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# The Stress-Intensity Factors for Two Exterior Griffith-Cracks in an Orthotropic Stress-Free Infinite Strip Opened by Body Forces

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**Abstract**

The exact expressions for stress-intensity factors at crack tips the and crack shape are obtained by using finite and integral Fourier transforms method while cracks are opened by symmetrical system of body forces in an infinite stress-free orthotropic strip. It is found that normal stress components possess Cauchy type singularity at crack tips while displacement is smooth.

**Keywords:** [1] Stress Intensity Factors (S.I.F.), [2] Crack-Opening Displacement (C.O.D.), [3] Body Forces, [4] Fourier Transform, [5] Orthotropy.

## 1. INTRODUCTION

Now–a-days composite materials are replacing the natural found materials from use. It is found analytically [1] that composite materials can be assumed as orthotropic continuum. The body forces in the medium are simulated by rivets or stiffeners used in structures.

Sneddon and Tweed [2, 3] and Tweed [4, 5] had solved the problems of crack opening in isotropic infinite medium due to body forces. Parihar and Kushwaha [6] had extended to rigidly lubricated strip. Kushwaha [7] extended to rectangular domain. Singh et.at. [8] had extended the problem to stress-free infinite orthotropic strip for an interior Griffith crack. Kushwaha and Jha [9] extended the problem of [8] to two interior Griffith cracks.

It is difficult to make interior cracks and then perform experiment. Therefore it is the need for further work. In the present research endeavour it is done for two similar exterior Griffith-cracks. It is easy to make exterior cracks by wedging.

Physically the cracks occupy the region $y = 0$, $b < |x| \leq a$, while the strip is of width $2a$ and the cracks lie over $x$-axis. $y$-axis passes through the middle of the strip. The cracks are of length $(a-b)$, see figure 1.

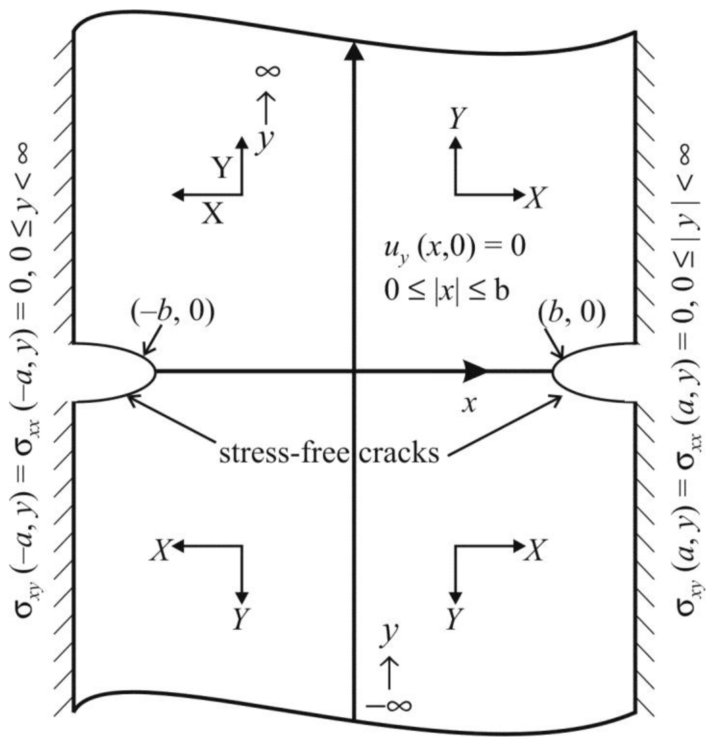


**Figure 1.** Stress-free orthotropic strip in the presence of Body Forces ($X$, $Y$).

It is assumed that plain-strain conditions prevail. The axes of material symmetry coincides with co-ordinate axes. The over all symmetry of problem reduces to solution domain as $[0,a]\cup[0,\infty)$. The physical problem is reduced to all following mixed-boundary value problem.

$$\sigma_{xx}(a,y) = \sigma_{xy}(a,y) = 0, \quad a \leq y < \infty \tag{1.1}$$

$$\sigma_{xy}(x,0) = 0, \quad 0 \leq x \leq a \tag{1.2}$$

$$\sigma_{yy}(x,0) = 0, \quad b < x \leq a \tag{1.3}$$

$$u_y(x,0) = 0, \quad 0 \leq x \leq b \tag{1.4}$$

The problem of finding the components of stress and of displacement at general point $(x,y)$ is divided into two. Namely (a) Body Force Problems (b) Elasticity Problem. Therefore,

$$\sigma_{ij}(x,y) = \sigma_{ij}^{(b)}(x,y) + \sigma_{ij}^{(e)}(x,y), i, j = x, y$$

$$u_i(x, y) = u_i^{(b)}(x, y) + u_i^{(e)}(x, y), \;\; i = x, y \tag{1.5}$$

Where super scripts (b) or (e) over quantities refere to body force problem or elasticity problem, respectively.

