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Heat and Mass Transfer on Flat Plate with Suction and Injection (Pr ≥ 1)

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Abstract

Governing Principle of Dissipative Process (GPDP) formulated by Gyarmati [1] into non equilibrium thermodynamics is applied to study the effects of heat transfer, two dimensional, laminar and constant property fluid flow in the boundary layer with suction and injection. The flow and temperature fields inside the boundary layer are approximated and the variational principle is formulated over the region of boundary layer. The Euler - Lagrange equations of the principle are obtained as polynomial equations in terms of momentum and thermal layer thicknesses. These equations are solvable for any given values of Prandtl number Pr and suction / injection parameter H. The obtained analytical solutions are compared with known numberical solutions (2), (3) and the comparison shows the fact that the accuracy is remarkable. In (10) Blasius flow on flat plate with suction and blowing was discussed.

1. Introduction

The effects of suction or injection in boundary layer flow are note worthy. Few practical examples, in nature and industrial processes are such as extursion of metals and plastics, cooling and drying of paper and textiles. Examples for Engineering applications are include heat exchanger, recovery of petroleum resources, fault zones, catalytic reactors, chemical reaction in a reactor chamber consisting of rectangular ducts, deposition of chemical vapor on surfaces and soon. Flow models in porous medium with applications in biological areas are such as diffusion in brain tissues, tissues generation process, blood flow in tumors, bio-heat transfer in tissues and bio-convection (4), (5), (6).

The present paper is concerned with the effect of uniform suction or injection on the flow of heat transfer characteristics of laminar boundary layers over a flat plate moving continuously in a quiescent ambient fluid. The analogous problem of boundary layers flow and heat transfer over a wedge with constant suction or injection has been treated recently by Wantanbe (7). Results were given for the velocity and temperature distributions, the coefficient of skin friction and Nussellt number for various values of the power law verification of the plate velocity, suction or injection parameter and different Prandtl number

2. Keywords

1. Prandtl Number (Pr) is a dimensionless number named after the German physicist "Ludwig Prandtl", defined as the ratio of momentum diffusivity to thermal diffusivity (8) i.e. Prandtl number is given as

$$\rho_r = \frac{\upsilon}{\propto} = \frac{\text{Viscons diffusoin rate}}{\text{Thermal diffusion rate}} = \frac{\frac{\mu}{e}}{K/_{Cp}\, e} = \frac{Cp\mu}{K}$$

2. Reynolds number $(R_{e1}) = \frac{VL}{\upsilon} = \text{Ratio of the inetia and Viscous forces}$

3. Velocity distribution is given by

$\frac{u}{U_\infty} = \frac{2y}{\propto} - \frac{2y^3}{\propto^3} + \frac{y^4}{\propto^4}$, where $\propto$ is the hydro dynamical boundary layer thickness.

4. Temperature distribution is given by

$\frac{T - T_\infty}{T_0 - T_\infty} = 1 - \frac{2y}{\beta} + \frac{2y^3}{\beta^3} - \frac{y^4}{\beta^4}$, where β is the thermal boundary layer thickness.

5. u, v are the velocity components along x and y axis.

6. T is the temperature

7. γ is Kinetic viscosity

8. LHTC - Local heat transfer coefficient.

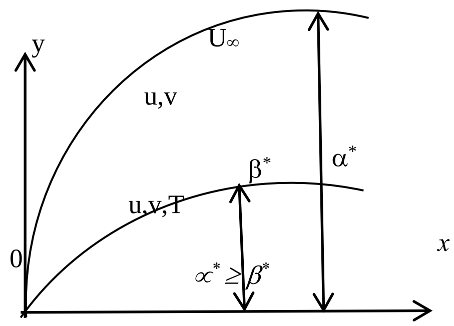
3. Mathematical modelling (P≥1) for heat and mass transfer

Calculation for the additional term υ_0 only due to injection and suction as in [10]

$$\frac{U}{U_\infty} = \frac{2y}{\alpha} - \frac{2y^3}{\alpha^3} + \frac{y^4}{\alpha^4}$$

$$\frac{T - T_\infty}{T_0 - T_\infty} = 1 - \frac{2y}{\beta} + \frac{2y^3}{\beta^3} - \frac{y^4}{\beta^4}$$

Boundary conditions are



$$y = 0, u = 0, \upsilon = \upsilon_0, T = T_0$$

$$y = \alpha,\ u = U_\infty, \frac{\partial u}{\partial y} = 0$$

$$y = \beta;\ T = T_\infty,\ \frac{\partial T}{\partial y} = 0$$

T_0 = constant, $\alpha = \alpha(x)$,

T_∞ = constant, $\beta = \beta(x)$,

U_∞ = constant

$\upsilon_0 = \upsilon_0(x)$,

$T_0 - T_\infty$ = constant

By governing boundary layer equation

$$\alpha\ \frac{\partial^2 T^*}{\partial y^2} = u\frac{\partial T}{\partial x} + \upsilon\frac{\partial T}{\partial y}$$

$$= \upsilon_0\ \frac{\partial T}{\partial y}$$

$$= \upsilon_0 (T - T_\infty)\left(-\frac{2}{\beta} + \frac{6y^2}{\beta^3} - \frac{4y^3}{\beta^4}\right)$$

$$\alpha\ \frac{\partial T^*}{\partial y} = \upsilon_0 (T_0 - T_\infty)\left(-\frac{2y}{\beta} + \frac{2y^3}{\beta^3} - \frac{y^4}{\beta^4}\right) + A$$

$$O = \upsilon_0 (T_0 - T_\infty)[-2 + 2 - 1] + A$$

$$\therefore A = \upsilon_0 (T_0 - T_\infty)$$

$$\alpha \frac{\partial T^*}{\partial y} = \upsilon_0 (T_0 - T_\infty)\left(1 - \frac{2y}{\beta} + \frac{2y^3}{\beta^3} - \frac{y^4}{\beta^4}\right)$$

$$T = (T_0 - T_\infty)\left(1 - \frac{2y}{\beta} + \frac{2y^3}{\beta^3} - \frac{y^4}{\beta^4}\right) + T_\infty$$

$$\frac{\partial T}{\partial y} = (T_0 - T_\infty)\left(-\frac{2}{\beta} + \frac{6y^2}{\beta^3} - \frac{4y^3}{\beta^4}\right) + 0$$

$$\int_0^\beta \left(\frac{\partial T}{\partial y}\right)\left(\frac{\partial T^*}{\partial y}\right) dy = \int_0^\beta \left[(T_0 - T_\infty)\left\{-\frac{2}{\beta} + \frac{6y^2}{\beta^3} - \frac{4y^3}{\beta^4}\right\}\right]\left[\frac{\upsilon_0 (T_0 - T_\infty}{\alpha}\left\{1 - \frac{2y}{\beta} + \frac{2y^3}{\beta^3} - \frac{y^4}{\beta^4}\right\}\right] dy$$

$$= \int_0^\beta \frac{\upsilon_0}{\alpha}(T_o - T_\infty)^2 \left\{-\frac{2}{\beta} + \frac{4y}{\beta^2} - \frac{4y^3}{\beta^4} + \frac{2y^4}{\beta^5} + \frac{6y^2}{\beta^3} - \frac{12y^3}{\beta^4} + \frac{12y^5}{\beta^6} - \frac{6y^6}{\beta^7} - \frac{4y^3}{\beta^4} + \frac{8y^4}{\beta^5} - \frac{8y^6}{\beta^7} + \frac{4y^7}{\beta^8}\right\} dy$$

$$= \frac{\upsilon_0}{\alpha}(T_o - T_\infty)^2 \int_0^\beta \left\{-\frac{2}{\beta} + \frac{4y}{\beta^2} - \frac{6y^3}{\beta^3} - \frac{20y^3}{\beta^4} + \frac{10y^4}{\beta^5} + \frac{12y^5}{\beta^6} - \frac{14y^6}{\beta^7} + \frac{4y^7}{\beta^8}\right\} dy$$

$$= \frac{\upsilon_0}{\alpha}(T_o - T_\infty)^2 \left[-\frac{2y}{\beta} + \frac{2y^2}{\beta^2} + \frac{2y^3}{\beta^3} - \frac{5y^4}{\beta^4} + \frac{2y^5}{\beta^5} + \frac{2y^6}{\beta^6} - \frac{2y^7}{\beta^7} + \frac{1}{2}\frac{y^8}{\beta^8}\right]_0^\beta$$

$$= \frac{\upsilon_0}{\alpha}(T_o - T_\infty)^2 \left[-2+2+2-5+2+2-2+\frac{1}{2}\right]$$

