Global Journal of Mathematical Sciences: Theory and Practical.
ISSN 0974-3200 Volume 10, Number 1 (2018), pp. 31-35

http://www.irphouse.com

Fuzzy Weakly Compact Continuous Functions

S. Daniel [1] and Dr.M. Jayaraman [2]

*[1]Research Scholar, J.J. College of Arts and Science,
Pudukkottai – 622422, Tamil Nadu, India.*

*[2]Assistant Professor, P.G. and Research Department of Mathematics,
Raja Doraisingam Government Arts College, Sivaganga – 630561,
Tamil Nadu, India.*

Definition :1.1

Throughout this paper X and Y represents fuzzy topological spaces on which no separation axioms are assumed unless explicitly stated. Let S be a fuzzy subset of a space X. The fuzzy closure and the fuzzy interior of S in X are denoted by $Fcl_x(S)$ and $Fint_x(S)$ or simply Fcl(S) and Fint(S) respectively. A fuzzy subset S is said to be fuzzy regular open (resp. fuzzy regular closed) if Fint (Fcl(S)) = S (resp. Fcl (Fint (S)) = S).

Definition : 2.1

An fuzzy open cover { V_α / $\alpha \in \nabla$ } of a fuzzy topological space X is said to be fuzzy regular if for each $\alpha \in \nabla$ there exists a nonempty regular closed set F_α in X, such that F_α & V_α and

X = U { Fint (F_α) / $\alpha \in \nabla$ }.

Definition : 2.2

A fuzzy topological space X is said to be fuzzy weakly compact (resp. fuzzy quasi H-closed) if every fuzzy regular (resp. fuzzy open) cover of X has a fuzzy finite subfamily whose fuzzy closure is a fuzzy cover of X.

Definition : 2.3

A fuzzy topological space X is said to be fuzzy almost- regular if for each fuzzy regular closed set F of X and each point x ∈ X - F there exist disjoint fuzzy open sets U and V, such that F & U and $x \in X$.

Definition : 2.4

A fuzzy subset K of a fuzzy topological space X is said to be fuzzy weakly compact relative to X if for each fuzzy cover $\{ V_\alpha / \alpha \in \nabla \}$ of K by fuzzy open set of X satisfying the following properties.

(i) There exists a finite fuzzy subset ∇_o of ∇, such that $K \,\square\, U\{ Fcl\,(V_\alpha) / \alpha \in \nabla_o \}$.

(ii) For each $\alpha \in \nabla$, V_α contains a nonempty fuzzy regular closed set F_α of x and

K & $U\{ Fint\,(F_\alpha) / \alpha \in \nabla \}$.

Definition : 2.5

Let $\mathcal{F}$ be a fuzzy filter on a space X. A point $x \in X$ is called a γ-fuzzy adherence point of $\mathcal{F}$ if $\mathcal{F}/\mathcal{U}(\overline{\mathcal{U}}_x) \neq 0$.

Definition : 2.6

Let A be a fuzzy subset of a space X. A point $x \in X$ is called a γ-fuzzy adherence point of A if $A \cap V \neq \emptyset$ for every $V \in \mathcal{U}(\overline{U}_x)$. The fuzzy set of all γ-fuzzy adherence points of A is called the γ-fuzzy closure of A. If A contains the γ-fuzzy closure of A then it is called γ-fuzzy closed.

Fuzzy sets Fuzzy Weakly Compact Relative to a Space .

Definition : 3.1

A fuzzy filter $\mathcal{F}$ on a space X is said to be fuzzy quasi-regular if there exists an fuzzy open filter ζ on X, such that $\mathcal{F} = \mathcal{U}(\bar{\zeta})$.

Theorem : 3.2

For a fuzzy subset A of a space X the following are equivalent.

(1) A is fuzzy weakly compact relative to X.

(2) Every fuzzy open filter ζ with $tr_A\zeta \neq 0$ has a γ-fuzzy adherence point in A.

(3) Every fuzzy filter $\bar{\zeta}$ such that ζ is an fuzzy open filter and $tr_A\zeta \neq 0$ has an γ-fuzzy adherence point in A.

(4) Every fuzzy quasi-regular filter $\mathcal{F} = \mathcal{U}(\bar{\zeta})$ such that $tr_A\zeta \neq 0$ has an fuzzy adherence point in A.