It is being checked through out the analysis, see Burniston [10],

$$u_y(x, 0) > 0, \qquad b < x \le a \tag{1.6}$$

The plan of the paper is as follows : In section 2 the problem is formulated. The section 3 will reduce the problem to dual series equation. Solution of this series equation will be reduced to Fredholm integral equation of second kind in section 4. The physical quantities will be reported, in terms of solution of Fredholm integral equation, in section 5. A special case of body force will be given in section 6. There will be graphs of stress-intensity factors and of crack shape. Discussion and conclusion will be in section 7. The references will be in the last.

## 2. FORMULATION

The body forces in the medium are producing stresses in the medium and specially at $y = 0,$ the normal stress produced will cause the cracks to open. Thus the body force problem is done first.

### Body Force Problem

The equations of equilibrium in the presence of body forces and no crack in the medium, is solved by taking appropriate finite Fourier transform with respect to *x* and Fourier integral transform w.r.t. *y*.

The equations of equilibrium

$$\frac{\partial \sigma_{xx}}{\partial_x} + \frac{\partial \sigma_{xy}}{\partial_y} + \rho X = 0, \; \frac{\partial \sigma_{xy}}{\partial_x} + \frac{\partial \sigma_{yy}}{\partial_y} + \rho Y = 0 \tag{2.1}$$

When (X, Y) are body force components and $\rho$ is mass density of the medium. The stress-strain relations are

$$\begin{vmatrix} e_{xx} \\ e_{yy} \\ e_{xy} \end{vmatrix} = \begin{bmatrix} a_{11} & a_{12} & 0 \\ a_{12} & a_{22} & 0 \\ 0 & 0 & a_{66} \end{bmatrix} \begin{vmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{vmatrix} \tag{2.2}$$

When $a_{11} \sim a_{16}$ are elastic constants & $e_{xx}$ etc, are strain components. Taking finite sine transform w.r.f. *x* and cosine integral transform w.r.t. *y* of Ist of (2.1), and finite cosine & integral sine transfer of 2$^{nd}$ of (2.1), then obtain the values of $\sigma_{xx,} \sigma_{xy}, \sigma_{yy}$ from (2.2) we after Fourier inversion

$$\pi a u_x^{(b)}(x,y) = 4\rho \sum_{n=1}^{\infty} \sin(\alpha_n x) \int_0^{\infty} \cos(sy)(w_1 X_{sc} - W_2 Y_{cs}) ds \tag{2.3}$$

$$\pi a u_y^{(b)}(x,y) = \pi a u_{yc}^{(b)}(a,y) + 4\rho \sum_{n=1}^{\infty} \cos(\alpha_n x) u_{yc}^{(b)}(\alpha_n, y) \tag{2.4}$$

$$u_{yc}(\alpha_n, y) = \int_0^{\infty} \sin(sy)(w_2 X_{sc} - w_3 Y_{cs}) ds \tag{2.5}$$

with

$$\left.\begin{aligned} & ww_1 = a_{11}a_{66}s^2 + \beta_0\alpha_n^2, w = (s^4 + 2B_1 s^2\alpha_n^2 + \beta_2\alpha_n^4) \\ & ww_2 = s\alpha_n\beta_1, ww_3 = \beta_0 s^2 + a_{22}a_{66}\alpha_n^2 \\ & \beta_1 = \beta_0 - a_{12}a_{66}, \beta_0 = a_{11}a_{22} - a_{12}^2 \\ & B_1 = \frac{2a_{12} + a_{66}}{a_{11}}, B_2 = \frac{a_{22}}{a_{11}} \end{aligned}\right\} \tag{2.6}$$

**Elasticity Problem**

The problem of crack opening in the medium is obtained by solving equations of equilibrium (2.1) with the absence of body forces. The method of Kushwaha [11] is used and thus the displacement components are given as

$$u_y^{(e)}(x,y) = \frac{1}{2}u_{yc}(0,y) + \sum_{n=1}^{\infty} \cos(\alpha_n x)\alpha_n^{-2}[a_{11}H,_{yyy} - (2a_{12} + a_{66})\alpha_n^2 H, y] + \int_0^{\infty} s^{-2}\cos(sy)$$

$$[a_{22}G,_{xxx} - a_{12}s^2 s,_x]ds, \tag{2.7}$$

$$u_x^{(e)}(x,y) = \sum_{n=1}^{\infty} \sin(\alpha_n x)\alpha_n^{-2}[a_{11}H,_{yy} - \alpha_n^2 a_{12}H]$$

$$+\int_0^{\infty} \sin(sy)s^{-2}[a_{22}G,_{xx} + s^2(2a_{12} + a_{66})G]ds \tag{2.8}$$