$$= -\frac{1}{2}\frac{\upsilon_0}{\alpha}(T_o - T_\infty)^2$$

$$\int_0^\beta \left(\frac{\partial T}{\partial y}\right)^2 dy = \int_0^\beta \left[(T_0 - T_\infty)\left(-\frac{2}{\beta} + \frac{6y^2}{\beta^3} - \frac{4y^3}{\beta^4}\right)\right]^2 dy$$

$$= (T_o - T_\infty)^2 \int_0^\beta \left[\frac{4}{\beta^2} - \frac{24y^2}{\beta^4} + \frac{16y^3}{\beta^5} + \frac{36y^4}{\beta^6} - \frac{48y^5}{\beta^7} + \frac{16y^6}{\beta^8}\right] dy$$

$$= (T_o - T_\infty)^2 \left[\frac{4y}{\beta^2} - \frac{8y^3}{\beta^4} + \frac{4y^4}{\beta^5} + \frac{36}{5}\frac{y^5}{\beta^6} - \frac{8y^6}{\beta^7} + \frac{16}{7}\frac{y^7}{\beta^8}\right]_0^\beta$$

$$= \frac{(T_o - T_\infty)^2}{\beta}\left[4-8+4+\frac{36}{5}-8+\frac{16}{7}\right]$$

$$= \frac{(T_o - T_\infty)^2}{\beta}\left(\frac{52}{35}\right)$$

$$= \frac{(T_o - T_\infty)^2}{\beta}(1.485714286)$$

$$\int_0^\beta\left(\frac{\partial T^*}{\partial y}\right)^2 dy = \int_0^\beta < \frac{\upsilon o^2 (T_0 - T_\infty)2}{\bar{\alpha}^2}\left\{1 - \frac{2y}{\beta} + \frac{2y^3}{\beta^3} - \frac{y^4}{\beta^4}\right\}^2_{(A)} + 2\upsilon_0\frac{(T_0 - T_\infty)}{\bar{\alpha}^2}\left(1 - \frac{2y}{\beta} + \frac{2y^3}{\beta^3} - \frac{y^4}{\beta^4}\right)$$

$$[U_\infty(T_0 - T_\infty)\beta^1\left\{-\frac{4}{15}\frac{\beta}{\alpha} + \frac{3}{35}\frac{\beta^3}{\alpha^3} - \frac{1}{36}\frac{\beta^4}{\alpha^4} + \frac{4}{3}\frac{y^3}{\alpha\beta^2} - \frac{12}{5}\frac{y^5}{\alpha\beta^4} + \frac{4}{3}\frac{y^6}{\alpha\beta^5} - \frac{4}{5}\frac{y^5}{\alpha^3\beta^2}\right.$$

$$\left.+\frac{12}{7}\frac{y^7}{\alpha^3\beta^4} - \frac{y^8}{\alpha^3\beta^5} + \frac{y^6}{3\alpha^4\beta^2} - \frac{3y^8}{4\alpha^4\beta^4} + \frac{4}{9}\frac{y^9}{\alpha^4\beta^5}\right\}_{(B)} + \alpha^1\left(\frac{2}{15}\frac{\beta^2}{\alpha^2} - \frac{9}{140}\frac{\beta^4}{\alpha^4} + \frac{1}{45}\frac{\beta^5}{\alpha^5} - \frac{2y^3}{3\alpha^2\beta}\right.$$

$$\left.\left.+\frac{6}{5}\frac{y^5}{\alpha^2\beta^3} - \frac{2y^6}{3\alpha^2\beta^4} + \frac{3}{5}\frac{y^5}{\alpha^4\beta} - \frac{9}{7}\frac{y^7}{\alpha^4\beta^3} + \frac{3}{4}\frac{y^8}{\alpha^4\beta^4} - \frac{4}{15}\frac{y^6}{\alpha^5\beta} + \frac{3}{5}\frac{y^8}{\alpha^5\beta^3} - \frac{16}{45}\frac{y^9}{\alpha^5\beta^4}\right)\right\}\Bigg]_{(c)} > dy$$

(A) $$= \int_0^\beta \upsilon_0^2\frac{(T_o - T_\infty)^2}{\bar{\alpha}^2}\left(1 + 4\frac{y^2}{\beta^2} + \frac{4y^6}{\beta^6} + \frac{y^8}{\beta^8} - \frac{4y}{\beta} - \frac{8y^4}{\beta^4} - \frac{4y^7}{\beta^7} + \frac{4y^3}{\beta^3} - \frac{2y^4}{\beta^4} + \frac{4y^5}{\beta^5}\right)dy$$

$$= \upsilon_0^2\frac{(T_o - T_\infty)^2}{\bar{\alpha}^2}\int_0^\beta\left(1 - 4\frac{y}{\beta} + \frac{4y^2}{\beta^2} + \frac{4y^3}{\beta^3} - \frac{10y^4}{\beta^4} + \frac{4y^5}{\beta^5} + \frac{4y^6}{\beta^6} - \frac{4y^7}{\beta^7} + \frac{y^8}{\beta^8}\right)dy$$

$$= \upsilon_0^2\frac{(T_o - T_\infty)^2}{\bar{\alpha}^2}\left(y - \frac{2y^2}{\beta} + \frac{4}{3} + \frac{y^3}{\beta^2} + \frac{y^4}{\beta^3} - \frac{2y^5}{\beta^4} + \frac{2}{3}\frac{y^6}{\beta^5} + \frac{4}{7}\frac{y^7}{\beta^6} - \frac{1}{2}\frac{y^8}{\beta^7} + \frac{1}{9}\frac{y^9}{\beta^8}\right)_0^\beta$$

$$= \upsilon_0^2\frac{(T_o - T_\infty)^2}{\bar{\alpha}^2}\beta\left(1 - 2 + + \frac{4}{3} + 1 - 2 + \frac{2}{3} + \frac{4}{7} - \frac{1}{2} + \frac{1}{9}\right)$$

$$= \upsilon_0^2\frac{(T_o - T_\infty)^2}{\bar{\alpha}^2}\beta N \text{ Where } N = -\frac{187}{126} + \frac{5}{3} = -\frac{187 + 210}{126} = \frac{23}{126} = 0.182539682$$

$$B = \int_0^\beta 2\upsilon_0\frac{(T_0 - T_\infty)^2}{\bar{\alpha}^2}U_\infty\beta^1\left(-\frac{4}{15}\frac{\beta}{\alpha} + \frac{3}{35}\frac{\beta^3}{\alpha^3} - \frac{1}{36}\frac{\beta^4}{\alpha^4} + \frac{4}{3}\frac{y^3}{\alpha\beta^2} - \frac{12}{5}\frac{y^5}{\alpha\beta^4} + \frac{4}{3}\frac{y^6}{\alpha\beta^5} - \frac{4}{5}\frac{y^5}{\alpha^3\beta^2}\right.$$

$$+\frac{12}{7}\frac{y^7}{\alpha^3\beta^4} - \frac{y^8}{\alpha^3\beta^5} + \frac{y^6}{3\alpha^4\beta^2} - \frac{3}{4}\frac{y^8}{\alpha^4\beta^4} + \frac{4}{9}\frac{y^9}{\alpha^4\beta^5} + \frac{8}{15}\frac{y}{\alpha} - \frac{6}{35}\frac{y\beta^2}{\alpha^3} + \frac{1}{18}\frac{y\beta^3}{\alpha^4} - \frac{8}{3}\frac{y^4}{\alpha\beta^3}$$

$$+\frac{24}{5}\frac{y^6}{\alpha\beta^5} - \frac{8}{3}\frac{y^7}{\alpha\beta^6} + \frac{8}{5}\frac{y^6}{\alpha^3\beta^3} - \frac{24}{7}\frac{y^8}{\alpha^3\beta^5} + \frac{2y^9}{\alpha^3\beta^6} - \frac{2y^7}{3\alpha^4\beta^3} + \frac{3}{2}\frac{y^9}{\alpha^4\beta^5} - \frac{8}{9}\frac{y^{10}}{\alpha^4\beta^6} - \frac{8}{15}\frac{y^3}{\beta^2\alpha}$$

$$+\frac{6}{35}\frac{y^3}{\alpha^3}-\frac{1}{18}\frac{y^3\beta}{\alpha^4}+\frac{8}{3}\frac{y^6}{\alpha\beta^5}-\frac{24}{5}\frac{y^8}{\alpha\beta^7}+\frac{8}{3}\frac{y^9}{\alpha\beta^8}-\frac{8}{5}\frac{y^8}{\alpha^3\beta^5}+\frac{24}{7}\frac{y^{10}}{\alpha^3\beta^7}-\frac{2y^{11}}{\alpha^3\beta^8}+\frac{2}{3}\frac{y^9}{\alpha^4\beta^5}$$