(5) Every fuzzy filter $\bar{\mathcal{F}}$ such that Ƒ is a fuzzy quasi-regular filter Ƒ = $\mathcal{U}$ ($\bar{\zeta}$) with $tr_A\zeta \neq 0$ has an fuzzy adherence point in A.

(6) Every fuzzy filter Ƒ such that Ƒ is a fuzzy quasi-regular filter Ƒ = $\mathcal{U}$ ($\bar{\zeta}$) with $tr_A\zeta \neq 0$ has an fuzzy adherence point in A.

(7) Every fuzzy open ultra filter ζ with $tr_A\zeta \neq 0$ γ-fuzzy converges.

(8) Let { C_α / α∈ ∇ } be a family of fuzzy closed set of X, such that for each α∈ ∇ ther exists an fuzzy open set $\wedge_\alpha$ *of* X satisfying C_α & A_α and ∩{ Fcl (A_α) / α∈ ∇}& X-A. Then there exists a finite fuzzy subset ∇_o of ∇ such that ∩{ Fint (C_α) / α∈ ∇_o}& X-A.

Proof : (1) ⟹ (2)

Let ζ be an fuzzy open filter on X with $tr_A\zeta \neq 0$. Suppose that ζ ∧ $\mathcal{U}$ ($\bar{\mathcal{U}}_x$) = 0 for every x ∈ A. Then there exists fuzzy open sets G_x∈ ζ , U_x∈ $\mathcal{U}_x$ and $\wedge_x$ ∈ $\mathcal{U}$ ($\bar{\mathcal{U}}_x$), such that G_x ∩ A_x = φ and U_x & Fcl (U_x) & $\wedge_x$. By G_x ∩ A_x = φ, we obtain

Fcl (G_x) ∩ $\wedge_x$ = φ and hence Fcl (G_x) ∩ Fcl (U_x) = φ.

Let us put B_x = X – Fcl (G_x), then Fcl (U_x) & B_x and B_x ∈ $\mathcal{U}$ ($\bar{\mathcal{U}}_x$) .

The family { B_x / x∈A } is a fuzzy cover of A by fuzzy open set of X and

A □ U{ Fint (Fcl ($\mathcal{U}_x$)) / x∈A }. Therefore , there exists a finite number of points x_1 , x_2 … x_n in A such that, A □U{ Fcl (B_{x_i}) / i = 1,2,…n} , Therefore we have ,

∩ { X – Fcl (B_{x_i}) / i = 1,2,… n } & X – A ⟶ (I)

For each i = 1,2,…n , G_{x_i}& Fint (Fcl (G_{x_i})) , Hence we have

X – Fcl (B_{x_i}) = Fint (X - B_{x_i})

= Fint (Fcl (G_{x_i})) ∈ ζ .

Therefore by (I) we obtain X – A ∈ ζ . This is a contradiction .

(2) ⟹(3) ⟹ (4) ⟹ (5) ⟹ (6) = (4) ⟹ (7) ⟹ (1)

<u>(4) ⟹ (8)</u> :

Let I(∇) be the family of all finite fuzzy subsets of ∇. Suppose that ,

∩{ Fint (C_α) / α∈ ∇ }∉ X – A for every ∇∈ I(∇).

Then $\mathcal{F}$= { $\bigcap_{\alpha\in\nabla} F$ int (C_α) / ∇∈ I(∇) } is an fuzzy open filter base with tr_A $\mathcal{F} \neq 0$. Thus $\mathcal{U}(\overline{\mathcal{F}})$ is a fuzzy quasi- regular filter on X, such that tr ≠ 0. By (4), there exists a point x∈A such that $\mathcal{U}(\overline{\mathcal{F}})$ / $\mathcal{U}_x$ ≠ 0. Put $\mathcal{P}$ = { $\bigcap_{\alpha\in\nabla}\wedge_\alpha$/ ∇∈ I(∇)}, then it is an fuzzy open filter base such that $\mathcal{U}(\overline{\mathcal{F}})$ & $\mathcal{P}$.

Therefore $\mathcal{P}$ ∧ $\mathcal{U}_x$ ≠ 0, and hence x ∈ Fcl ($\wedge_\alpha$) for every α∈ ∇.

Thus we obtain x∈∩{ Fcl (A_α) / α∈ ∇ }. This is a contradiction because

$\cap\{$ Fcl (A_α) / $\alpha\in\nabla$ }& X – A .