with,

$$\left.\begin{aligned} & (r_1 - r_2)H(\alpha_n, y) = [(r_1 - r_2)A_n - B_n]e^{-\alpha_n r_1 y} + \beta_n e^{-\alpha_n r_2 y} \\ & (r_3 - r_4)G(x,s) = [(r_3 - r_4)C - D]\cos(sr_3 x) + D\cos(sr_4 x) \end{aligned}\right\} \tag{2.9}$$

where $r_1, r_2$ and $r_3, r_4$ are roots of

$$r^4 - 2B_1 r^2 + B_2 = 0, \ r^4 - 2B_1' r^2 + B_2' = 0 \tag{2.10}$$

respectively. And

$$B_1' = (2a_{11} + a_{66})/a_{22}, B_2' = B_2^{-1} \tag{2.11}$$

while $B_1$ and $B_2$ are defined in last of (2.6).

## 3. REDUCTION TO DUAL SERIES

The geometrical symmetry and symmetrical system of body force will give

$$\sigma_{xy}^{(b)}(x,0)=0,\ \ u_y^{(b)}(x,0)=0,\ \ 0\le x\le a \tag{3.1}$$

$$\sigma_{xy}^{(b)}(a,y)=0,\quad 0\le y<\infty \tag{3.2}$$

Then using (1.5) and (3.1) – (3.2) in second of (1.1), (1.2), (1.4) it reduce to

$$\sigma_{xy}^{(e)}(x,0)=0,\qquad\qquad 0\le x\le a \tag{3.3}$$

$$\sigma_{xy}^{(e)}(a,y)=0,\qquad\qquad 0\le y<\infty \tag{3.4}$$

$$u_y^{(e)}(x,0)=0,\qquad\qquad 0\le x\le b \tag{3.5}$$

and (1.3) gives

$$\sigma_{yy}^{(e)}(x,0)=-\sigma_{yy}^{(b)}(x,0),\quad b<x\le a \tag{3.6}$$

Thus the series equations (3.5) and (3.6) constitutes the mixed-boundary value problem. Now making one of (2.7) – (2.8) and the relations (2.2) the boundary conditions (3.3) - (3.4) gives,

$$r_1A_n=B_n \tag{3.7}$$

$$D=\frac{[Cr_3(r_3-r_4)\sinh(ar_3s)]}{P_1(\alpha_n)} \tag{3.8}$$

$$P_1(\alpha_n)=r_3\sinh(ar_3s)+r_4\sinh(ar_4s) \tag{3.8)a}$$

The boundary condition first of (1.1) gives

$$C=[\alpha_nB_n+\sigma_{yycc}^{(b)}(\alpha_n,s)](-1)^n/P(\alpha_n) \tag{3.9}$$

$$P(\alpha_n)=\frac{\alpha_n^2r_3[\cosh(ar_3\alpha_n)-r_3(r_3^2-r_4^2)\cosh(a\alpha_nr_4)]}{P_1(\alpha_n)} \tag{3.10}$$

Thus out of four constants $A_n, B_n; C(s), D(s),$ we obtained three relations (3.7) – (3.10) which will determine three constants interms of one constants i.e., $B_n$.

Thus the mixed-boundary conditions (3.5) – (3.6) will reduce to

$$\frac{\phi_0}{2}+\sum_{n=1}^{\infty}\phi_n\cos(\alpha_nx)=0,\quad 0\le x\le b, \tag{3.11}$$

$$\sum_{n=1}^{\infty}\alpha_n\phi_n\cos(\alpha_nx)=P(x),\quad b<x\le a, \tag{3.12}$$

with $\phi_n=B_n\alpha_n,\quad \phi_0=A_0/d,$ (3.13)

$$\left.\begin{aligned} &d = d_2 - d_1(r_1 + r_2)/[r_1(r_1 - r_2)] \\ &(r_1 - r_2)d_1 = r_1[(r_1 + r_2)(a_{12} + a_{66} - a_{12}^2 r_1^2)] \\ &(r_1 - r_2)d_2 = (r_1 + r_2)[a_{12} + a_{66} - a_{11}(r_1^2 + r_2^2 - r_1 r_2)] \end{aligned}\right\} \tag{3.14}$$

$$P(x) = -\sigma_{yy}^{(b)}(x,0) + \sum_{n=1}^{\infty} \phi_n \cos(\alpha_n x) M(\alpha_n, x) \tag{3.15}$$

$$\left.\begin{aligned} &M(\alpha_n, x) = (-1)^n \alpha_n^2 \int_0^{\infty} f_3(s\alpha_n, x)\, ds \\ &f_3(s\alpha_n, x) = f_2(\alpha_n s) F_2(s\alpha_n, x) / F_1(sa) \\ &r_3(r_3 - r_4) F_1(sa) = 2(r_3 + r_4) s^2 \cosh(sar_4) \\ &f_2(\alpha_n s) = \frac{r_4^2}{\alpha_n^2 r_4^2 + s^2} + \frac{r_3^2}{\alpha_n^2 r_3^2 + s^2} \end{aligned}\right\} \tag{3.16}$$