$$-\frac{3}{2}\frac{y^{11}}{\alpha^4\beta^7}+\frac{8}{9}\frac{y^{12}}{\alpha^4\beta^8}+\frac{4}{15}\frac{y^4}{\beta^3\alpha}-\frac{3}{35}\frac{y^4}{\alpha^3\beta}+\frac{1}{36}\frac{y^4}{\alpha^4}-\frac{4}{3}\frac{y^7}{\alpha\beta^6}+\frac{12}{5}\frac{y^9}{\alpha\beta^8}-\frac{4}{3}\frac{y^{10}}{\alpha\beta^9}+\frac{4}{5}\frac{y^9}{\alpha^3\beta^6}$$

$$\left.-\frac{12}{7}\frac{y^{11}}{\alpha^3\beta^8}+\frac{y^{12}}{\alpha^3\beta^9}-\frac{y^{10}}{3\alpha^4\beta^6}+\frac{3}{4}\frac{y^{12}}{\alpha^4\beta^8}-\frac{4}{9}\frac{y^{13}}{\alpha^4\beta^9}\right]dy$$

$$(B) = 2\upsilon_0\frac{(T_0-T_\infty)^2}{\bar{\alpha}^2}U\infty\beta^1\left[-\frac{4}{15}\frac{\beta y}{\alpha}+\frac{3}{35}\frac{\beta^3 y}{\alpha^3}-\frac{1}{36}\frac{\beta^4 y}{\alpha^4}+\frac{1}{3}\frac{y^4}{\alpha\beta^2}-\frac{2}{5}\frac{y^6}{\alpha\beta^4}+\frac{4}{21}\frac{y^7}{\alpha\beta^5}-\frac{2}{15}\frac{y^6}{\alpha^3\beta^2}\right.$$

$$+\frac{3}{14}\frac{y^8}{\alpha^3\beta^4}-\frac{1}{9}\frac{y^9}{\alpha^3\beta^5}+\frac{1}{21}\frac{y^7}{\alpha^4\beta^2}-\frac{1}{12}\frac{y^9}{\alpha^4\beta^4}+\frac{2}{45}\frac{y^{10}}{\alpha^4\beta^5}+\frac{4}{15}\frac{y^2}{\alpha}-\frac{3}{35}\frac{y^2\beta^2}{\alpha^3}+\frac{1}{36}\frac{y^2\beta^3}{\alpha^4}$$

$$-\frac{8}{15}\frac{y^5}{\alpha\beta^4}+\frac{24}{35}\frac{y^7}{\alpha\beta^5}-\frac{1}{3}\frac{y^8}{\alpha\beta^6}+\frac{8}{35}\frac{y^7}{\alpha^3\beta^3}-\frac{8}{21}\frac{y^9}{\alpha^3\beta^5}+\frac{1}{5}\frac{y^{10}}{\alpha^3\beta^6}-\frac{1}{12}\frac{y8}{\alpha^4\beta^3}+\frac{3}{20}\frac{y^{10}}{\alpha^4\beta^5}$$

$$-\frac{8}{99}\frac{y^{11}}{\alpha^4\beta^6}-\frac{2}{15}\frac{y^4}{\beta^2\alpha}+\frac{3}{70}\frac{y^4}{\alpha^3}-\frac{1}{72}\frac{y^4\beta}{\alpha^4}+\frac{8}{21}\frac{y^7}{\alpha\beta^5}-\frac{8}{15}\frac{y^9}{\alpha\beta^7}+\frac{4}{15}\frac{y^{10}}{\alpha\beta^8}-\frac{8}{45}\frac{y^9}{\alpha^3\beta^5}+\frac{24}{77}\frac{y^{11}}{\alpha^3\beta^7}$$

$$-\frac{1}{6}\frac{y^{12}}{\alpha^3\beta^8}+\frac{1}{15}\frac{y^{10}}{\alpha^4\beta^5}-\frac{1}{8}\frac{y^{12}}{\alpha^4\beta^7}+\frac{8}{117}\frac{y^{13}}{\alpha^4\beta^8}+\frac{4}{75}\frac{y^5}{\beta^3\alpha}-\frac{3}{175}\frac{y^5}{\alpha^3\beta}+\frac{1}{180}\frac{y^5}{\alpha^4}-\frac{1}{6}\frac{y^8}{\alpha\beta^6}$$

$$\left.+\frac{6}{25}\frac{y^{10}}{\alpha\beta^8}-\frac{4}{33}\frac{y^{11}}{\alpha\beta^9}+\frac{2}{25}\frac{y^{10}}{\alpha^3\beta^6}-\frac{1}{7}\frac{y^{12}}{\alpha^3\beta^8}+\frac{1}{13}\frac{y^{13}}{\alpha^3\beta^9}-\frac{1}{33}\frac{y^{11}}{\alpha^4\beta^6}+\frac{3}{52}\frac{y^{13}}{\alpha^4\beta^8}-\frac{2}{63}\frac{y^{14}}{\alpha^4\beta^9}\right]_0^\beta$$

$$(B) = 2\upsilon_0\frac{(T0-T\infty)^2}{\bar{\alpha}^2}U_\infty\beta\beta^1\left[-\frac{4}{15}\Delta+\frac{3}{35}\Delta^3-\frac{1}{36}\Delta^4+\frac{1}{3}\Delta-\frac{2}{5}\Delta+\frac{4}{21}\Delta-\frac{2}{15}\Delta^3+\frac{3}{14}\Delta^3-\frac{1}{9}\Delta^3+\frac{1}{21}\Delta^4\right.$$

$$-\frac{1}{12}\Delta^4+\frac{2}{45}\Delta^4+\frac{4}{15}\Delta-\frac{3}{35}\Delta^3+\frac{1}{36}\Delta^4-\frac{8}{15}\Delta+\frac{24}{35}\Delta-\frac{1}{3}\Delta+\frac{8}{35}\Delta^3-\frac{8}{21}\Delta^3+\frac{1}{5}\Delta^3-\frac{1}{12}\Delta^4+\frac{3}{20}\Delta^4$$

$$-\frac{8}{99}\Delta^4-\frac{2}{15}\Delta+\frac{3}{70}\Delta^3-\frac{1}{72}\Delta^4+\frac{8}{21}\Delta-\frac{8}{15}\Delta+\frac{4}{15}\Delta-\frac{8}{45}\Delta^3+\frac{24}{77}\Delta^3-\frac{1}{6}\Delta^3+\frac{1}{15}\Delta^4-\frac{1}{8}\Delta^4$$

$$+\frac{8}{117}\Delta^4+\frac{4}{75}\Delta-\frac{3}{175}\Delta^3+\frac{1}{180}\Delta^4-\frac{1}{6}\Delta+\frac{6}{25}\Delta-\frac{4}{33}\Delta+\frac{2}{25}\Delta^3-\frac{1}{7}\Delta^3+\frac{1}{13}\Delta^3-\frac{1}{33}\Delta^4$$

$$\left.+\frac{3}{52}\Delta^4-\frac{2}{63}\Delta^4\right]\text{Where }\Delta=\frac{\beta}{\alpha}$$

$$(B) = 2\upsilon_0 \frac{(T_0\text{-}T_\infty)^2}{\bar{\alpha}^2} U_\infty \beta\beta^1 \Big\lfloor \Delta\Big(-\frac{4}{15}+\frac{1}{3}-\frac{2}{5}+\frac{4}{21}+\frac{4}{15}-\frac{8}{15}+\frac{24}{35}-\frac{1}{3}-\frac{2}{15}+\frac{8}{21}-\frac{8}{15}+\frac{4}{15}$$

$$+\frac{4}{75}-\frac{1}{6}+\frac{6}{25}-\frac{4}{33}\Big)+\Delta^3\Big(\frac{3}{35}-\frac{2}{15}+\frac{3}{14}-\frac{1}{9}-\frac{3}{35}+\frac{8}{35}-\frac{8}{21}+\frac{1}{5}+\frac{3}{70}-\frac{8}{45}$$

$$+\frac{24}{77}-\frac{1}{6}-\frac{3}{175}+\frac{2}{25}-\frac{1}{7}+\frac{1}{13}\Big)+\Delta^4\Big(-\frac{1}{36}+\frac{1}{21}-\frac{1}{12}+\frac{2}{45}+\frac{1}{36}-\frac{1}{12}+\frac{3}{20}-\frac{8}{99}$$

$$-\frac{1}{72}+\frac{1}{15}-\frac{1}{8}+\frac{8}{117}+\frac{1}{180}-\frac{1}{33}+\frac{3}{52}-\frac{2}{63}\Big)$$

