alpha \in \Delta \cup \lambda\alpha_0\right\}$$

$\Rightarrow$ X is fuzzy semi closed.

Remarks (2.4):

For a fuzzy subset λ of a space X, the following conditions are equivalent.

(i) λ is semi closed relative to X .

(ii) Every cover of by fuzzy semi open subsets of X has a finite sub cover.

(iii) Every cover of $\tilde{\lambda}$ by fuzzy semi regular subsets of X has a finite sub cover.

Theorem (2.5) :

Let $\tilde{\lambda}$ and $\tilde{\mu}$ be two fuzzy subsets of a space X such that $\tilde{\lambda} \le \tilde{\mu} \le X$, where $\tilde{\mu}$ is a fuzzy semi open subset, then if $\tilde{\lambda}$ be fuzzy semi closed relative to X, it is fuzzy semi closed relative to $\tilde{\mu}$ also.

Proof :

Let $X = \left\{ \lambda \tilde{\alpha} : \alpha \in \Delta \right\}$ be a cover of λ and λα be a fuzzy semi open subset of μ for all $\alpha \in \Delta$. Since is a fuzzy open subset of X, so is $\lambda\alpha \,\forall\, \alpha \in \Delta$.

Since λ is fuzzy semi closed relative to X, a finite sub family ;α_0 such that

$$\tilde{\lambda} \leq \vee \left\{ \left[cl(\lambda\alpha_0) \right]_X \alpha_0 \in \Delta \right\}$$

$$\Rightarrow \tilde{\lambda} \leq \vee \left\{ \left[cl(\lambda\alpha_0) \right]_X \cap \mu \right\}$$

Hence $\tilde{\lambda} \leq \vee \left\{ \left[cl(\lambda\alpha_0) \right]_\mu , \alpha_0 \in \Delta \right\}$

$\Rightarrow \tilde{\lambda}$ is fuzzy semi closed relative to sub space $\tilde{\mu}$.

3. DISCONNECTED AND SEMI HAUSDORFF SPACE :

Definition (3.1):

A fuzzy topological space is said to be extremely disconnected if $cl\left(\tilde{U}\right)$ is fuzzy open in X for every fuzzy[5] open set $\tilde{U}$ of X.

Remarks (3.2):

If x is an extremely disconnected fuzzy topological space and λ is fuzzy semi regular subset of X, then λ is fuzzy closed and fuzzy open in X.

Remarks (3.3):

A fuzzy open set of a space X is fuzzy semi-closed as sub space of X, iff it is fuzzy semi closed relative to X

Theorem (3.4):

An extremely disconnected fuzzy topological space X is fuzzy semi closed if every fuzzy semi-regular subset of X is a fuzzy semi closed sub-space of X.

Proof :

Let $C=\left\{\lambda\tilde{\alpha}:\alpha\Delta\right\}$ be a fuzzy cover of X, where $\lambda\alpha$ is a fuzzy semi open subset of $X\forall\alpha\in\Delta$.

Suppose that $1\neq\left\lfloor cl_X\left(\lambda\tilde{\beta}\right)\right\rfloor_S\neq0$

Since $\lambda\tilde{\beta}$ is a fuzzy semi open subset of X, so $\left\lfloor cl_X\left(\lambda\tilde{\beta}\right)\right\rfloor_S$ is a fuzzy semi regular subset of X.

$\Rightarrow 1-\left\lfloor cl_X\left(\lambda\tilde{\beta}\right)\right\rfloor_S$ is a fuzzy semi regular subset of X.

$\Rightarrow 1-\left\lfloor cl_X\left(\lambda\tilde{\beta}\right)\right\rfloor_S$ is a fuzzy semi closed subset and hence both fuzzy semi open and semi closed in X, by remark (3.2), so $1-\left\lfloor cl_X\left(\lambda\tilde{\beta}\right)\right\rfloor_S$ is fuzzy semi closed relative to X, be remarks (3.3),

$$\Rightarrow 1-\left\lfloor cl_X\left(\lambda\tilde{\beta}\right)\right\rfloor_S\leq\vee\left\{\lambda\tilde{\alpha}:\alpha\in\Delta\right\},\exists$$

a finite sub family $\lambda\alpha_0$ such that,

$$X=\vee\left\{\left\lfloor cl\left(\lambda\tilde{\alpha}\right)\right\rfloor_S\cup\lambda\tilde{\beta}\right\}$$

$$\Rightarrow X \text{ is fuzzy semi closed}.$$

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