$$\left.\begin{aligned} &F_2(s\alpha_n, x) = [r_4^2 \cosh(sr_4 x) + (r_3^2 + r_4 - r_3)\cosh(sr_3 x)] F_0(sa) \\ &F_0(sa) = sa\left[\frac{r_3 \cosh(sar_3) - r_4 \cosh(sar_4)}{r_3(r_3 - r_4)\cosh(sar_3)}\right] + \int_0^{\infty} \sigma_{xx}^{(b)}(a, y)\cos(sy)\, dy \\ &\sigma_{xx}^{(b)}(x, y) = \frac{4}{\pi a}\left[\frac{1}{2}\sigma_{yyc}^{(b)}(0, y) - \sum_{n=1}^{\infty} \cos(\alpha_n x)\sigma_{yyc}^{(b)}(\alpha_n)\right] \\ &\sigma_{yyc}^{(b)}(\alpha_n, y) = \int_0^{\infty} \cos(sy)[w_1 Y_{cs} + w_2 X_{sc}]\, ds \end{aligned}\right\} \tag{3.15)b}$$

Thus the physical problem is reduced to dual series equations (3.11) – (3.15).

## 4. SOLUTION OF DUAL SERIES

The solution of dual series equation is obtained by the method of Parihar [12] we assume the trial solution as

$$\alpha_n \phi_n = 2\int_b^a g(t)\sin(\alpha_n t)\, dt \tag{4.1}$$

$$\phi_0 = 2\int_b^a (t - a) g(t)\, dt \tag{4.2}$$

and then using the series

$$\frac{x}{2} + \sum_{n=1}^{\infty} \frac{\sin(nx)\cos ny}{n} = \begin{cases} \pi/2, & x > y \\ \pi/4, & x > y \\ 0, & x < y \end{cases} \tag{4.3}$$

Then using (4.1) – (4.3) into (3.11) which satisfies it identically. Now it is assumed that

$$g(a) = 0 \tag{4.4}$$

There is no loss of generality. The substitution of (4.1) into (3.12) and using

$$\sum \frac{\sin(\alpha_n x)\sin\alpha_n y}{\alpha_n} = \frac{1}{2q}\log\left|\frac{\sin q\dfrac{(x-y)}{2}}{\sin q\dfrac{(x+y)}{2}}\right| \tag{4.5}$$

and then using [12] to invert this

$$g(t) = \frac{2}{a^2}\frac{\cos(qt/2)}{\sqrt{G(b,c)}}\left[\Delta_0(t) + \int_b^a g(s)K(s,t)ds\right] \tag{4.6}$$

with

$$\Delta_0(t) = -\int_b^a \frac{\sin(qx/2)\sqrt{G(b,x)}\sigma_{yy}^{(b)}(x,0)dx}{G(x,t)} \tag{4.7}$$

$$G(b,t) = \left|\cos(qb) - \cos(qt)\right| \tag{4.8}$$

$$K(x,t) = \int_b^a \frac{\sin(qx/2)\sqrt{G(b,x)}}{G(x,t)}N(s,x)dx \tag{4.9}$$

Where

$$N(s,x) = \sum_{n=1}^{\infty}(-1)^n \alpha_n \sin(\alpha_n s)\cos(\alpha_n x)\int_0^{\infty} f_3(\beta\alpha_n, x)d\beta \tag{4.10}$$

While $f_3$ is given in (3.15). The equation (4.6) is Fredholm integral equation of second kind. Thus the physical problem is reduced to solution (4.6).

## 3. PHYSICAL QUANTITIES

The physical quantities which are important in fracture mechanics are normal stress and then stress-intensity factor and the crack shape.

### Crack Shape

The crack shape or crack opening displacement is obtained through the value of left hand side of (3.11) for $b < x \le a$. Using (4.1) - (4.3) in (3.11) it is given as

$$u_y^{(e)}(x,0) = d\left[-\int_x^a g(t)dt + \int_b^a g(t)dt\right] \tag{5.1}$$

where d is given by (3.14).