$$(B) = 2\frac{\upsilon_0(T_0-T_\infty)^2}{\bar{\alpha}^2} U_\infty\beta\beta^1 < -(0.070735211)\Delta + (0.024484405)\Delta^3 - (0.008058613)\Delta^4 >$$

$$(C) = \int_0^\beta 2\frac{\upsilon_0(T_0-T_\infty)^2}{\bar{\alpha}^2} U_\infty\alpha^1 \Big\{\Big[\Big(\frac{2}{15}\frac{\beta^2}{\alpha^2}-\frac{9}{140}\frac{\beta^4}{\alpha^4}+\frac{1}{45}\frac{\beta^5}{\alpha^5}-\frac{2}{3}\frac{y^3}{\alpha^2\beta}+\frac{6}{5}\frac{y^5}{\alpha^2\beta^3}-\frac{2y^6}{3\alpha^2\beta^4}+\frac{3}{5}\frac{y^5}{\alpha^4\beta}-\frac{9}{7}\frac{y^7}{\alpha^4\beta^3}$$

$$+\frac{3}{4}\frac{y^8}{\alpha^4\beta^4}-\frac{4}{15}\frac{y^6}{\alpha^5\beta}+\frac{3}{5}\frac{y^8}{\alpha^5\beta^3}-\frac{16}{45}\frac{y^9}{\alpha^5\beta^4}\Big]-\frac{2y}{\beta}\Big(\frac{2}{15}\frac{\beta^2}{\alpha^2}-\frac{9}{140}\frac{\beta^4}{\alpha^4}+\frac{1}{45}\frac{\beta^5}{\alpha^5}-\frac{2}{3}\frac{y^3}{\alpha^2\beta}+\frac{6}{5}\frac{y^5}{\alpha^2\beta^3}$$

$$-\frac{2}{3}\frac{y^6}{\alpha^2\beta^4}+\frac{3}{5}\frac{y^5}{\alpha^4\beta}-\frac{9}{7}\frac{y^7}{\alpha^4\beta^3}+\frac{3}{4}\frac{y^8}{\alpha^4\beta^4}-\frac{4}{15}\frac{y^6}{\alpha^5\beta}+\frac{3}{5}\frac{y^8}{\alpha^5\beta^3}-\frac{16}{45}\frac{y^9}{\alpha^5\beta^4}\Big]+\frac{2y^3}{\beta^3}\Big(\frac{2}{15}\frac{\beta^2}{\alpha^2}-\frac{9}{140}\frac{\beta^4}{\alpha^4}$$

$$+\frac{1}{45}\frac{\beta^5}{\alpha^5}-\frac{2}{3}\frac{y^3}{\alpha^2\beta}+\frac{6}{5}\frac{y^5}{\alpha^2\beta^3}-\frac{2}{3}\frac{y^6}{\alpha^2\beta^4}+\frac{3}{5}\frac{y^5}{\alpha^4\beta}-\frac{9}{7}\frac{y^7}{\alpha^4\beta^3}+\frac{3}{4}\frac{y^8}{\alpha^4\beta^4}-\frac{4}{15}\frac{y^6}{\alpha^5\beta}+\frac{3}{5}\frac{y^8}{\alpha^5\beta^3}$$

$$-\frac{16}{45}\frac{y^5}{\alpha^5\beta 4}\Big]-\frac{y^4}{\beta^4}\Big[\frac{2\beta^2}{15\alpha^2}-\frac{9}{140}\frac{\beta^4}{\alpha^4}+\frac{1}{45}\frac{\beta^5}{\alpha^5}-\frac{2}{3}\frac{y^3}{\alpha^2\beta}+\frac{6}{5}\frac{y^5}{\alpha^2\beta^3}-\frac{2}{3}\frac{y^6}{\alpha^2\beta^4}+\frac{3}{5}\frac{y^5}{\alpha^4\beta}$$

$$-\frac{9}{7}\frac{y^7}{\alpha^4\beta^3}+\frac{3}{4}\frac{y^8}{\alpha^4\beta^4}-\frac{4}{15}\frac{y^6}{\alpha^5\beta}+\frac{3}{5}\frac{y^8}{\alpha^5\beta^3}-\frac{16}{45}\frac{y^9}{\alpha^5\beta^4}\Big]\Big]\Big\}dy$$

$$(C) = \int_0^\beta 2\frac{\upsilon_0(T_0-T_\infty)^2}{\bar{\alpha}^2} U_\infty\alpha^1 \Big\{\Big[\Big(\frac{2}{15}\frac{\beta^2}{\alpha^2}-\frac{9}{140}\frac{\beta^4}{\alpha^4}+\frac{1}{45}\frac{\beta^6}{\alpha^5}-\frac{2}{3}\frac{y^3}{\alpha^2\beta}+\frac{6y^5}{5\alpha^2\beta^3}-\frac{2}{3}\frac{y^6}{\alpha^2\beta^4}+\frac{3}{5}\frac{y^5}{\alpha^4\beta}$$

$$-\frac{9}{7}\frac{y^7}{\alpha^4\beta^3}+\frac{3}{4}\frac{y^8}{\alpha^4\beta^4}-\frac{4}{15}\frac{y^6}{\alpha^5\beta}+\frac{3}{5}\frac{y^8}{\alpha^5\beta^3}-\frac{16}{45}\frac{y^9}{\alpha^5\beta^4}-\frac{4}{15}\frac{y\beta}{\alpha^2}+\frac{18}{140}\frac{y\beta^3}{\alpha^4}-\frac{2}{45}\frac{y\beta^4}{\alpha^5}$$

$$+\frac{4}{3}\frac{y^4}{\alpha^2\beta^2}-\frac{12}{5}\frac{y^6}{\alpha^2\beta^4}+\frac{4}{3}\frac{y^7}{\alpha^2\beta^5}-\frac{6}{5}\frac{y^6}{\alpha^4\beta^2}+\frac{18}{7}\frac{y^8}{\alpha^4\beta^4}-\frac{6}{4}\frac{y9}{\alpha^4\beta^5}+\frac{8}{15}\frac{y^7}{\alpha^5\beta^2}-\frac{6}{5}\frac{y^9}{\alpha^5\beta^4}+\frac{32}{45}\frac{Y^{10}}{\alpha^5\beta^5}$$

$$+\frac{4}{15}\frac{y^3}{\beta\alpha^2}-\frac{18}{140}\frac{y^3\beta^2}{\alpha^4}+\frac{2}{45}\frac{y^3\beta^2}{\alpha^5}-\frac{4}{3}\frac{y^6}{\alpha^2\beta^4}+\frac{12}{5}\frac{y^8}{\alpha^2\beta^6}-\frac{4}{3}\frac{y^9}{\alpha^2\beta^7}+\frac{6}{5}\frac{y8}{\alpha^4\beta^4}-\frac{18}{7}\frac{y^{10}}{\alpha^4\beta^6}$$

$$+\frac{6}{4}\frac{y^{11}}{\alpha^4\beta^7}-\frac{8}{15}\frac{y^9}{\alpha^5\beta^4}+\frac{6}{5}\frac{y^{11}}{\alpha^5\beta^6}-\frac{32}{45}\frac{y^{12}}{\alpha^5\beta^7}-\frac{2}{15}\frac{y^4}{\beta^2\alpha^2}+\frac{9}{140}\frac{y^4}{\alpha 4}-\frac{1}{45}\frac{y^4\beta}{\alpha^5}+\frac{2}{3}\frac{y^7}{\alpha^2\beta^5}$$

$$-\frac{6}{5}\frac{y9}{\alpha^2\beta^7}+\frac{2}{3}\frac{y^{10}}{\alpha^2\beta^8}-\frac{3}{5}\frac{y9}{\alpha^4\beta^5}+\frac{9}{7}\frac{y^{11}}{\alpha^4\beta^7}-\frac{3}{4}\frac{y^{12}}{\alpha^4\beta^8}+\frac{4}{15}\frac{y^{10}}{\alpha^5\beta^5}-\frac{3}{5}\frac{y^{12}}{\alpha^5\beta^7}+\frac{16}{45}\frac{y^{13}}{\alpha^5\beta^8}\Bigg]dy$$

$$(C)=2\frac{\upsilon_0(T_0-T_\infty)^2}{\alpha^{-2}}U_\infty\alpha^1\Bigg[\frac{2}{15}\frac{\beta^2}{\alpha^2}y-\frac{9}{140}\frac{\beta 4}{\alpha^4}y+\frac{1}{45}\frac{\beta^5}{\alpha^5}y-\frac{1}{6}\frac{y^4}{\alpha^2\beta}+\frac{1}{5}\frac{y^6}{\alpha^2\beta^3}-\frac{2}{21}\frac{y^7}{\alpha^2\beta^4}+\frac{1}{10}\frac{y^6}{\alpha^4\beta}$$

