International Journal of Business Management and Economics Research.
ISSN 2349-2333 Volume 3, Number 1 (2016), pp. 19-27

http://www.irphouse.com

Unemployment – Discussion with a Mathematical Model

Gulbanu Pathan[1] and P.H.Bhathawala[2]

[1]*ExAsst.Prof. Science & Humanities Department, Merchant Institute of Technology, Piludara, India.*
[2]*Retd. Prof. & Head, Department of Mathematics, VNSGU, Surat, India.*

Abstract

In this paper we have proposed and analyzed a nonlinear mathematical model for unemployment considering the situation of job competition between native unemployed & new immigrants. We assumed the situation where government and private sector both made the efforts for creating new vacancies without delay but there is job competition between unemployed and new immigrants. We also consider that unemployed and new migrants try for self-employment by their independent work. The stability of equilibrium point of model is studied. Numerical simulation is carried out to compare with analytical result.

Keywords: Employed persons, unemployed persons, self-employment, migration, dynamic variables.

INTRODUCTION

Unemployment is the problem which expand day by day. It is not limited to a particular state or country but it disturbed the economy of whole world. In present time unemployed raised such a way that people are ready to leave their own territory for getting a job. People moving to other countries for searching a job. So, that government territory feels more burden of unemployment because of native unemployed and new migrants.

Nikolopoulos and Tzanetis ([6]) presented a model for a housing allocation of homeless families due to natural disaster. Using some concept of this paper, Misra and Singh ([1, 2]) developed a nonlinear mathematical model for unemployment. In ([2]) the model considered three dynamic variables number of unemployed persons, employed persons and newly created vacancies by government intervention. Inspired by this paper G.N.Pathan and P.H.Bhathawala ([7]) developed a mathematical model for unemployment with effect of self-employment. In ([4]) M. Neamtu presented a

model for unemployment based on some concept of ([2]) with adding two new variables number of present jobs in the market and number of immigrants.
Based on concept of above models we developed a new model of unemployment with four dynamic variables (i) Number of unemployed persons *U(t)*,(ii) Number of new migrant workers *M(t)*, (iii) Number of Employed persons *E(t)* and (iv) Number of newly created vacancies by government and private sector *V(t).* We assumed that native unemployed and new migrants both can apply for available vacancies equally. Therefore, new migrants attracts to the territory and government realized the burden of unemployment of native workers and migrant workers. So, government tried to take a step of creating new vacancies with the help of private sector without delay. We consider the situation that native unemployed and immigrants both try for their independent work and taking a step of self-employment to survive.
The paper is organized as follows: Section 2 describes Model for unemployment, Section 3 describes an equilibrium analysis, Section 4 describes the stability of equilibrium point, Numerical simulation describes in section 5 and Conclusion is given in section 6.

MATHEMATICAL MODEL:
In the process of developing a model we assume that all entrants of the category unemployment are fully qualified to do any job at any time *t*. Number of unemployed persons *U,* increases with constant rate a_1. The rate of movement from unemployed class to employed class is jointly proportional to *U* and *(P+V - E)*. Where present jobs in the market provided by government and private sector is constant denoted by *P*. Government and private sector try to create new vacancies without delay denoted by *V* and number of employed persons denoted by *E.* So, total available vacancies in the market are *P+V-E.*
We assumed that job search is open for native unemployed as well as new immigrants. So, new migrants also become part of the labor workforce of the territory denoted by *M*. Number of immigrants increases with constant rate m_1. The rate of movement of immigrants in employment is jointly proportional to *M* and (*P+V-E)*. Native unemployed and immigrants both try for self-employment to survive which is proportional to number of unemployed and immigrants with the rate a_5 and a_7 respectively. Employed persons joint unemployed class with rate a_4 because of fired from the job or leave the job. The death and retirement rate of employed person is a_8 .The death rate of unemployed and immigrants are a_3 and m_3 respectively. The rate of immigrants who registered as unemployed or fired from their job is a_6.

$$\frac{dU}{dt} = a_1 - a_2U(P+V-E) - a_3U + a_4E - a_5U + a_6M \qquad \text{\_\_\_\_\_(1)}$$

$$\frac{dM}{dt} = m_1 - m_2 M(P+V-E) - a_6 M - m_3 M - a_7 M \qquad \text{\_\_\_\_\_(2)}$$

$$\frac{dE}{dt} = a_2 U(P+V-E) + m_2 M(P+V-E) - a_4 E + a_5 U + a_7 M - a_8 E \qquad \text{\_\_\_\_\_(3)}$$

$$\frac{dV}{dt} = \alpha U + \beta M - \delta V \qquad \text{\_\_\_\_\_(4)}$$

Here, α and β is the rate of newly created vacancies by government and private sector respectively and δ is the diminution rate of newly created vacancies.