**Normal Stress**

The normal stress $\sigma_{yy}^{(e)}(x,0)$ at $y = 0$ for $0 \le x < b$ is obtained through the values of series in left hand side of (3.12) after transforming $P(x)$ on left hand side, and it is given as

$$\sigma_{yy}^{(e)}(x,0) = \frac{2}{\pi r_1}\left[\int_b^a \frac{g(t)\sin(\alpha_n t)dt}{G(x,t)} + F(x)\right] 0 \le x < b \tag{5.2}$$

$$F(x) = \sigma_{yy}^{(b)}(x,0) + 2\int_b^a g(t)F_1(t,x)dt, \tag{5.3}$$

$$F_1(t,x) = \sum_{n=1}^{\infty} \sin(\alpha_n t)\cos(\alpha_n x)M(\alpha_n, x) \tag{5.4}$$

Where in $M(\alpha_n, x)$ is given in (3.15). Now using the value of $g(t)$ from (4.6) into (5.2) and evaluating the integrals it is given as

$$\sigma_{yy}^{(e)}(x,0) = [\pi r_1 \delta(x)]^{-1}\left[\Delta_0(x) + \int_c^a g(y)K(y,x)dy + F(x)\right], 0 \le x < b \tag{5.5}$$

$$\delta(x) = \frac{\sqrt{G(x,b)}}{\cos(qx/2)} \tag{5.6}$$

$\Delta_0(x)$ is defined in (4.7).

**STRESS-INTENSITY FACTORS**

The stress-intensity factor at crack tip is defined as

$$K_b = \lim_{x \to 0^+} \sqrt{b-x}\ \sigma_{yy}(x,0) \tag{5.7}$$

The component $\sigma_{yy}(x,0)$ does not possess square root singularity, therefore (5.7) will reduce to

$$K_b = \lim_{x \to 0^+} \sqrt{b-x}\,\sigma_{yy}^{(e)}(x,0) \tag{5.8}$$

Now using (5.8) in (5.5) we get,

$$K_b = [\pi r_1 \delta_1(b)]^{-1}\Delta_1(b),\ \delta_1(b) = \left[2q\tan\left(\frac{qb}{2}\right)\right]^{1/2} \tag{5.9}$$

$$\Delta_1(b) = \Delta_0(b) + \int_b^a g(y)K(y,b)dy \tag{5.10}$$

where the function $F(x)$ does not possess singularity at crack tips.

## 6. A SPECIAL TYPE OF BODY FORCE

The point body force is assumed as see figure 2.

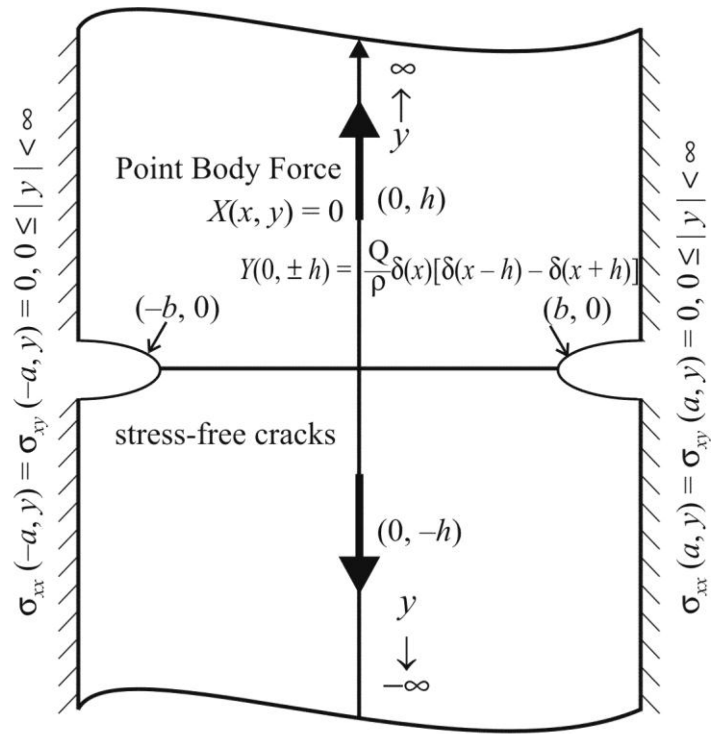


**Figure 2:** Special point body forces are acting at $(0,\pm h)$ is positive and negative y-directions, respectively.

$$X(x,y)=0,\ \ Y(x,y)=\frac{Q}{\rho}\delta(x)[\delta(y-h)^{1}-\delta(y+h)] \tag{6.1}$$

where $\rho$ is mass-density of the medium.

The point force is acting at points $(0,\pm h)$ and of intensity Q. Now, making use of (6.1) into (2.3) – (2.4) and then using (2.2), it is easy to evaluate the value of $\sigma_{yy}^{(b)}(x,0)$. Therefore, it is easy to evaluate $\Delta_0(t)$ which is given as,

$$\Delta_0(t)=aQ\left[e_1\frac{\sinh(qhe_3/2)\sqrt{R(qhe_3,b}}{R(qhe_3,t)}+\frac{e_2\sinh(qhe_4/2)\sqrt{R(qhe_4,b)}}{R(qhe_4,t)}\right],b<t\le a \tag{6.2}$$

$$R(\alpha,\beta)=\cosh\alpha-\cos\beta$$

The value of $\Delta_0(t)$ will be used for crack shape through the evaluation of $g(t)$.