$$-\frac{9}{56}\frac{y^8}{\alpha^4\beta^3}+\frac{1}{12}\frac{y^9}{\alpha^4\beta^4}-\frac{4}{105}\frac{y^7}{\alpha^5\beta}+\frac{1}{15}\frac{y^9}{\alpha^5\beta^3}-\frac{8}{225}\frac{y^{10}}{\alpha^5\beta^4}-\frac{2}{15}\frac{y^2\beta}{\alpha^2}+\frac{9}{140}\frac{y^2\beta^3}{\alpha^4}-\frac{1}{45}\frac{y^2\beta^4}{\alpha^5}+\frac{4}{15}\frac{y^5}{\alpha^2\beta^2}$$

$$-\frac{12}{35}\frac{y^7}{\alpha^2\beta^4}+\frac{1}{6}\frac{y^8}{\alpha^2\beta^5}-\frac{6}{35}\frac{y^7}{\alpha^2\beta^2}+\frac{2}{7}\frac{y^9}{\alpha^4\beta^4}-\frac{3}{20}\frac{y^{10}}{\alpha^4\beta^5}+\frac{1}{15}\frac{y^8}{\alpha^5\beta^2}-\frac{3}{25}\frac{y^{10}}{\alpha^5\beta^4}+\frac{32}{495}\frac{y^{11}}{\alpha^5\beta^5}$$

$$+\frac{1}{15}\frac{y^4}{\beta\alpha^2}-\frac{9}{280}\frac{y^4\beta}{\alpha^4}+\frac{1}{90}\frac{y^4\beta^2}{\alpha^5}-\frac{4}{21}\frac{y^7}{\alpha^2\beta^4}+\frac{4}{15}\frac{y^9}{\alpha^2\beta^6}-\frac{2}{15}\frac{y^{10}}{\alpha^2\beta^7}+\frac{2}{15}\frac{y^9}{\alpha^4\beta^4}-\frac{18}{77}\frac{y^{11}}{\alpha^4\beta^6}$$

$$+\frac{1}{8}\frac{y^{12}}{\alpha^4\beta^7}-\frac{4}{75}\frac{y^{10}}{\alpha^5\beta^4}+\frac{1}{10}\frac{y^{12}}{\alpha^5\beta^6}-\frac{32}{585}\frac{y^{13}}{\alpha^5\beta^7}-\frac{2}{75}\frac{y^5}{\beta^2\alpha^2}+\frac{9}{700}\frac{y^5}{\alpha^4}-\frac{1}{225}\frac{y^5}{\alpha^5}\beta$$

$$+\frac{1}{12}\frac{y^8}{\alpha^2\beta^5}-\frac{3}{25}\frac{y^{10}}{\alpha^2\beta^7}+\frac{2}{33}\frac{y^{11}}{\alpha^2\beta^8}-\frac{3}{50}\frac{y^{10}}{\alpha^4\beta^5}+\frac{3}{28}\frac{y^{12}}{\alpha^4\beta^7}-\frac{3}{52}\frac{y^{13}}{\alpha^4\beta^8}+\frac{4}{165}\frac{y^{11}}{\alpha^5\beta^5}-\frac{3}{65}\frac{y^{13}}{\alpha^5\beta^7}+\frac{8}{315}\frac{y^{14}}{\alpha^5\beta^8}\Bigg]_0^\beta$$

$$(C)=2\frac{\upsilon_0(T_0-T_\infty)^2}{\alpha^{-2}}U_\infty\alpha\alpha^1\Bigg[\frac{2}{15}\Delta^3-\frac{9}{140}\Delta^5+\frac{1}{45}\Delta^6-\frac{1}{6}\Delta^3+\frac{1}{5}\Delta^3-\frac{2}{21}\Delta^3+\frac{1}{10}\Delta^5-\frac{9}{56}\Delta^5+\frac{1}{12}\Delta^5$$

$$-\frac{4}{105}\Delta^6+\frac{1}{15}\Delta^6-\frac{8}{225}\Delta^6-\frac{2}{15}\Delta^3+\frac{9}{140}\Delta^5-\frac{1}{45}\Delta^6+\frac{4}{21}\Delta^3-\frac{12}{35}\Delta^3+\frac{1}{6}\Delta^3-\frac{6}{35}\Delta^5$$

$$+\frac{2}{7}\Delta^5-\frac{3}{20}\Delta^5+\frac{1}{15}\Delta^6-\frac{3}{25}\Delta^6+\frac{32}{495}\Delta^6+\frac{1}{15}\Delta^3-\frac{9}{280}\Delta^5+\frac{1}{90}\Delta^6-\frac{4}{21}\Delta^3+\frac{4}{15}\Delta^3$$

$$-\frac{2}{15}\Delta^3+\frac{2}{15}\Delta^5-\frac{18}{77}\Delta^5+\frac{1}{8}\Delta^5-\frac{4}{75}\Delta^6+\frac{1}{10}\Delta^6-\frac{32}{585}\Delta^6-\frac{2}{75}\Delta^3+\frac{9}{700}\Delta^5-\frac{1}{225}\Delta^6$$

$$+\frac{1}{12}\Delta^3-\frac{3}{25}\Delta^3+\frac{2}{33}\Delta^3-\frac{3}{50}\Delta^5+\frac{3}{28}\Delta^5-\frac{3}{52}\Delta^5+\frac{4}{165}\Delta^6-\frac{3}{65}\Delta^6+\frac{8}{315}\Delta^6\Big]$$

$$(C)=2\frac{\upsilon_0(T_0-T_\infty)^2}{\bar{\alpha}^2}U_\infty\alpha\alpha^1\Big[\Delta^3\left(\frac{2}{15}-\frac{1}{6}+\frac{1}{5}-\frac{2}{21}-\frac{2}{15}+\frac{4}{15}-\frac{12}{35}+\frac{1}{6}+\frac{1}{15}-\frac{4}{21}+\frac{4}{15}-\frac{2}{15}-\frac{2}{75}+\frac{1}{12}+\frac{2}{33}-\frac{3}{25}\right)$$

$$+\Delta^5\left(-\frac{9}{140}+\frac{1}{10}-\frac{9}{56}+\frac{1}{12}+\frac{9}{140}-\frac{6}{35}+\frac{2}{7}-\frac{3}{20}-\frac{9}{280}+\frac{2}{15}-\frac{18}{77}+\frac{1}{8}+\frac{9}{700}-\frac{3}{50}+\frac{3}{28}-\frac{3}{52}\right)$$

$$+\Delta^6\left(\frac{1}{45}-\frac{4}{105}+\frac{1}{15}-\frac{8}{225}-\frac{1}{45}+\frac{1}{15}-\frac{3}{25}+\frac{32}{495}+\frac{1}{90}-\frac{4}{75}+\frac{1}{10}-\frac{32}{585}-\frac{1}{225}-\frac{4}{165}-\frac{3}{65}+\frac{8}{315}\right)$$

$$(C)=2\upsilon_0\frac{(T_0-T_\infty)^2}{\bar{\alpha}^2}U_\infty\alpha\alpha^1<(0.035367965)\Delta^3-(0.018363302)\Delta^5+(0.006446886)\Delta^6>$$

$$=2\upsilon_0\frac{(T_0-T_\infty)^2}{\bar{\alpha}^2}U_\infty\alpha\alpha^1<q_1\Delta^3+q_2\Delta^5+q_3\Delta^6>$$

Where $q_1 = 0.035367965$

$q_2 = -0.018363302$

$q_3 = 0.006446886$

$$\delta\int_0^\beta\left(\frac{\partial T^*}{\partial y}\right)^2 dy=\upsilon_0^2\frac{(T_0-T_\infty)^2}{\bar{\alpha}^2}\beta N+2\upsilon_0\frac{(T_0-T_\infty)^2}{\bar{\alpha}^2}U_\infty[\beta\beta^1<P_1\Delta+P_2\Delta^3+P_3\Delta^4>$$
$$+\alpha\alpha^1<q_1\Delta^3++q_2\Delta^5+q_3\Delta^6>]$$