<u>(8)⟹(1) :</u>

Let { $\wedge_\alpha$/ $\alpha\in\nabla$ } be an fuzzy open cover of A with property (i). For each $\alpha\in\nabla$ there exists a nonempty fuzzy regular closed set C_α such that C_α & A_α and

A & $\cup\{$ Fint (C_α) / $\alpha\in\nabla\}$. We consider the family {X - A_α / $\alpha\in\nabla$} of fuzzy closed sets. For each $\alpha\in\nabla$, X - C_α is fuzzy open in X, X - C_α □ X - A_α and $\cap\{$ Fcl (X - C_α) / $\alpha\in\nabla$ }

X - $\cup\{$ Fint (C_α) / $\alpha\in\nabla\}$ & X – A , by (8) there exists a finite fuzzy subset ∇_o of ∇ such that

$\cap$ { Fint (X $-$ $\wedge_\alpha$ / $\alpha\in\nabla_o$} & X – A , Therefore we obtain A & $\cup\{$ Fcl (A $_\alpha$) / $\alpha\in\nabla_o\}$. This shows that A is fuzzy weakly compact relative to X.

Fuzzy Weakly Compact Continuous Functions.

Definition : 4.1

A function f : X → Y is said to be fuzzy weakly compact continuous if for each x∈ X and each fuzzy open neighbourhood V of f(x) having the complement fuzzy weakly compact relative to Y, there exists an fuzzy open neighbourhood U of X such that f (U) & V (i.e FWCC – Fuzzy Weakly Compact Continuous Functions).

Theorem : 4.2

For a function f : X → Y the following are equivalent.

(1) f is FWC – continuous.

(2) If V is fuzzy open in Y and Y – V is fuzzy weakly compact relative to Y, then $f^{-1}(V)$ is fuzzy open in X.

(3) If F is fuzzy closed in Y and fuzzy weakly compact relative to Y, then $f^{-1}(F)$ is fuzzy closed in X.

Proof : <u>(1)⟹(2)</u>

Let V be an fuzzy open set of Y. such that Y – V is fuzzy weakly compact relative to Y. Let

x ∈ $f^{-1}(V)$. Then f (x) ∈ V and there exists an fuzzy open neighbourhood U of x such that

f(U) & V. Therefore we have x ∈ ∪ & $f^{-1}(V)$.

This shows that $f^{-1}(V)$ is fuzzy open in X.

<u>(2) ⟹ (3)</u>

This is obvious.

(3)⟹ (1) :

Let $x \in X$ and V an fuzzy open neighbourhood of f(x) such that Y -V is fuzzy weakly compact relative to Y. By (3), $f^{-1}(Y - V)$ is fuzzy closed in X and hence

$U = f^{-1}(V)$ is an fuzzy open set containing x such that f (U) & V.

Lemma : 4.3

If A_1 and A_2 are fuzzy weakly compact relative to a space X, then $A_1 \cup A_2$ is fuzzy weakly compact relative to X.

Proof :

Let $\mathcal{V} = \{ V_\alpha / \alpha \in \nabla \}$ be a fuzzy cover of $A_1 \cup A_2$ by fuzzy open sets of X satisfying property (P). Then $\mathcal{V}$ is a fuzzy cover of A_1, A_2 satisfying (P), and hence for each i = 1,2 there exists a fuzzy finite subset ∇_i of ∇ such that A_i & $\cup\{ Fcl (V_\alpha) / \alpha \in \nabla_i \}$.

Therefore, we have $A_1 \cup A_2$ & $\cup\{ Fcl (A_\alpha) / \alpha \in \nabla_1 \cup \nabla_2 \}$.

This shows that $A_1 \cup A_2$ is fuzzy weakly compact relative to X.

REFERENCES

[1] K.K. Azad, On fuzzy semi continuous, fuzzy almost continuous, and fuzzy weakly continuous, J. Math. Anal. Appl. 82(1981), 14-32.

[2] N. Biswas, on Characterizations of Semi Continuous functions, Abti, Accad, Na2, Lincei Rend Cl.Sci Fis Mat Natur(8) 48, 399-402 (1970)

[3] Hu Cheng-Ming, A class of fuzzy topological space I, Neimenggu Daxue Xuabao I (1980), 101

[4] sR. Lowen, Fuzzy topological spaces and fuzzy compactness, J. Math, Appl. 56(1976), 621-633.