$$\Delta_0(x) = \frac{a}{2}\left[\frac{e_1 \sinh(qhe_3)}{R(qhe_3, x)}\left\{\delta(x) + \frac{\sqrt{R(qhe_3, b)}}{\cosh(qhe_3/2)}\right\}\right.$$

$$\left. + e_3 \frac{\sinh(qhe_4}{R(qhe_4, x)}\left\{\delta(x) + \frac{\sqrt{R(qhe_4, b)}}{\cosh(qhe_4/2)}\right\}\right], 0 \le x < b \tag{6.4}$$

The value of $\Delta_0(x)$ will be used in normal stress component. Thus $K_b$ is given as

$$K_b = [\delta_1(b)\pi r_1]^{-1}\Delta_1(b)$$

$$\Delta_1(b) = a\left[\frac{e_1 \sinh(qhe_3/2)}{\sqrt{R(qhe_3, b)}} + \frac{e_2 \sinh(qhe_4/2)}{\sqrt{R(qhe_4, b)}} + \int_b^a g(y)K(y, b)dy\right], \tag{6.5}$$

$$\left.\begin{array}{l} e_1 = (a_{12} + a_{66} - a_{11}e_3^2)[a_{11}(e_3^2 - e_4^2)]^{-1} \\ e_2 = [a_{11} + e_4^2 - a_{12} - a_{66}][a_{11}(e_3^2 - e_4^2)]^{-1} \\ e_3 = r_1 + (r_1^2 - r_2^2)^{1/2}, e_4 = r_1 - (r_1^2 - r_2^2)^{1/2} \end{array}\right\} \tag{6.6}$$

## SOLUTION OF FREDHOLM INTEGRAL EQUATION

The solution of Fredholm integral equation, for special point force body force is obtained numerically by the method of fox and Goodwin [13] In the equation ( . ), the following substitution is made

$$\cos(qt/2) = p\cos(qb/2) \tag{6.7}$$

$$\left.\begin{array}{ll} \text{where} & p_i = p_{i-1} + \frac{1}{2},\ i = 1, -10, \\ \text{with} & p_0 = 0, \quad p_{10} = 1. \end{array}\right\} \tag{6.8}$$

The limit of integration ($b$, $a$) is changed to (0, 1).

Thus a system of ten linear equations as

$$A_{ij}(p_i) = d_j; \quad i, j = 1, 2, 3.....10, \tag{6.9}$$

with

$$A_{ii}(p_i) = \frac{2}{a^2}\{\delta(p_i)\}\beta_i; \ \ i, j = 1, 2, 3.....10, \ \ \beta_i = 1 - K(p_i, t_i) \tag{6.10}$$

$$A_{ij} = -\frac{2}{a^2}\{\delta(p_i)\}^{-1} K(p_j, t_i), \ d_j = -\frac{2}{a^2}\langle\delta(t_j)\rangle^{-1}\Delta_0(t_j), \tag{6.11}$$

Where $\Delta_0(t_j)$ is given by (6.2) – (6.3) and (6.6) the variation of stress-intensity with respect to $h$, the distance of point of application of point body force.

## 7. DISCUSSION AND CONCLUSION

### Discussion

The crack shapes for different $(h/a)$ are shown in figures 3 and 4. The stress-intensity factors are plotted in figures 5 and 6.