Lemma:

The set $\Omega = \{(U,M,E,V) : 0 \le U + M + E \le \frac{a_1 + m_1}{\gamma}, 0 \le V \le \frac{(\alpha+\beta)(a_1+m_1)}{\gamma\delta}\}$, where $\gamma = \min(a_3, m_{3,} a_8)$ is a region of attraction for the system (1) – (4) and it attracts all solutions initiating in the interior of the positive octant.

Proof:

From equation (1) – (3) we get,

$$\frac{d}{dt}(U(t) + M(t) + E(t)) = a_1 + m_1 - a_3 U(t) - m_3 M(t) - a_8 E(t)$$

Which gives

$$\frac{d}{dt}(U(t) + M(t) + E(t)) \le a_1 + m_1 - \gamma(U(t) + M(t) + E(t))$$

Where $\gamma = \min(a_3, m_3, a_8)$.

By taking limit supremum

$$\lim_{t\to\infty} \sup(U(t) + M(t) + E(t)) \le \frac{a_1 + m_1}{\gamma}$$

from (4) we have

$$\frac{dV}{dt} = \alpha U(t) + \beta M(t) - \delta V(t)$$

$$\therefore \frac{dV}{dt} \le (\alpha + \beta) U(t) - \delta V(t)$$

By taking limit supremum which leads to,

$$\lim_{t\to\infty} \sup V(t) \le \frac{(\alpha+\beta)(a_1+m_1)}{\delta\gamma}$$

This proves the lemma.

EQUILIBRIUM ANALYSIS:

The model system (1) - (4) has only one non negative equilibrium point $E_0(U^*, M^*, E^*, V^*)$ which obtained by solving the following set of algebraic equations.

$$a_1 - a_2U(P+V-E) - a_3U + a_4E - a_5U + a_6M = 0 \qquad \text{\_\_\_\_\_(5)}$$

$$m_1 - m_2M(P+V-E) - a_6M - m_3M - a_7M = 0 \qquad \text{\_\_\_\_\_(6)}$$

$$a_2U(P+V-E) + m_2M(P+V-E) - a_4E + a_5U + a_7M - a_8E = 0 \qquad \text{\_\_\_\_\_(7)}$$

$$\alpha U + \beta M - \delta V = 0 \qquad \text{\_\_\_\_\_(8)}$$

By taking addition of equation (5), (6) and (7)

$$a_1 + m_1 - a_3U - m_3M - a_8E = 0$$

$$\therefore\ E = \frac{a_1 + m_1 - a_3U - m_3M}{a_8} \qquad \text{\_\_\_\_\_\_(9)}$$

From (8)

$$V = \frac{\alpha U + \beta M}{\delta} \qquad \text{\_\_\_\_\_(10)}$$

$$\therefore\ P + V - E = \frac{aa_8U + ba_8M - (a_1 + m_1 - Pa_8)}{a_8} \qquad \text{\_\_\_\_\_(11)}$$

Where $a = \dfrac{\alpha}{\delta} + \dfrac{a_3}{a_8}$, $b = \dfrac{\beta}{\delta} + \dfrac{m_3}{a_8}$

Put values of equations (9) and (11) in (5) and (6) we get,

$$A_0U^2 + A_1UM - A_2U - A_3M - A_4 = 0 \qquad \text{\_\_\_\_\_(12)}$$

$$B_0M^2 + B_1UM - B_2M - B_3 = 0 \qquad \text{\_\_\_\_\_(13)}$$

Where,

$A_0 = aa_2a_8$, $A_1 = a_2a_8b$,

$A_2 = [a_2(a_1 + m_1 - Pa_8) - a_3(a_4 + a_8) - a_5a_8]$,

$A_3 = [a_6a_8 - m_3a_4]$, $A_4 = [a_1a_8 + a_4(a_1 + m_1)]$,

$B_0 = ba_8 m_2$, $B_1 = aa_8 m_2$,

$B_2 = [m_2(a_1 + m_1 - Pa_8) - a_8(a_6 + m_3 + a_7)]$, $B_3 = m_1 a_8$.

equation (12) and (13) represent the equation of hyperbolas.