Where $N = 0.182539682$, $P_1 = -0.070735211$, $q_1 = 0.035367965$

$P_2 = 0.024484405$, $q_2 = -0.0818363302$

$P_3 = -0.008058613$, $q_3 = 0.006446886$

Variational principle (for additional term υ_0 only

$$\delta\int_{y=0}^{\beta}\int_{x=0}^{L}\left[\frac{\partial T}{\partial y}\frac{\partial T^*}{\partial y}-\frac{1}{2}\left(\frac{\partial T}{\partial y}\right)^2-\frac{1}{2}\left(\frac{\partial T^*}{\partial y}\right)^2\right]dxdy=0$$

$$\delta\int_{x=0}^{L} < -\frac{1}{2}\frac{\upsilon_0}{\alpha}(T_0 - T_\infty)^2 - \frac{1}{2}\frac{(T_0 - T_\infty)^2}{\beta}(1.485714286) - \frac{1}{2}\left\{\upsilon_0^2\frac{(T_0 - T_\infty)^2}{\overset{-2}{\alpha}}\beta N\right.$$

$$+2\upsilon_0\frac{(T_0 - T_\infty)^2}{\overset{-2}{\alpha}}U_\infty\left(\beta\beta^1(P_1\Delta + P_2\Delta^3 + P_3\Delta^4\right) + \alpha\alpha^1(q_1\Delta^3 + q_2\Delta^5 + q_3\Delta^6)]\} > dx = 0$$

$$\delta\int_{x=0}^{L} < -\frac{1}{2}\frac{\upsilon_0}{\alpha}(T_0 - T_\infty)^2 - \frac{1}{2}\frac{(T_0 - T_\infty)^2}{\beta}(1.485714286) - \frac{1}{2}\upsilon_0^2\frac{(T_0 - T_\infty)^2}{\overset{-2}{\alpha}}\beta(0.182539682)$$

$$\frac{-\upsilon_0(T_0 - T_\infty)^2}{\overset{-2}{\alpha}}U_\infty\left[\beta\beta^1\left(-0.070735211\Delta + 0.024484405\Delta^3 - 0.008058613\Delta^4\right)\right.$$
$$+\alpha\alpha^1(0.035367965\Delta^3 - 0.018363302\Delta^5 + 0.006446886\Delta^6)] > dx = 0$$

<u>GPDP variational principle (due to injection and suction)</u> :

$$\delta\int_{x=0}^{L} < -\frac{1}{2}\upsilon_0\frac{\rho}{\gamma}(T_0 - T_\infty)^2 - \frac{1}{2}\frac{(T_0 - T_\infty)^2}{\beta}(1.485714286) - \frac{1}{2}\upsilon_0^2(T_0 - T_\infty)^2\frac{\rho^2}{\gamma^2}\beta(0.182539682)$$

$$-\upsilon_0(T_0 - T_\infty)^2\frac{\rho^2}{\gamma^2}U_\infty\left[\beta\beta^1\left(-0.070735211\Delta + 0.024484405\Delta^3 - 0.008058613\Delta^4\right)\right.$$
$$+\alpha\alpha^1(0.035367965\Delta^3 - 0.018363302\Delta^5 + 0.006446886\Delta^6)]$$

$$+\rho(T_0 - T_\infty)^2\frac{U_\infty^2}{\gamma}\left[\alpha^1\left(-0.1063492\Delta^2 + 0.05757576\Delta^4 - 0.02046842\Delta^5\right)\right.$$
$$+\beta^1(0.2126984\Delta - 0.07676768\Delta^3 + 0.02558553\Delta^4)]$$

$$-\rho^2(T_0 - T_\infty)^2\frac{U_\infty^2}{2\gamma^2}\left[\alpha^{1^2}\beta\left(0.007636807\Delta^4 - 0.008296465\Delta^6 + 0.002949939\Delta^7\right)\right.$$
$$+0.002312761\Delta^8 - 0.001659019\Delta^9 + 0.0002984636\Delta^{10})]$$
$$+\beta^{1^2}\beta(0.03054724\Delta^2 - 0.02212391\Delta^4 + 0.007374847\Delta^5$$
$$+0.004111575\Delta^6 - 0.002765032\Delta^7 + 0.0004663494\Delta^8)$$
$$+\alpha^1\beta^1\beta(-0.03054724\Delta^3 + 0.02765488\Delta^5 - 0.009587301\Delta^6$$
$$-0.006167362\Delta^7 + 0.00428584\Delta^3 - 0.000746159\Delta^9)] > dx = 0$$

Where $\frac{1}{\alpha} = \frac{\rho}{\gamma}$

$$\alpha = \alpha^* \sqrt{\frac{\gamma x}{U_\infty}}, \qquad \beta = \beta^* \sqrt{\frac{\gamma x}{U_\infty}}, \qquad \Delta = \frac{\beta}{\alpha} = \frac{\beta^*}{\alpha^*}$$

$$\alpha^1 = \frac{\alpha^*}{2}\sqrt{\frac{\gamma}{U_\infty x^*}}, \qquad \beta^1 = \frac{\beta^*}{2}\sqrt{\frac{\gamma}{U_\infty x}}, \qquad H = \frac{\upsilon_0 \sqrt{Rx}}{U_\infty}$$

$$\alpha\alpha^1 = \frac{\alpha^{*2}}{2}\frac{\gamma}{U_\infty} \qquad \beta\beta^1 = \frac{\beta^{*2}}{2}\frac{\gamma}{U_\infty} \qquad H = \frac{\upsilon_0}{U_\infty}\frac{\sqrt{U_\infty x}}{\gamma}$$

$$\upsilon_0 = HU_\infty \sqrt{\frac{\gamma}{U_\infty x}}$$

Euler - Lagrange Equation

$$\frac{\partial L}{\partial \beta^x} = 0$$

$$\partial \int_{x=0}^{L} < -\frac{1}{2} H \; U\infty \sqrt{\frac{\gamma}{U_\infty x}}\frac{\rho}{\gamma}(T_0 - T_\infty)^2 - \frac{1}{2}(T_0 - T_\infty)^2 \frac{1}{\beta^*}\sqrt{\frac{U_\infty}{\gamma x}}(1.485714286)$$

$$-\frac{1}{2} H^2 U_\infty^2 \frac{\gamma}{U_\infty x}(T_0 - T_\infty)^2 \frac{\rho^2}{\gamma^2}\beta^* \sqrt{\frac{\gamma x}{U_\infty}}(0.182539682)$$

$$-HU_\infty \sqrt{\frac{\gamma}{U_\infty x}}(T_0 - T_\infty)^2 \frac{\rho^2}{\gamma^2} U_\infty \left[\frac{\beta^{*2}}{2}\frac{\gamma}{U_\infty}(-0.070735211\Delta + 0.024484405\Delta^3 - 0.008058613\Delta^4)\right]$$

$$\frac{\alpha^{*2}}{2}\frac{\gamma}{U_\infty}(0.035367965\Delta^3 - 0.018363302\Delta^5 + 0.006446886\Delta^6)\Big]$$

$$+\rho(T_0 - T_\infty)^2 \frac{U_\infty}{\gamma}\left|\frac{\alpha^*}{2}\sqrt{\frac{\gamma}{U_\infty x}}(-0.1063492\Delta^2 + 0.05757576\Delta^4 - 0.02046842\Delta^5)\right.$$

$$+\frac{\beta^*}{2}\sqrt{\frac{\gamma}{U_\infty x}}(0.2126984\Delta - 0.07676768\Delta^3 + 0.02558553\Delta^4)\Big]$$

$$-\rho^2(T_0 - T_\infty)^2 \frac{U_\infty^2}{2\gamma^2}\left|\frac{\alpha^{*2}}{4}\cdot\frac{\gamma}{U_\infty x}\beta^* \sqrt{\frac{\gamma x}{U_\infty}}(0.007636807\Delta^4 - 0.008296465\Delta^6 + 0.002949939\Delta^7\right.$$

$$+0.002312761\Delta^8 - 0.001659019\Delta^9 + 0.0002984636\Delta^{10})$$

$$+\frac{\beta^{*2}}{4}\frac{\gamma}{U_\infty x}\beta^* \sqrt{\frac{\gamma x}{U_\infty}}(0.03054724\Delta^2 - 0.02212391\Delta^4 + 0.007374847\Delta^5$$

$$+0.004111575\Delta^6 - 0.002765032\Delta^7 + 0.0004663494\Delta^8)$$

$$+\frac{\alpha^*}{2}\sqrt{\frac{\gamma}{U_\infty x}}\square\frac{\beta^*}{2}\sqrt{\frac{\gamma}{U_\infty x}}\square\beta^*\sqrt{\frac{\gamma x}{U_\infty}}(-0.03054724\Delta^3+0.02765488\Delta^5-0.009587301\Delta^6$$
$$-0.006167362\Delta^7+0.0042858\Delta^8-0.000746159\Delta^9)] > dx = 0$$