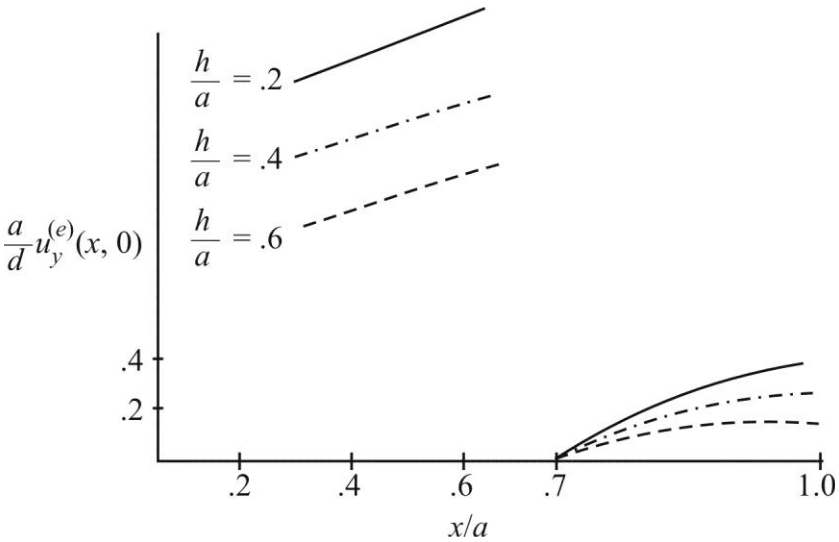


**Figure 3a:** Crack shape for different values of *h*/*a* with crack length = 0.3

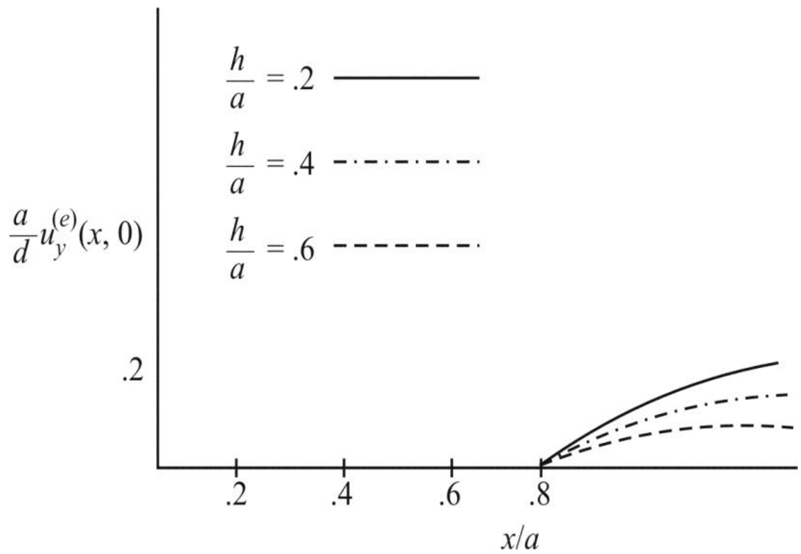


**Figure 3b:** Crack shape for different values of *h*/*a* for crack length = 0.2

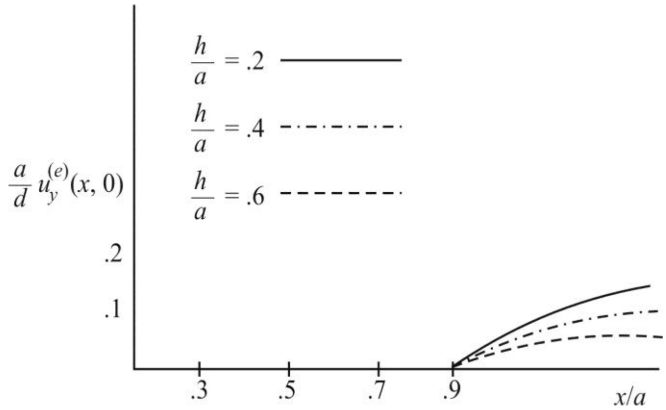


**Figure 3c:** Crack shape for different values of *h*/*a* for crack length = 0.2

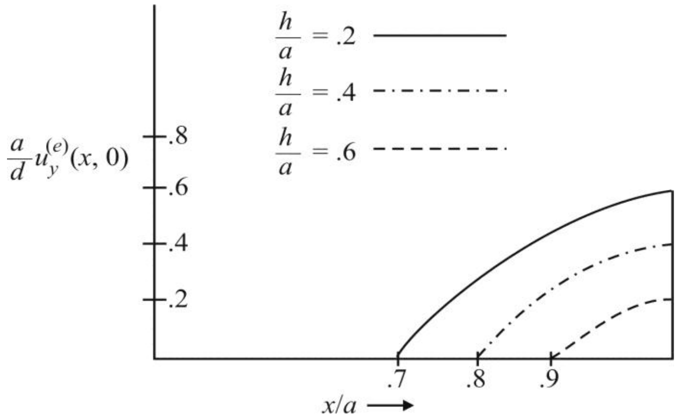


Figure 4a : $\frac{a}{d}u_y$ is plotted against $x/a$ for different values of $h/a$ when grains or $a_{11}$ are parallel to $x$-axis

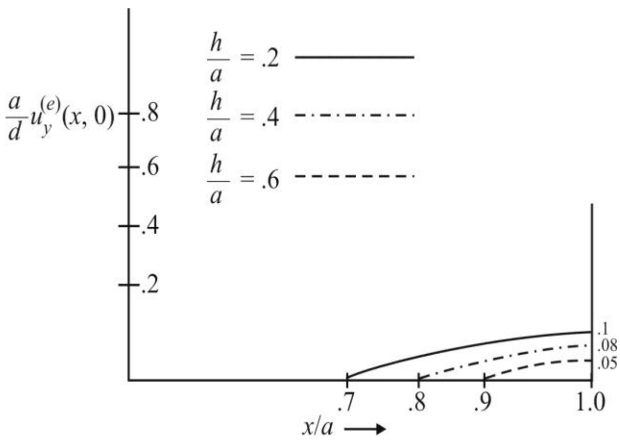


**Figure 4b:** $\frac{a}{d}u_y^{(e)}(x,0)$ is plotted against $x/a$ when grains or $a_{11}$ are parallel to $y$-axis for different values of $h/a$.