from equation (12) ,

$$M = \frac{A_4 + A_2 U - A_0 U^2}{A_1 U - A_3} \qquad \text{\_\_\_\_\_(14)}$$

put value of equation (14) in (13) we get,

$$H_0 U^3 + H_1 U^2 - H_2 U - H_3 = 0 \qquad \text{\_\_\_\_\_(15)}$$

$+ \quad + \quad - \quad -$

1

Where

$H_0 = A_2 A_1 B_1 - A_0(A_3 B_1 + A_1 B_2)$,

$H_1 = A_0 B_0 A_4 - B_0 A_2^{\ 2} + A_3(A_2 B_1 + A_0 B_2) + A_1 A_2 B_2 + B_3 A_1^{\ 2}$,

$H_2 = 2A_2 A_4 B_0 - A_4(A_3 B_1 + A_1 B_2) + A_2 A_3 B_2 + 2A_1 A_3 B_3$,

$H_3 = B_0 A_4^{\ 2} + A_3 A_4 B_2 - A_3^{\ 2} B_3$.

Since H_i , *i=1,2,3,4* all are positive and number of changes in signs of equation (15) is only one. By Descart's rule equation (15) has only one positive solution say $U*$. So, we get the non-negative equilibrium point of model with coordinates:

$$M* = \frac{A_4 + A_2 U* - A_0 (U*)^2}{A_1 U* - A_3}$$

$$E* = \frac{a_1 + m_1 - a_3 U* - m_3 M*}{a_8}$$

$$V* = \frac{\alpha U* + \beta M*}{\delta}$$

So, $E_0(U*, M*, E*, V*)$ is required non negative solution of the Model.

STABILITY ANALYSIS:

To check the local stability of equilibrium point $E_0(U*, M*, E*, V*)$ we calculate the variational matrix T of the model system (1) – (4) corresponding to $E_0(U*, M*, E*, V*)$.

$$T = \begin{bmatrix} C_{11} & a_6 & C_{13} & -p_3 \\ 0 & C_{23} & p_4 & -p_4 \\ C_{31} & C_{32} & C_{33} & C_{34} \\ \alpha & \beta & 0 & -\delta \end{bmatrix}$$

Where,
$p_1 = a_2(P+V-E)$, $p_2 = m_2(P+V-E)$,
$p_3 = a_2U$, $p_4 = m_2M$,
$C_{11} = -p_1 - a_3 - a_5$, $C_{13} = p_3 + a_4$, $C_{23} = -p_2 - a_6 - m_3 - a_7$,
$C_{31} = p_1 + a_5$, $C_{32} = p_2 + a_7$, $C_{33} = -p_3 - p_4 - a_4 - a_8$,
$C_{34} = p_3 + p_4$.

The characteristic equation of above matrix is

$$\lambda^4 + d_1\lambda^3 + d_2\gamma^2 + d_3\lambda + d_4 = 0 \qquad \text{\_\_\_\_\_(16)}$$

Where,

$$d_1 = \delta - C_{33} - C_{23} - C_{11},$$
$$d_2 = C_{11}(C_{23}+C_{33}) - \delta(C_{11}+C_{33}+C_{23}) + C_{23}C_{33} + p_4(\beta - C_{32}) - C_{13}C_{31} + \alpha p_3,$$
$$d_3 = C_{33}\delta(C_{11}+C_{23}) + C_{23}C_{11}(\delta - C_{33}) - \beta p_4(C_{11}+C_{34}+C_{33}) + C_{32}p_4(C_{11}-\delta)$$
$$+ C_{13}C_{31}(C_{23}-\delta) + p_4a_6(\alpha - C_{31}) - \alpha(C_{13}C_{34} + p_3C_{33} + p_3C_{23}),$$
$$d_4 = (C_{34}+C_{33})(\beta p_4C_{11} - \alpha a_6p_4) + (p_3 - C_{13})(\beta p_4C_{31} - \alpha p_4C_{32}) + C_{11}\delta(C_{32}p_4 - C_{23}C_{33})$$
$$+ C_{31}\delta(C_{13}C_{23} - p_4a_6) + \alpha C_{23}(C_{13}C_{34} + p_3C_{33}).$$

Since, d_1, d_2, d_3, d_4 are positive then all coefficients of equation (16) are positive and some algebraic manipulation convey that $d_1d_2 > d_3$ and $d_1d_2d_3 > d_3^{\,2} + d_1^{\,2}d_4$. So,by Routh Hurwitz criteria all roots of equation (16) are negative or having a negative real part. Therefore, equilibrium point $E_0 = (U^*, M^*, E^*, V^*)$ is locally asymptotically stable.