$$\text{Where } \Delta = \frac{\beta^*}{\alpha^*}$$

$$\delta\int_{x=0}^{L} < \frac{(T_0-T_\infty)^2}{2}\sqrt{\frac{U_\infty}{\gamma x}}\Big\{-HP-\frac{1}{\beta^*}(1.485714286)-H^2\rho^2\beta^*(0.182539682)$$

$$-H\rho^2\Big[\beta^{*2}(-0.070735211\Delta+0.024484405\Delta^3-0.008058613\Delta^4)$$
$$+\alpha^{*2}(0.035367965\Delta^3-0.018363302\Delta^5+0.006446886\Delta^6\Big]$$

$$+\rho\Big[\alpha^*(-0.1063492\Delta^2+0.05757576\Delta^4-0.02046842\Delta^5)$$
$$+\beta^*(0.2126984\Delta-0.07676768\Delta^3+0.02558553\Delta^4)\Big]$$

$$-\rho^2\Big[\frac{\alpha^{*2}\beta^*}{4}(0.007636807\Delta^4-0.008296465\Delta^6+0.002949939\Delta^7$$
$$+0.002312761\Delta^8-0.001659019\Delta^9+0.0002984636\Delta^{10})$$
$$+\frac{\beta^{*3}}{4}(0.03054724\Delta^2-0.02212391\Delta^4+0.007374847\Delta^5$$
$$+0.004111575\Delta^6-0.002765032\Delta^7+0.0004663494\Delta^8)$$

$$+\frac{\alpha^*\beta^{*2}}{4}(-0.03054724\Delta^3 0+0.02765488\Delta^5-0.009587301\Delta^6$$
$$-0.006167362\Delta^7+0.0042858\Delta^8-0.000746159\Delta^9)\Big]\Big\} > dx = 0$$

$$\delta\int_{x=0}^{L} < -H\rho-\frac{1}{\beta^*}(1.485714286)-H^2\rho^2\beta^*(0.182539682)$$

$$-H\rho^2\Big[-0.070735211\frac{\beta^{*3}}{\alpha^*}+0.024484405\frac{\beta^{*5}}{\alpha^{*3}}-0.008058613\frac{\beta^{*6}}{\alpha^{*4}}$$
$$+0.035367965\frac{\beta^{*3}}{\alpha^*}-0.018363302\frac{\beta^{*5}}{\alpha^{*3}}+0.006446886\frac{\beta^{*6}}{\alpha^{*4}}\Big]$$

$$+\rho[-0.1063492\frac{\beta^{*2}}{\alpha^{*}}+0.05757576\frac{\beta^{*4}}{\alpha^{*3}}-0.02046842\frac{\beta^{*5}}{\alpha^{*4}}$$

$$+0.2126984\frac{\beta^{*2}}{\alpha^{*}}-0.07676768\frac{\beta^{*4}}{\alpha^{*3}}+0.02558553\frac{\beta^{*5}}{\alpha^{*4}}\Bigg]$$

$$-\frac{\rho^{2}}{4}[0.007636807\frac{\beta^{*5}}{\alpha^{*2}}-0.008296465\frac{\beta^{*7}}{\alpha^{*4}}+0.002949939\frac{\beta^{*8}}{\alpha^{*5}}$$

$$+0.002312761\frac{\beta^{*9}}{\alpha^{*6}}-0.001659019\frac{\beta^{*10}}{\alpha^{*7}}+0.0002984636\frac{\beta^{*11}}{\alpha^{*8}}$$

$$+0.03054724\frac{\beta^{*5}}{\alpha^{*2}}-0.02212391\frac{\beta^{*7}}{\alpha^{*4}}+0.007374847\frac{\beta^{*8}}{\alpha^{*5}}$$

$$+0.004111575\frac{\beta^{*9}}{\alpha^{*6}}-0.002765032\frac{\beta^{*10}}{\alpha^{*7}}+0.0004663494\frac{\beta^{*11}}{\alpha^{*8}}$$

$$-0.03054724\frac{\beta^{*5}}{\alpha^{*2}}+0.02765488\frac{\beta^{*7}}{\alpha^{*4}}-0.009587301\frac{\beta^{*8}}{\alpha^{*5}}$$

$$-0.006167362\frac{\beta^{*9}}{\alpha^{*6}}+0.0042858\frac{\beta^{*10}}{\alpha^{*7}}-0.000746159\frac{\beta^{*11}}{\alpha^{*8}}\Bigg]>dx=0$$

$$\delta\int_{x=0}^{L}<-H\rho-\frac{1}{\beta^{*}}(1.485714286)-H^{2}\rho^{2}\beta^{*}(0.182539682)$$

$$-H\rho^{2}[-0.035367246\frac{\beta^{*3}}{\alpha^{*}}+0.006121103\frac{\beta^{*5}}{\alpha^{*3}}-0.001611727\frac{\beta^{*6}}{\alpha^{*7}}\Bigg]$$

$$+\rho[+0.1063492\frac{\beta^{*2}}{\alpha^{*}}-0.01919192\frac{\beta^{*4}}{\alpha^{*3}}+0.00511711\frac{\beta^{*5}}{\alpha^{*4}}\Bigg]$$

$$-\frac{\rho^2}{4}\left[0.007636807\frac{\beta^{*5}}{\alpha^{*2}}-0.002765495\frac{\beta^{*7}}{\alpha^{*4}}+0.000737485\frac{\beta^{*8}}{\alpha^{*5}}\right.$$

$$\left.+0.000256974\frac{\beta^{*9}}{\alpha^{*6}}-0.000138251\frac{\beta^{*10}}{\alpha^{*7}}+0.000018653\frac{\beta^{*11}}{\alpha^{*8}}\right]>dx=0$$

$$\frac{\partial L}{\partial \beta^*}=\frac{1}{\beta^{*2}}(1.4857142867 - H^2\rho^2(0.182539682)$$

$$-H\rho^2\left(0.106101738\frac{\beta^{*2}}{\alpha^*}+0.030605515\frac{\beta^{*4}}{\alpha^{*3}}-0.009670362\frac{\beta^{*5}}{\alpha^{*4}}\right)$$

$$+\rho\left(0.2126984\frac{\beta^*}{\alpha^*}-0.07676768\frac{\beta^{*3}}{\alpha^{*3}}+0.02558555\frac{\beta^{*4}}{\alpha^{*4}}\right)$$

$$-\frac{\rho^2}{4}\left(0.038184035\frac{\beta^{*4}}{\alpha^{*2}}-0.019358465\frac{\beta^{*6}}{\alpha^{*4}}+0.00589988\frac{\beta^{*7}}{\alpha^{*5}}\right.$$

$$\left.+0.002312766\frac{\beta^{*8}}{\alpha^{*6}}-0.00138251\frac{\beta^{*9}}{\alpha^{*7}}+0.000205183\frac{\beta^{*10}}{\alpha^{*8}}\right)=0$$

Suction or Injection case :

$$1.485714286-H^2\rho^2\beta^{*2}(0.182539682)$$

$$-H\rho^2\beta^{*3}(-0.106101738\Delta+0.030605515\Delta^3-0.009670362\Delta^4)$$

$$+\rho\beta^{*2}(0.2126984\Delta-0.07676768\Delta^3+0.0255855\Delta^4)$$

$$-\frac{\rho^2}{4}\beta^{*4}(0.038184035\Delta^2-0.019358465\Delta^4+0.00589988\Delta^5)$$

$$+0.002312766\Delta^6-0.00138251\Delta^7+0.000205183\Delta^8)=0,(P\geq 1)$$

$$1.485714286-H^2\rho^2\beta^{*2}(0.182539682)$$

$$-H\rho^2\beta^{*3}\left(-0.106101738\frac{\beta^*}{\alpha^*}+0.030605515\frac{\beta^{*3}}{\alpha^{*3}}-0.009670362\frac{\beta^{*4}}{\alpha^{*4}}\right)$$

$$+\rho\beta^{*2}\left(0.2126984\frac{\beta^{*}}{\alpha^{*}} - 0.07676768\frac{\beta^{*3}}{\alpha^{*3}} + 0.0255855\frac{\beta^{*4}}{\alpha^{*4}}\right)$$

$$-\frac{\rho^2}{4}\beta^{*4}\left(0.038184035\frac{\beta^{*2}}{\alpha^{*2}} - 0.019358465\frac{\beta^{*4}}{\alpha^{*4}} + 0.00589988\frac{\beta^{*5}}{\alpha^{*5}}\right.$$

$$\left. +0.002312766\frac{\beta^{*6}}{\alpha^{*6}} - 0.00138251\frac{\beta^{*7}}{\alpha^{*7}} + 0.000205183\frac{\beta^{*8}}{\alpha^{*8}}\right) = 0$$

$$1.485714286\alpha^{*8} - H^2\rho^2\beta^{*2}\alpha^{*8}(0.182539682)$$

$$+H\rho^2\beta^{*4}\alpha^{*7}(-0.106101738) - H\rho^2\beta^{*6}\alpha^{*5}(0.030605515) + H\rho^2\beta^{*7}\alpha^{*4}(0.009670362)$$

$$+\rho\beta^{*3}\alpha^{*7}(0.2126984) - \rho\beta^{*5}\alpha^{*5}(0.07676768) + \rho\beta^{*6}\alpha^{*4}(0.0255855)$$

$$-\rho^2\beta^{*6}\alpha^{*6}(0.0095460087) + \rho\beta^{*8}\alpha^{*4}(0.0048396162) - \rho^2\beta^{*9}\alpha^{*3}(0.00147497)$$

$$-\rho^2\beta^{*10}\alpha^{*2}(0.0005781915) + \rho^2\beta^{*11}\alpha^{*}(0.0003456275) - \rho^2\beta^{*12}(0.00005129575) = 0$$