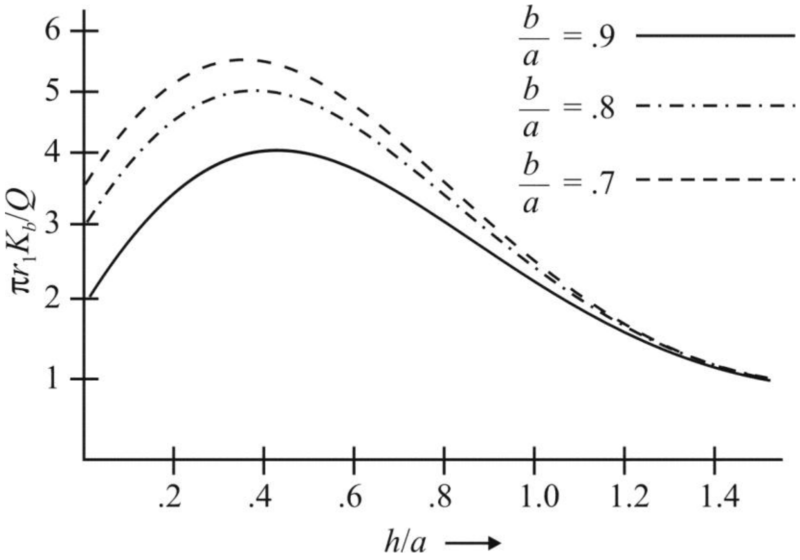


**Figure 5:** $K_b$ is plotted against $h/a$ for different values of $b/a$ when grains or $a_{11}$ are parallel to $x$-axis.

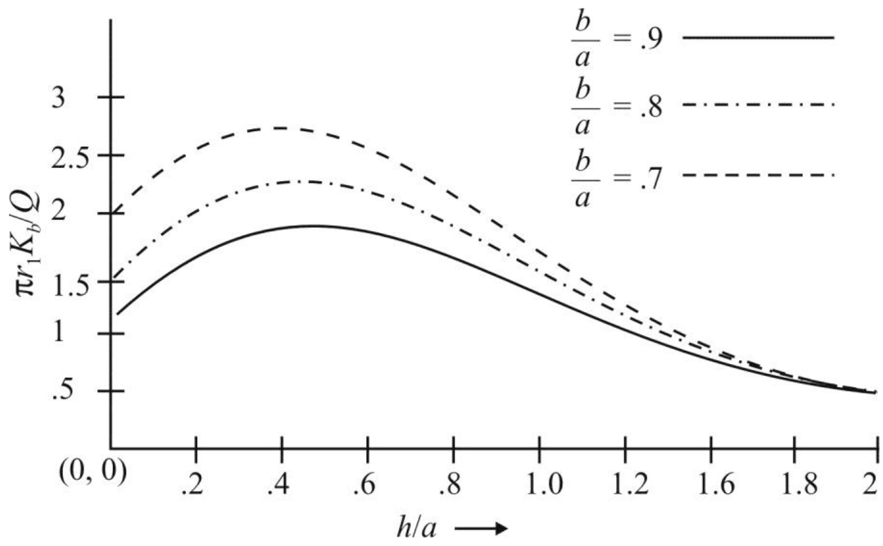


**Figure 6:** $K_b$ is plotted against $h/a$ for different values of $b/a$ when grains or $a_{11}$ are parallel to $y$-axis.

The elastic constants for wood of Oak and given as

$$a_{11} = s_{11} - s_{13}^2 / s_{33}, a_{22} = s_{22} - s_{23}^2 / s_{33}$$

$$a_{12} = s_{12} - s_{13}s_{23} / s_{33}, \ a_{66} = s_{66}$$

where

$$s_{11} = 1040, \ s_{22} = 175, \ s_{33} = 468, \ s_{66} = 1320$$
$$s_{12} = -88.4, \ s_{13} = -303, \ s_{23} = -59.4$$

The principle of cross-linear super position is being used in obtaining the solution of elasticity problem, see [19]

## CONCLUSION

(i) From figures 3 it is observed that as point of application of point force goes away from the crack axis, crack opening becomes less.

(ii) It is also observed that when crack length is less than crack opening is also small.

(iii) From Figures 4 it is observed that when crack axis is along x-axis and elastic constant $a_{11}$ has greater value than that of $a_{22}$, then crack opening is more.

(iv) If grains i.e. $a_{11}$ is along y-axis than crack opening is less.

(v) From figures 5 and 6 it is observed that when crack length is less $K_b$ is also less.

(vi) It is also observed from figure 5 & 6 that when $a_{11}$ is along crack axis then $K_b$ is smaller then that of $k_b$ when $a_{11}$ lies along perpendicular to crack axis.

(vii) This method can be extended for multiple cracks ie. *n*-cracks in strip or rectangle.

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