NUMERICAL SIMULATION:

For the Numerical simulation using MATLAB 7.6.0 we consider the following data,
$a_1 = 5000$, $a_2 = 0.00004$, $a_3 = 0.04$, $a_4 = 0.004$, $a_5 = 0.03$, $a_6 = 0.1$, $a_7 = 0.01$, $a_8 = 0.006$, $m_1 = 3000$, $m_2 = 0.00002$, $m_3 = 0.05$, $\alpha = 0.04$, $\beta = 0.002$, $\delta = 0.008$, $P = 10000$

The equilibrium values of the model are:
$U^* = 92826$, $M^* = 28118$, $E^* = 480176$, $V^* = 471159$.

The eigenvalues of the variational matrix corresponding to the equilibrium point $E_0 = (U^*, M^*, E^*, V^*)$ of model system (1) - (4) are:
-4.315, $-0.0068851, -0.079415$, -0.18108. All eigenvalues are negative. so, equilibrium $E_0 = (U^*, M^*, E^*, V^*)$ is locally asymptotically stable.

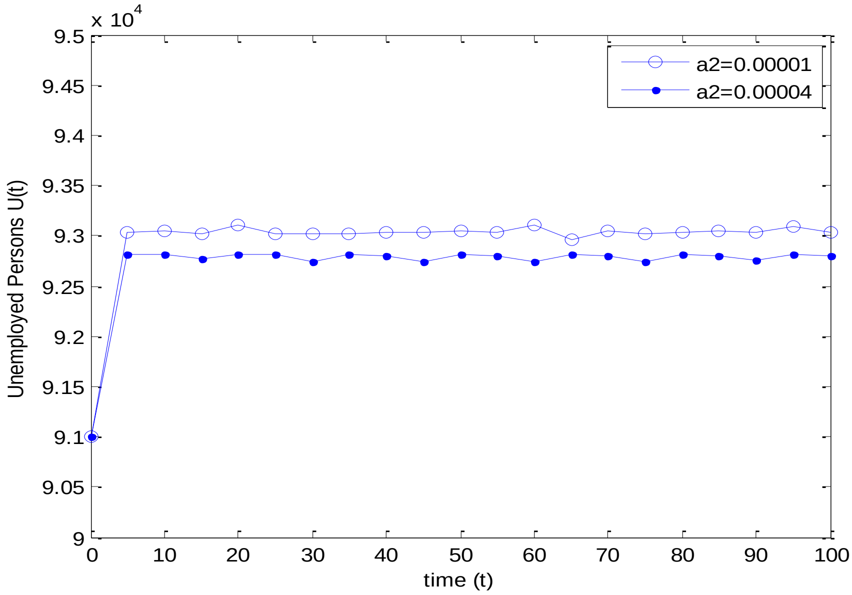

Figure-1

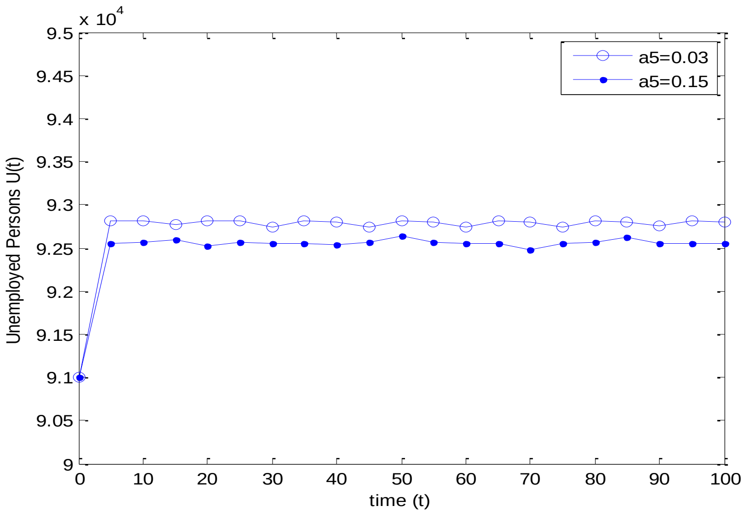

Figure-2

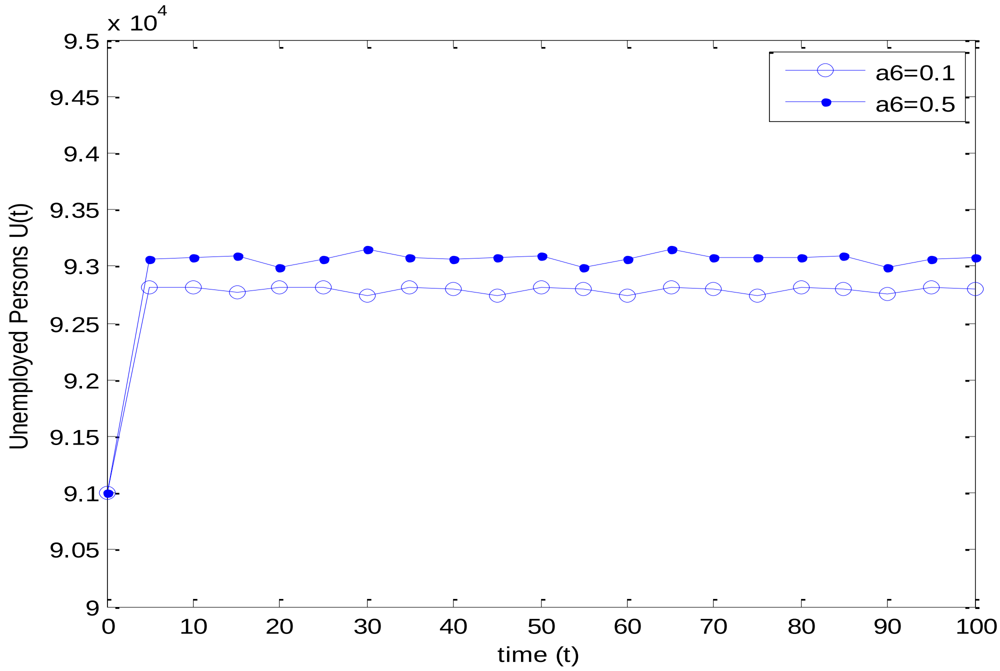

Figure-3

CONCLUSION

The paper proposed and analyzed a nonlinear mathematical model for unemployment using four dynamic variables number of unemployed persons, number of new immigrants, number of employed persons and newly created vacancies. We find that equilibrium point is locally asymptotically stable. Theoretical calculation is verified by Numerical simulation which is done using MATLAB 7.6.0

From figure-1 we observe that if rate of movement of unemployed persons to join employed class is increased then rate of unemployed is decreased. Figure-2 shown that unemployment decreased with the higher rate of self-employment. Figure-3 Shown that if rate of new unemployed migrants increased then rate of unemployed increased. We analyzed if number of new immigrants increased and join the labor workforce of the territory then they either join the unemployed class or employed class of the territory. That is number of unemployed persons of the territory is account by both native unemployed and new