Removing $-\frac{1}{2}$ throughout

$$H^2\rho^2\beta^{*2}\alpha^{*8}(0.091269841) - H\rho^2\beta^{*4}\alpha^{*7}(0.053050869) + H\rho^2\beta^{*6}\alpha^{*5}(0.015302757)$$

$$-H\rho^2\beta^{*7}\alpha^{*4}(0.004835181) + \rho^2\beta^{*12}(0.0000256475) - \rho^2\beta^{*11}\alpha^{*}(0.0001728135)$$

$$+\rho^2\beta^{*10}\alpha^{*2}(0.0002890955) + \rho^2\beta^{*9}\alpha^{*3}(0.000737485) - \rho^2\beta^{*8}\alpha^{*4}(0.002419808)$$

$$+\rho^2\beta^{*6}\alpha^{*6}(0.004773004) - \rho\beta^{*6}\alpha^{*4}(0.01279275) + \rho\beta^{*5}\alpha^{*5}(0.03838384)$$

$$-\rho\beta^{*3}\alpha^{*7}(0.1063492) - 0.742857143\alpha^{*8} = 0$$

Multiplying throughout by 10000

$$H^2\rho^2\beta^{*2}\alpha^{*8}(912.69841) - H\rho^2\beta^{*4}\alpha^{*7}(530.50869) + H\rho^2\beta^{*6}\alpha^{*5}(153.02757)$$

$$-H\rho^2\beta^{*7}\alpha^{*4}(48.35181) + \rho^2\beta^{*12}(0.256475) - \rho^2\beta^{*11}\alpha^{*}(1.728135)$$

$$+\rho^2\beta^{*10}\alpha^{*2}(2.890955) + \rho^2\beta^{*9}\alpha^{*3}(7.37485) - \rho^2\beta^{*8}\alpha^{*4}(24.19808)$$

$$+\rho^2\beta^{*6}\alpha^{*6}(47.73004)-\rho\beta^{*6}\alpha^{*4}(127.9275)+\rho\beta^{*5}\alpha^{*5}(383.8384)$$
$$-\rho\beta^{*3}\alpha^{*7}(1063.492)-7428.57143\alpha^{*8}=0,\ \ \rho\geq 1$$

When H = 0

$$\rho^2\beta^{*12}(0.256475)-\rho^2\beta^{*11}\alpha^{*}(1.728135)+\rho^2\beta^{*10}\alpha^{*2}(2.890955)$$

$$+\rho^2\beta^{*9}\alpha^{*3}(7.37485)-\rho^2\beta^{*8}\alpha^{*4}(24.19808)+\rho^2\beta^{*6}\alpha^{*6}(47.73004)$$

$$-\rho\beta^{*6}\alpha^{*4}(127.9275)+\rho\beta^{*5}\alpha^{*5}(383.8384)-\rho\beta^{*3}\alpha^{*7}(1063.492)$$

$$-7428.57143\alpha^{*8}=0,\ (\rho\geq 1)$$

Verification purpose

When H = 0, Actual calculated accepted value of α^* = 5.595119, present value of α^* = 5.590312. A comparative study of actual calculated accepted value of β^* and present value of β^* with different Prandtl numbers is given below .

Pr	Calculated values of β^*	Present values of β^*
1	5.648534	5.647215
10	2.525947	2.525252
100	1.162451	1.162114
1000	0.53852	0.5383499
10000	0.2498518	0.2497456

4. Result and Discussion

It is observed the velocity along the plate increases which intern decreases thickness of the momentum boundary layer for both slip and no slip conditions.

H>O shows the suction and H<O shows the blowing. For H>O, fluid velocity increases as the fluid particles are sucked in the porous wall, which intern reduces both the fluid boundary layer and the thickness of the momentum boundary layer. In short we can summaries that due to suction behavior, the velocity increases and the fluid and thickness of momentum of boundary layer decreases. On the other hand for the case of blowing i.e. H<O, the opposite trend is observed.

When suction is increased H>O, that refers to bring the fluid close to the wall; this causes a decrease in temperature profile and also decreases the thermal boundary layer. This entire phenomenon causes an increase in the rate of heat transfer. An opposite trend can be seen for the case of blowing H<O.

The increase in suction H parameter causes a decrease in the concentration of fluid. An opposite trend can be seen for the case of blowing H<O.

The decrease in concentration causes the increase in the mass transfer from the fluid to the porous medium. For the values of H from 0.10 to 0.60, hydrodynamial boundary layer thickness $(\propto^*)$, thermal boundary layer thickness (β^*) and local heat transfer coefficient (LHTC) values are given in Appendix A for various values of Prandtl numbers, 1,10,100,1000 and 10,000.

5. CONCLUSION

The aim of the present paper was to study boundary layers over a permeable and isothermal surface that moves continuously in its plane. Heat transfer coefficient are computed for wide range of suction or injection parameters and prandtl number and the surface forced velocity.

It was shown that the effect of suction or injection on both velocity profile and temperature distribution is significant. The thermal boundary layer thickness is substantially affected by changes in Pr. When H=O, for the prandtl number 100, the present computed local heat transfer coefficient is - 2.115418 which is nearly equal to the accepted LHTC of - 2.2620584.

Finally, it should be emphasized that the principal aim of the paper was to investigate the effects of a wall mass transfer parameter on boundary layers occurring on a continuous moving surface by using a well known calculation technique. The agreement of the present results with those of an impermeable surface (similarity solutions) reported by Banks [9] is good.

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APPENDIX -A

Table I (Pr = 1)

H	$\propto^*$	β^*	**LHTC**
0.10	6.24597500	6.23542800	-3.66941100
0.20	6.95423900	6.87485800	-4.38730300
0.30	7.70938200	7.56203200	-5.25828600
0.40	8.50563000	8.29273400	-6.29610800
0.50	9.33761600	9.06245800	-7.51267900
0.60	10.20059000	9.86675800	-8.91810800

Table II (Pr = 10)

H	$\propto^*$	β^*	**LHTC**
0.10	6.24597500	3.37703000	-5.50646900
0.20	6.95423900	4.27476400	-10.01991000
0.30	7.70938200	5.19686700	-16.56021000
0.40	8.50563000	0.78264770	3.97286300
0.50	9.33761600	0.59259670	4.99534000
0.60	10.20059000	0.48468250	6.00045700

Table III (Pr = 100)

H	$\propto^*$	β^*	**LHTC**
0.10	6.24597500	0.30850120	10.03879000
0.20	6.95423900	0.14370160	20.01406000
0.30	7.70938200	4.76080100	-121.55800000
0.40	8.50563000	2.68765700	15.86197000
0.50	9.33761600	0.05707811	50.00199000
0.60	10.20059000	0.04755735	60.0012700

Table VI (Pr = 1000)

H	**$\propto^*$**	**β^***	**LHTC**
0.10	6.24597500	2.41706200	-130.29440000
0.20	6.95423900	-0.01426514	200.00150000
0.30	7.70938200	0.00951010	300.00030000
0.40	8.50563000	0.00713236	399.99980000
0.50	9.33761600	0.00570586	499.99960000
0.60	10.20059000	0.00475507	599.99940000

Table V (Pr = 10000)

H	**$\propto^*$**	**β^***	**LHTC**
0.10	6.24597500	0.00285343	999.99580000
0.20	6.95423900	0.00142855	1999.99500000
0.30	7.70938200	0.00095101	2999.99500000
0.40	8.50563000	0.00071432	3999.99400000
0.50	9.33761600	0.00057073	4999.99300000
0.60	10.20059000	0.00047987	5999.99300000