unemployed immigrants. To control the unemployment if government and private sector create new vacancies then it should be proportional to number of native unemployed and also proportional to number of unemployed immigrants. That is new vacancies should be created proportional to both U and M for reduced the unemployment at some level. Also, the model proposed that as self-employment rate goes higher unemployment rate goes lower for both native and immigrants unemployed.

REFERENCES

[1] A.K.Misra, A.K.Singh, 4, 2013,"A mathematical model for unemployment", IJTPC, pp.2322-3138.
[2] A.K.Misra, A.K.Singh, 2013, " A Delay mathematical model for the control of unemployment", Differ Equ Dyn Syst,21(3), pp.291-307.
[3] N.Sirghi, M.Neamtu, 1,2014, "A dynamic model for unemployment control with distributed delay", Mathematical Methods in finance and business Administration, Proceeding of the International Business Administration conference, Tenerife Spain, pp.42-48
[4] M.Neamtu, 1,2014, " A dynamic model for unemployment control with migration and distributed delay", Mathematical Methods in finance and business Administration, Proceeding of the International Business Administration conference, Tenerife, Spain, pp.23-31.
[5] N. Sirghi, M.Neamtu, 2013,"Deterministic and stochastic advertising diffusion model with delay", WSEAS Transaction on system and control, 4(8), pp. 141-150.
[6] C.V.Nikolopoulos, DE Tzanetis, 4,2003, "A model for housing allocation of a homeless population due to natural disaster", Nonlinear Anal.,pp.561-579.
[7] G.N.Pathan, P.H.Bhathawala, 2015, "A Mathematical Model of unemployment with effect of self-employment", IOSR- JM, 11(6), pp.37-43.
[8] M. Neamtu, M.Pirtea, G. Mircea, D. Opris, 2010,"Stochastic, Fuzzy, Hybrid delayed Dynamics Heterogeneous competitions with Product Differentiation", WSEAS Transaction on mathematics, 7(9), pp.581-591.